AN EFFICIENT ALGORITHM FOR CHANNEL ESTIMATION AND RESOURCE ALLOCATION IN OFDMA DOWNLINK NETWORKS

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ABSTRACT

The dynamic allocation of resources in OFDMA networks has recently been an active area of research. Most of the given solutions in the literature assume however that perfect channel state information (CSI) is available at the transmitter side. In this paper, we present a novel algorithm that *jointly* estimates the channel and allocates resources in OFDMA networks. Unlike the previous OFDMA channel estimation algorithms, we consider here the downlink case. The proposed approach transforms the original non-convex optimization problem into a set of consecutive smaller sub-problems that can be efficiently solved in *an independent manner*. The algorithm minimizes the mean squared error of the estimate of the channel while maximizing the average user utility function of the OFDMA system.

Index Terms— OFDMA, Multi-user OFDM, PSAM, Channel Estimation, Resource Allocation, downlink channel, WiMAX

1. INTRODUCTION

Multiuser orthogonal frequency division multiplexing (OFDM) systems, also known as Orthogonal Frequency Division Multiple Access (OFDMA), are quickly becoming the system of choice for many new communication standards. For example, OFDMA is currently used in WiMAX (IEEE 802.16) [1] and in 3GPP long-term evolution (LTE) [2]. OFDMA is essentially a multi-user communication system that utilizes standard OFDM modulation and demodulation blocks, and thus preserve the attractive benefits of these multi-carrier transmission systems. Unlike the conventional OFDM case where all sub-carriers are allocated to a single user, each sub-carrier is exclusively assigned to a user in an OFDMA network. The communication link between each user and the base station is modelled as a time varying channel response, and differs from one user to the other. OFDMA algorithms exploit this spectral diversity to dynamically allocate the available system resources such as constellation size, power, and subcarrier assignment among the different users to maximize the system efficiency and ensure fairness in resource distribution. The dynamic resource allocation of an OFDMA network is currently an active area of research where the assignments of these resources is chosen optimally in order to maximize a so-called utility function (see [2], [3], [4], and [5] to name a few). However, for these algorithms to be effective, perfect channel state information (CSI) is required at the transmitter. The problem of channel estimation in OFDMA systems has been considered recently in [6], [7], and [8]. All of these research contributions study the uplink case and focus mainly on reducing the pilot overhead, a challenging problem in channel estimation of OFDMA uplink systems. In this paper, we propose a completely different paradigm for OFDMA channel estimation. In specific, we study the channel estimation problem in the downlink case and propose the deployment of the pilot symbol assisted modulation (PSAM) technique where a subset of the sub-carriers are reserved for training during every OFDM frame [9] instead of the pre-amble technique where all sub-carriers are used for transmitting training symbols in specific OFDM frames [9]. The motivation for doing this is two folds: first, in the downlink case, the same set of training subcarriers or pilots can be employed by all users, eliminating therefore the problem of pilot overhead. Second, the PSAM approach, which is usually preferred to the pre-amble one when the channels are highly time-varying, can be now exploited to estimate the channels [10]. Interestingly enough, and unlike the uplink case, the channel estimation problem is not anymore decoupled from the dynamic resource allocation problem and a solution that achieves a balanced tradeoff between the two is now required. The main contribution of this paper is therefore the derivation of a new algorithm for the *joint* channel estimation and dynamic resource allocation of OFDMA downlink systems. Although the proposed algorithm is sub-optimal (the joint optimization problem is non convex), it has linear complexity and exhibits an elegant layered structure. In specific, the new algorithm is partitioned into simple blocks that are independent from each other. The effect of each block on the overall performance can then be effectively monitored and future updates can be easily incorporated in the algorithm to improve performance.

