

MULTIUSER MIMO TRANSCEIVER STRATEGY FOR TDD UPLINK AND DOWNLINK IN TIME-VARYING CHANNEL

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ABSTRACT

Block diagonalization (BD) is a downlink multiuser multiple-input multiple-output (MIMO) strategy that utilizes transmitter channel state information (CSI). This paper proposes a practical uplink MIMO scheme for time division duplex (TDD) systems to co-exist with BD, so that the CSI of the BD channels is used also in the uplink transmission. It is shown that the precoded pilot symbols are sufficient in both uplink and downlink to satisfy the needs of both transmission and reception. The capacity of the system is analyzed in conjunction with linear receivers in a time-varying fading channel. According to the results the proposed uplink strategy provides capacity gain over non-precoded transmission, without being sensitive to Doppler.

Index Terms— Multiuser MIMO, time division duplex (TDD), space division multiaccess (SDMA), block diagonalization (BD)

1. INTRODUCTION

In order to attain all the capacity gains available to multiple-input multiple-output (MIMO) communication systems, channel state information (CSI) should be utilized in the transmitter. CSI for transmitter is available in time division duplex (TDD) systems, provided that the channel does not change significantly between the receive and transmit periods. Due to channel reciprocity, the receiving node can estimate the state of the channel during one frame, and use that knowledge for the purposes of MIMO transmission in the next one. CSI can be estimated from pilot symbols that are known to the receiver. The pilots are also necessary for performing coherent demodulation in the receiver side. In order to keep pilot overhead as low as possible, it is desirable that the same pilot symbols are a useful reference for both reception and transmission.

In a cellular multiuser MIMO system, the downlink comprises a broadcast channel (BC), whereas the uplink is a multiple access channel (MAC). The channel reciprocity leads into duality properties between the BC and MAC [1] [2]. When designing the MIMO user and data multiplexing strategy for a system, both directions need to be taken into account together. One distinctive difference between the base node and the user terminals is that the base node can have the CSI of the channels to all the terminals, while the terminals only have access to the CSI of their individual radio channels. Thus, the base node is capable to centralized processing to attain space division multiple access (SDMA). On the other hand, the terminals can attempt SDMA like transmission only based on the information contained in the signal received in the downlink.

Block diagonalization (BD) is a non-iterative scheme for SDMA in the downlink direction [3]. It orthogonalizes the MIMO channels

of different users, so that the received signals are free from multiuser interference. This allows precoding based on singular value decomposition (SVD) to be carried out individually for each user. Transmission with SVD precoding is a capacity achieving scheme for point-to-point MIMO with CSI in the transmitter [4]. The channel matrix is decomposed into eigenmodes, and each eigenmode is loaded with a data stream associated with an optimal transmit power according to the waterfilling (WF) principle. This method is robust against channel correlation and even matrix singularity, as the weak eigenmodes are typically not allocated with transmit power at all. Furthermore, the receiver for SVD based transmission is easy to construct, since the eigenmodes are orthogonal in the receiver.

In a time-varying fading radio channel the CSI obtained during the TDD receive frame is already partially outdated when the transmit frame starts. Therefore the CSI contains a delay error that has decremental impact on the system capacity. The effect of delayed CSI in case of single-user MIMO was studied [5], and in case of downlink multiuser MIMO in [6]. In addition to delay error, the effect of noisy CSI estimation on multiuser multiple antenna systems was analyzed in [7].

In this paper we propose a practical uplink MIMO scheme for time division duplex (TDD) systems to co-exist with BD, so that the CSI of the BD channels is used also in the uplink transmission. In this strategy, the precoded pilot symbols are sufficient in both uplink and downlink to satisfy the needs of both transmission and reception. In Section 2 the generic uplink-downlink multiuser MIMO system model is described, and in Section 3 the generic receiver structures and the proposed transmitters are presented. Finally, in Section 4 numerical capacity analysis results are given.