2. PROBLEM FORMULATION

We assume an OFDMA network consisting of a single base station and M independent users, all with single antennas. The set of users is given by $\mathcal{M} = \{1, \ldots, M\}$. The available bandwidth, B, is divided among K sub-carriers, each with a bandwidth of $B_s = B/K$. The set of available sub-carriers is then $\mathcal{K} = \{1, \ldots, K\}$. Assuming the cyclic prefix is of adequate length, the OFDMA signal is

$$\mathbf{y}_m[n] = \mathbf{X}_m[n]\mathbf{G}_m[n]\mathbf{h}_m[n] + \mathbf{w}_m[n] \tag{1}$$

where n is the time index, $\mathbf{y}_m$ is the received OFDMA signal for user m, $\mathbf{X}_m$ is a diagonal matrix with the transmitted symbols along the diagonal entries, $\mathbf{G}_m$ is a diagonal matrix with diagonal entries given by $[\mathbf{G}_m]_{k,k} = \sqrt{p_{m,k}}$ with $p_{m,k}$ being the transmit power for user m on subcarrier k, $\mathbf{w}_m$ is circularly symmetric white gaussian noise with variance $\sigma_w^2 = N_0 B/K$ and noise floor N_0, and $\mathbf{h_m}$ is a vector of the frequency domain channel path gain for user m with

$$h_{m,k}[n] = \frac{1}{\sqrt{K}} \sum_{i=1}^{L} g_{m,i}[n] e^{-j2\pi ik/K} \tag{2}$$

where $g_{m,i}$ is the i^{th} channel tap for user m, and L is the length of the channel impulse response. The achievable rate on any sub-carrier for a given user is given in [11] as

$$c_{m,k}[n] = log_2\left(1 + \beta p_{m,k}[n]\frac{|h_{m,k}[n]|^2}{\sigma_w^2}\right) \tag{3}$$

where the product $p_{m,k}[n]|h_{m,k}[n]|^2/\sigma_w^2$ represents the signal to noise ratio experienced by user m on subcarrier k, and β is a constant defined by the acceptable bit error rate (BER) as follows

$$\beta = \frac{1.5}{-ln(5BER)} \tag{4}$$

A utility function is typically assigned to each user and quantifies the "satisfaction" of each individual. User utility must be monotonically increasing, i.e., $U_m(x+\Delta) \geq U_m(x)\ \forall\ \Delta \geq 0$ [12]. Given (3) and for every time frame index 'n', the resource allocation that provides maximum average utility is found by solving the following problem

$$\begin{aligned} U_{avg} &= \underset{\mathbf{P}\in\mathcal{P}}{argmax}\ \frac{1}{M}\sum_{m=1}^{M} U_m(R_m) \qquad (5)\\ \text{where }\ R_m &= \sum_{k=1}^{K} B_s log_2\left(1+\beta p_{m,k}\frac{|h_{m,k}|^2}{\sigma_w^2}\right)\\ s.t.\quad & \textstyle\sum_{m=1}^{M}\sum_{k=1}^{K} p_{m,k} \leq P_T\\ & p_{m,k}p_{a,k} = 0\ \forall\ m \neq a \end{aligned}$$

where $U_m(.)$ is the chosen utility function for user m, $\mathbf{P}$ is a matrix of the transmit power for the user and subcarrier pairs, i.e. $[\mathbf{P}]_{m,k} = p_{m,k}$, and $\mathcal{P}$ is the set of all possible power allocations. The constraints are such that the total transmit power is within the power budget and only one user is assigned to each subcarrier. It should be noted that the optimization in (5) maximizes the average utility of the system users, which is a function of the rate and not the rate itself. In general, a solution which maximizes the average utility does not guarantee maximum system throughput. If maximum total rate is desired, the utility function should be chosen equal to the user rate.

The PSAM approach. Training symbols, known to both the transmitter and receiver, are inter-mixed with the data symbols during each frame. N_p subcarriers are therefore chosen from the set of all available subcarriers to transmit these training symbols. The set of pilots, the subcarriers used for training symbol transmission, is given by $\mathbf{z} = [z_1, \ldots, z_{N_p}] \in \mathcal{K}$, where z_i is the location of the i^{th} training subcarrier. By using only the chosen sub-carriers, (1) becomes

$$\mathbf{y}_m^p[n] = \mathbf{X}_m^p[n]\mathbf{G}_m^p[n]\mathbf{h}_m^p[n] + \mathbf{w}_m^p[n] \tag{6}$$

where $()^p$ denotes the subset corresponding to the training subcarrier set. The diagonal of $\mathbf{X}_m^p[n]$ contains the pilot symbols and is thus known at the transmitter and receiver. The frequency domain channel path gains is estimated using a least squares approach

$$\hat{\mathbf{h}}_m^p[n] = \left(\mathbf{X}_m^p[n]\mathbf{G}_m^p[n]\right)^{\dagger}\mathbf{y}_m^p[n] \tag{7}$$

where † represents the pseudo inverse operator. The time domain channel response is obtained by using the inverse discrete Fourier transform (IDFT). From (2), we can write

$$\mathbf{h}_m[n] = \mathbf{Q}_L[n]\mathbf{g}_m[n] \tag{8}$$

where $\mathbf{Q}_L[n]$ is the $K \times L$ DFT matrix. To derive an expression that includes only the pilot symbols, we choose the rows of $\mathbf{Q}_L$ that correspond to the pilot subcarrier set $\mathbf{z}$, and create the DFT matrix $\mathbf{Q}_p[n]$. It follows that

$$\hat{\mathbf{g}}_m[n] = \mathbf{Q}^p[n]^{-1}\hat{\mathbf{h}}_m^p[n] \tag{9}$$

From (9) and (7), we can express the mean squared error (MSE) as

$$\mathrm{E}\left[|\hat{\mathbf{g}}_m[n] - \mathbf{g}_m[n]|^2\right] = \sigma_w^2\, tr\left(\mathbf{Q}^p[n]^{\mathcal{H}}\mathbf{\Theta}[n]\mathbf{Q}^p[n]\right)^{-1} \tag{10}$$

where $^{\mathcal{H}}$ represents the hermitian transpose operator and $\mathbf{\Theta}[n]$ is a diagonal matrix with the transmit power of each training subcarrier along the diagonal entries. The matrix $\mathbf{Q}^p[n]$, and thus the MSE, is entirely dependent upon the choice of $\mathbf{z}$. The effect of pilot placement on user utility is studied by incorporating the indicator function

$$b_k = \begin{cases} 0 & if\ k\ \in\ \mathbf{z} \\ 1 & otherwise \end{cases} \tag{11}$$

into the objective function of (5) to get

$$U_{avg} = \frac{1}{M}\sum_{m=1}^{M} U_m\left(B_s\sum_{k=1}^{K} c_{m,k}b_k\right) \tag{12}$$

Clearly, a decrease in average user utility occurs when training subcarriers are chosen in the specific sub-carrier locations that are capable of higher transmission rates. The average user utility is therefore highly sensitive to the choice of pilot placement when a large amount of rate diversity exists, i.e., when the choice of sub-carriers produces a wide range of transmission rates. This in particular happens when there is substantial frequency selectivity in the underlying channel.

3. MINIMIZING UTILITY LOSS

The optimization problem (5) with the modified average user utility function (12) is a combinatorial optimization problem in which the decision variables b_k are discrete and the constraints are non linear and non convex. Furthermore, the problem has prohibitively high computational complexity. In the remainder of this paper, we will be therefore interested in developing a suboptimal algorithm that generates adequate solutions at an *acceptable* computational cost. Our approach is as follows: from (10) and (12), a tradeoff exists between minimum MSE (MMSE) and the maximum average user utility. The choice of pilot placement for maximum utility may not give reliable channel estimates and errors in channel estimation can potentially degrade the system performance. It is therefore reasonable to first restrict the pilot placement to the locations that provides the MMSE channel estimate. In [10], the authors show that for the single user OFDM case, this is achieved by equally spaced pilot sub-carriers

$$\mathbf{z} = [i, i+\frac{K}{N_p}, \ldots, i+\frac{(N_p-1)K}{N_p}] \tag{13}$$

where $i \in [0, \ldots, \frac{K}{N_p} - 1]$. Additionally, the solution requires that equal power be used for all pilot subcarriers, i.e. $[\mathbf{\Theta}]_{c,c} = \theta$. As noted in [10], the result holds independent from the choice of i. In other words, *any* value of $i \in [0, \ldots, \frac{K}{N_p} - 1]$ produces an MMSE estimate of the channel, independent of the specific channel response. In our case however, while the MSE remains constant for all choices of i, the user utility is dependent on i, thus *the choice of i is no longer independent of the channel response*. Since the channel is time-varying, the maximum average user utility can be achieved by assigning the pilot placement dynamically for each channel realization. This dynamic pilot placement can be used in conjunction with a dynamic subcarrier assignment (DSA) and adaptive power allocation (APA) algorithm to obtain a joint resource allocation and channel estimation algorithm. Before describing the details of the new algorithm, we illustrate the above ideas with a simple example.