2. SYSTEM MODEL

We consider a MIMO system with one base node having N_b antenna elements, and K user terminals each with N_u antenna elements. Furthermore, we assume each user is allocated with N_s data streams in both uplink and downlink.

The downlink MIMO signal received by the terminal of user $k \in \{1, 2, \dots, K\}$, can be written as

$$\mathbf{x}_k^d = \mathbf{H}_k \sum_{i=1}^K \mathbf{M}_i^d \mathbf{A}_i^d \mathbf{b}_i^d + \mathbf{n}_k^d \quad (1)$$

where $\mathbf{H}_k \in \mathbb{C}^{N_u \times N_b}$ is the channel matrix, $\mathbf{M}_k^d \in \mathbb{C}^{N_b \times N_s}$ is the downlink transmit precoder matrix with unit length column vectors, $\mathbf{A}_k^d = \text{diag}(\sqrt{p_{k,1}^d}, \dots, \sqrt{p_{k,N_s}^d})$ is the real-valued diagonal transmit amplitude matrix, $\mathbf{b}_k^d \in \mathbb{C}^{N_s \times 1}$ is the data symbol vector, and $\mathbf{n}_k^d \in \mathbb{C}^{N_u \times 1}$ is white Gaussian noise vector with variance N_0 .

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Similarly, the uplink signal received by the base node becomes

$$\mathbf{x}^u = \sum_{i=1}^K \mathbf{H}_i^H \mathbf{M}_i^u \mathbf{A}_i^u \mathbf{b}_i^u + \mathbf{n}^u \quad (2)$$

where $\mathbf{M}_k^u \in \mathbb{C}^{N_u \times N_s}$ is the uplink transmit precoder matrix with unit length column vectors, and \mathbf{A}_k^u is the diagonal transmit amplitude matrix. Note that for notational convenience, we denote the uplink channel as conjugate transpose \mathbf{H}_k^H instead of transpose \mathbf{H}_k^T .

We also define generic linear receivers $\mathbf{W}_k^d \in \mathbb{C}^{N_u \times N_s}$ and $\mathbf{W}_k^u \in \mathbb{C}^{N_b \times N_s}$. Depending on the transmit precoders and receivers, signal-to-interference+noise ratio (SINR) can be calculated for each stream. Let $\mathbf{m}_{k,s}^u$ be the s th column of \mathbf{M}_k^u and $\mathbf{w}_{k,s}^u$ the s th column of \mathbf{W}_k^u . Assuming the data streams are uncorrelated, SINR for stream s of user k in uplink direction is

$$\gamma_{k,s}^u = \frac{p_{k,s}^u |\mathbf{w}_{k,s}^u{}^H \mathbf{H}_k \mathbf{m}_{k,s}^u|^2}{\sum_{(i,j) \neq (k,s)} p_{i,j}^u |\mathbf{w}_{k,s}^u{}^H \mathbf{H}_i \mathbf{m}_{i,j}^u|^2 + N_0 \|\mathbf{w}_{k,s}^u\|^2} \quad (3)$$

and similarly

$$\gamma_{k,s}^d = \frac{p_{k,s}^d |\mathbf{w}_{k,s}^d{}^H \mathbf{H}_k \mathbf{m}_{k,s}^d|^2}{\sum_{(i,j) \neq (k,s)} p_{i,j}^d |\mathbf{w}_{k,s}^d{}^H \mathbf{H}_i \mathbf{m}_{i,j}^d|^2 + N_0 \|\mathbf{w}_{k,s}^d\|^2} \quad (4)$$

in downlink. Furthermore, by assuming Gaussian symbol alphabets the mutual information between the transmitted sequence and decision statistics becomes

$$R_{k,s} = \log_2(1 + \gamma_{k,s}) \quad \text{bits/s/Hz} \quad (5)$$

which is also the upper limit for the achievable data rate. In ergodic sense, the achievable data rate is obtained by averaging (5) over channel realizations.