Example. Assume that the number of users $M = 2$. Let the number of subcarriers be $K = 4$, and the number of training symbols per frame be $N_p = 2$. The set of possible rates for each user are given as $[2, 2, 0, 2]$ for $user_1$ and $[2, 0, 4, 2]$ for $user_2$. The utility function chosen for this example is given as $U_m = \ln(R_m)$ for all m and the subcarrier bandwidth is equal to 1 KHz. If a fixed pilot placement is used, then, the high rate sub-carriers might be used for pilot symbols. To illustrate a worst case scenario, assume that the fixed pilot placement is chosen such that the pilots are always assigned to the subcarriers 1 and 3 and $U_{AVG} = 0.693$. By contrast, the dynamic pilot placement in this case would choose subcarriers 2 and 4 and the corresponding $U_{AVG} = 1.039$.

4. ALGORITHM AND COMPLEXITY ISSUES

We start with a dynamic subcarrier assignment (DSA) algorithm to allocate sub-carriers among the users. Using (3), and a fixed power allocation, the rates for each user can be calculated for each subcarrier. The maximum total rate is found by adding all possible subcarrier rates for each user. There are only K/N_p possible choices of i, and an exhaustive search in this case is computationally acceptable. For each of these choices, the possible rates for the corresponding subcarriers can be subtracted to determine the total rate for each user assuming the given pilot placement. Finding the total rate and then subtracting the rates lost due to the pilot symbols makes use of the inherent redundancy to reduce the complexity, as compared to finding the total rate for each pilot placement. From this adjusted rate, the average user utility can be found. The choice of i that maximizes the average user utility is then used to assign the pilot subcarriers. After the pilot placement has been found, the power can be distributed among the nonpilot subcarriers using an adaptive power allocation (APA) procedure. Since the pilot subcarriers are required to have a fixed power, the total power constraint used by the APA algorithm must be reduced accordingly. A summary of the algorithm is depicted below and clearly shows the modular nature of the proposed approach. The solution of the joint optimization problem is given through a union of *independent blocks*, namely a channel estimation stage, a DSA block, the DPP algorithm, and an APA procedure over the remaining sub-carriers. It is assumed that a separate channel is available to relay resource and pilot assignments to the individual users of the system. From the outline of the proposed algorithm, the channel estimate at time '$n-1$' is used for resource allocation and pilot placement at time 'n'. Finally, a fixed pilot placement is used to initialize the system when no previous channel estimate is available. It is important to keep in mind at this point that the algorithm given in the table is valid for *any* admissible choice of the utility function.

Joint Algorithm
`Previous Channel Estimation Available`
`iterative loop`
DSA algorithm
DPP
$R_m = \sum_{k=1}^{K} c_{m,k} \;\forall\, m$
`for` $s = 0 : N_p - 1$
$R'_{m,s} = R_m - \sum_{v=0}^{N_p-1} c_{m,s+\frac{K}{N_p}v} \;\; \forall\, m$
$U_s = \frac{1}{M}\sum_{m=1}^{M} U_m(R'_{m,s})$
end
$i = argmax \;\forall\, s \;\; U_s$
$p_{m,k} = \theta \;\;\forall\;\; k \in \mathbf{z}$
APA over remaining subcarriers
`end loop`

Complexity Issues. Finding the total possible rates for each user requires MK operations. The linear search of possible choices of i requires K/N_p iterations. During each iteration, the N_p lost rates must be subtracted. Therefore, the number of operations for the linear search is K. From this, the overall complexity of the dynamic placement algorithm is $\mathcal{O}(MK)$. The DSA and APA algorithms used here are the ones proposed in [5]. In general, the APA algorithm has a complexity of $\mathcal{O}(K)$ and the DSA algorithm has a complexity of $\mathcal{O}\left((M-1)^2 K log(K)\right)$. However, if the utility function is chosen to be linear, e.g., the maximum sum of weighted rates [2], the complexity of the DSA algorithm is reduced to $\mathcal{O}\left((M-1)^2 K\right)$. Since $K \gg M$, the algorithm has linear complexity in such a case.

5. SIMULATION RESULTS

We considered an OFDMA network with $M = 2$ users and a subcarrier bandwidth of approximately 10 KHz. Rayleigh fading channels with different power delay spreads were used in the simulations. For each channel realization, the resources were allocated and the average user utility found. The dynamic pilot placement algorithm was compared to a fixed pilot placement (FPP) algorithm, where $i = 3$ was utilized regardless of the channel realizations. Both the FPP and DPP methods used the DSA and APA algorithms proposed in [5] to distribute the system resources. The first set of simulations were performed over 100 channel realizations with a fixed SNR of 15 dB, with the utility found for each channel realization. This illustrates the potential utility improvement of the DPP algorithm. The results shown in Fig. 1 were found using the best effort traffic utility function defined in [13] with a bad urban delay spread and a doppler frequency of 350 Hz. Fig. 2 were also performed using the best effort traffic utility function and a doppler frequency of 350 Hz, but employed a flat delay spread, or equal power on all channel taps. Fig.3 shows results found using a weighted sum of rates utility function, where the utility function for each user is simply a weighting factor [2]. The weighting for this simulation was chosen as $[.3, .7]$ and a bad urban delay spread was used with a doppler frequency of 100 Hz. The results in Fig. 4 again employed a weighted sum of rates utility function and used a flat delay spread with a doppler frequency of 100 Hz. The final set of simulations were performed over a range of different SNR values. For each value, resource allocation and pilot assignment were performed for 100 channel realizations. The average utility was found for each channel realization and the results in Fig. 5 shows the average utility improvement in this case.

6. REFERENCES

[1] E. Yoon, D. Tujkovic, and A. Paulraj, "Exploiting channel statistics to improve the average sum rate in OFDMA systems," in *Proc. VTC*, 2005, pp. 1053–1057.

[2] I.C. Wong and B.L. Evans, "Optimal OFDMA resource allocation with linear complexity to maximize ergodic weighted sum capacity," in *Proc. ICASSP*, April 2007.

[3] Z. Han, Z. Ji, and K.J.R. Lui, "Low-complexity OFDMA channel allocation with nash bargaining solution fairness," in *Proc. IEEE Glob. Telecom. Conf.*, December 2004, pp. 3726–3731.

[4] K. Seong, M. Mohseni, and J. Cioffi, "Optimal resource allocation for OFDMA downlink systems," in *Proc. ISIT*, 2006.

[5] G. Song and Y. Li, "Cross-layer optimization for OFDM wireless networks - part II: Algorithm development," *IEEE Trans. on Wireless Comm.*, vol. 4, no. 2, pp. 625–634, March 2005.

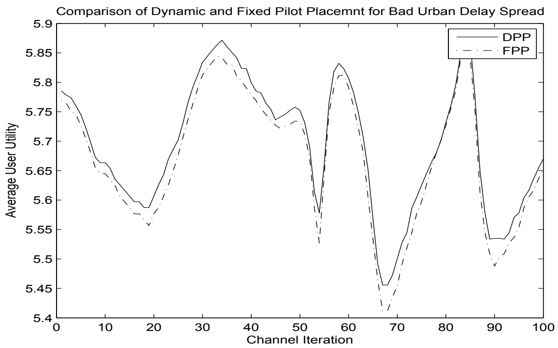

Fig. 1. Performance of the dynamic pilot placement algorithm and the fixed pilot placement for a bad urban delay spread

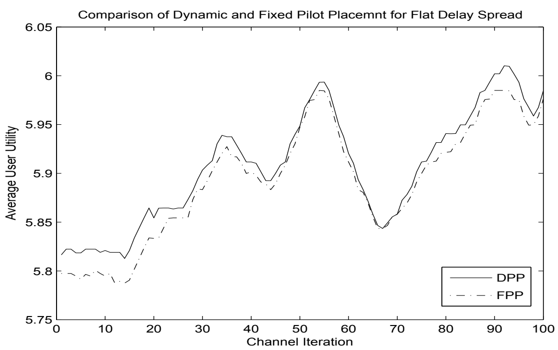

Fig. 2. Performance of the dynamic pilot placement algorithm and the fixed pilot placement for a flat time domain spread

[6] J. Lee, J. Chang, and W. Sung, "Performance improvement of channel estimation based on pilot structure variations for cellular OFDMA systems," in *Proc. VTC*, 2004, pp. 989–993.