3. TRANSMITTER AND RECEIVER DESIGN

3.1. Pilot responses

In order to facilitate coherent detection, pilots transmitted with beamforming via the same precoders as data are necessary. However, unlike data, we propose the pilots have equal power allocation per stream. This way the channel gains can be correctly observed from the received signal without getting mixed with the amplitude adjustment caused by power allocation, and the pilot responses can be utilized for the purpose of transmit precoding as well.

Furthermore, the pilot symbol sequences associated with different streams and users are all mutually orthogonal, which accommodates interference free channel estimation. Due to pilot precoding, neither the base node nor the terminals have explicit knowledge of channel matrices \mathbf{H}_k but only the pilot responses, which are

$$\begin{aligned} \mathbf{R}_{k,j}^d &= \mathbf{H}_k \mathbf{M}_j^d \in \mathbb{C}^{N_u \times N_s} \\ \mathbf{R}_k^u &= \mathbf{H}_k^H \mathbf{M}_k^u \in \mathbb{C}^{N_b \times N_s} \end{aligned} \quad (6)$$

for downlink and uplink respectively. In the downlink, $\mathbf{R}_{k,j}^d$ denotes the response seen by user k of the signal transmitted to user j .

3.2. Block diagonalization

Block diagonalization has the restriction for the number of antennas that $KN_u \leq N_b$. Furthermore, as the number of streams per user must satisfy $N_s \leq \max(N_u, N_b)$, it follows that $N_s \leq N_u$. Here, we fix $N_s = N_u$ so that all the degrees of freedom available are used.

As a downlink SDMA strategy, the objective of BD is to make the signals received by each terminal free from the signals dedicated to others. Let \mathbf{C}_k denote matrix whose columns span the null space of the stacked channel matrix

$$\tilde{\mathbf{H}}_k = [\mathbf{H}_1^T \cdots \mathbf{H}_{k-1}^T \mathbf{H}_{k+1}^T \cdots \mathbf{H}_K^T]^T \quad (7)$$

so that

$$\mathbf{H}_k \mathbf{C}_j = \mathbf{0}, \quad j \neq k. \quad (8)$$

The block diagonalized channel can now be expressed as

$$\tilde{\mathbf{H}}_k = \mathbf{H}_k \mathbf{C}_k \quad (9)$$

which is the effective MIMO subchannel seen by user k . Since the knowledge of the channel in base node comprises only $\mathbf{R}_k^u = \mathbf{H}_k^H \mathbf{M}_k^u$, we may set

$$\mathbf{R}_k^u \mathbf{C}_j = \mathbf{M}_k^u{}^H \mathbf{H}_k \mathbf{C}_j = \mathbf{0}, \quad j \neq k \quad (10)$$

as the block diagonalization condition. This condition is equivalent with (8) if the uplink precoding matrix was chosen to be unitary so that $\mathbf{M}_k^u \mathbf{M}_k^{uH} = \mathbf{I}$. Thus we can equally solve the BD matrices \mathbf{C}_k directly from the estimated pilot responses.

3.3. SVD precoding and waterfilling

In a single-user case, the linear transmit precoding based on SVD decomposes the $N_u \times N_b$ channel matrix into $\mathbf{H} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^H$, from which the eigenmode precoding vectors are those columns of matrix \mathbf{V} that associate with non-zero singular values. The corresponding optimal receiver filters are columns of matrix \mathbf{U} . Transmit power is then allocated to the $\max(N_u, N_b)$ eigenmodes according to the waterfilling principle depending on the singular values located on the diagonal of the matrix $\mathbf{\Lambda}$. In the reverse link the eigenmodes are the same, as $\mathbf{H}^H = \mathbf{V} \mathbf{\Lambda}^H \mathbf{U}^H$. Thus the power allocation is the same, the precoding vectors being now columns of matrix \mathbf{U} .