[7] P. Fertl and G. Matz, "Multi-user channel estimation in OFDMA uplink systems based on irregular sampling and reduced pilot overhead," in *Proc. ICASSP*, 2007, pp. 297–300.

[8] M. Pun, C. Kuo, and M. Morelli, "Joint synchronization and channel estimation in uplink OFDMA systems," in *Proc. ICASSP*, 2005, pp. 857–860.

[9] S. Ohno, "Preamble and pilot symbol design for channel estimation in OFDM," in *Proc. ICASSP*, April 2007, pp. 281–284.

[10] R. Negi and J. Cioffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Trans. on Consumer Electronics*, vol. 44, no. 3, pp. 1122–1128, August 1998.

[11] X. Qui and K. Chawla, "On the performance of adaptive modulation in cellular systems," *IEEE Transactions on Communications*, vol. 47, no. 6, pp. 884–895, June 1999.

[12] G. Song and Y. Li, "Cross-layer optimization for OFDM wireless networks - part I: Theoretical framework," *IEEE Trans. on Wireless Comm.*, vol. 4, no. 2, pp. 614–624, March 2005.

[13] Z. Jiang, Y. Ge, and Y. Li, "Max-utility wireless resource managment for best effort traffic," *IEEE Transactions on Wireless Communications*, vol. 4, no. 1, pp. 100–111, January 2005.

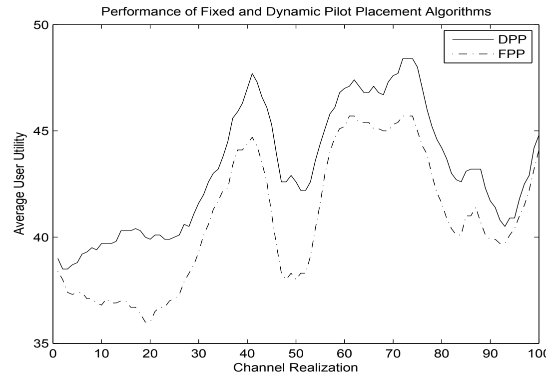

Fig. 3. Performance of the dynamic pilot placement algorithm and the fixed pilot placement a weighted sum of rates utility function and K=64 subcarriers

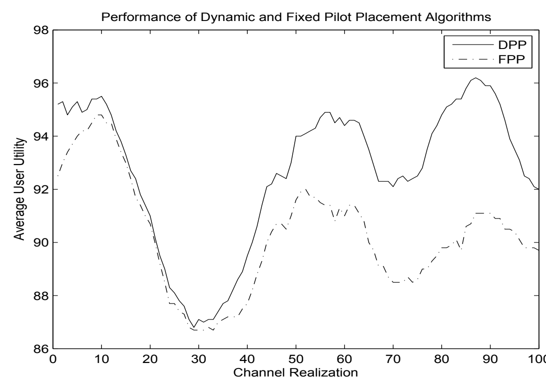

Fig. 4. Performance of the dynamic pilot placement algorithm and the fixed pilot placement for a flat time domain spread employing weighted sum of rates utility function and K=128 subcarriers.

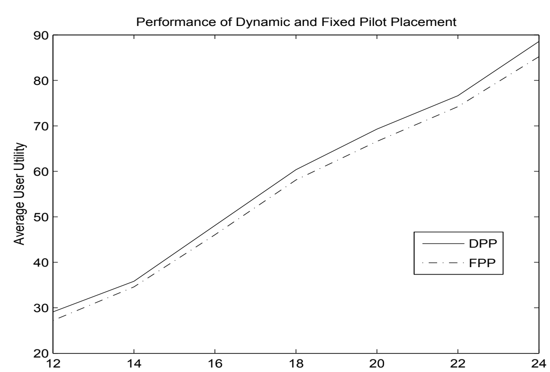

Fig. 5. Average utility improvement of the dynamic pilot placement algorithm and the fixed pilot placement averaged over 100 channel realizations for a range of SNR values.