Similarly, in a multiuser MIMO system, the user-specific block diagonalized channels can be further decomposed as

$$\tilde{\mathbf{H}}_k = \mathbf{H}_k \mathbf{C}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{V}_k^H \quad (11)$$

into eigenmodes, each of which represents a data stream. Due to the constraint on the number of antennas, i.e. $KN_u \leq N_b$, the maximum number of useful eigenmodes in $\tilde{\mathbf{H}}_k$ is N_u . Here we exclude the null space from the decomposition, so that the maximum number of columns in both \mathbf{U}_k and \mathbf{V}_k is N_u . In physical channels the number of non-zero singular values is almost always N_u , and in this case \mathbf{U}_k is a $N_u \times N_u$ unitary matrix.

We can employ the reciprocity of each of the decoupled MIMO channels $\tilde{\mathbf{H}}_k$, and decompose them for the uplink as well as downlink transmission. In the downlink the precoding matrix over BD channel becomes \mathbf{V}_k , and in the uplink \mathbf{U}_k . Thus the downlink precoder over the physical channel \mathbf{H}_k is $\mathbf{M}_k^d = \mathbf{C}_k \mathbf{V}_k$, whereas the uplink precoder remains $\mathbf{M}_k^u = \mathbf{U}_k$. On the other hand in the uplink, matrix \mathbf{C}_k serves as user-specific receiver frontend that rejects the interfering uplink signals transmitted by other users. That is,

the responses from other users after the receiver of user k disappear, since

$$(\mathbf{C}_k \mathbf{V}_k)^H \mathbf{H}_j^H \mathbf{M}_j^u = \mathbf{0}, \quad j \neq k \quad (12)$$

due to condition (8). Here, $\mathbf{C}_k \mathbf{V}_k$ is a zero-forcing receiver. Note that matrix \mathbf{C}_k does not enhance noise, since its column vectors are orthonormal.

The eigenmodes are further allocated with transmit power with the objective to maximize data rate. The waterfilling power allocation becomes

$$p_{k,s} = \max \left(0, \mu_k - \frac{N_0}{\lambda_{k,s}^2} \right) \quad (13)$$

where N_0 is the noise variance, $\lambda_{k,s}$ is the s th singular value of $\tilde{\mathbf{H}}_k$, and μ_k is chosen to satisfy the per-user power constraint $P_k = \sum_s p_{k,s}$. Due to reciprocity, given the same per-user transmit power constraints in both directions, the capacities of the uplink and downlink are equal. Alternatively, in the downlink the base node can use a common power constraint $P = \sum_{s,k} p_{k,s}$ in order to maximize cell throughput.

Due to our chosen downlink piloting strategy, the uplink precoding has to be based on the downlink pilot response $\mathbf{R}_{k,k}^d = \mathbf{H}_k \mathbf{M}_k^d$, which has fewer dimensions than the physical MIMO channel itself. Assuming the downlink precoder was chosen by the base node to be $\mathbf{M}_k^d = \mathbf{C}_k \mathbf{V}_k$, the pilot response is

$$\mathbf{R}_{k,k}^d = \mathbf{H}_k \mathbf{C}_k \mathbf{V}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{I} \quad (14)$$

in which the last equality is obtained by substituting (11). The last expression presents the SVD of the pilot response, from which the uplink precoding matrix becomes $\mathbf{M}_k^u = \mathbf{U}_k$ and the per-user power allocation is calculated from $\mathbf{\Lambda}_k$, as with actual BD channel knowledge. Thus we suggest UL precoding based on SVD of the pilot response. Furthermore, one can see that the received waveforms are in fact weighted columns of \mathbf{U}_k . Therefore SVD precoding in uplink is equal to just normalizing the received waveforms into transmit precoders.

3.4. Time-varying channel

The treatment in the previous sections considered static channel conditions. In practice the transmit precoders have to be constructed based on channel response experienced during the latest receive frame prior to transmission. In a time-varying channel this results in a delay error in transmit CSI. As a result the orthogonality between users and streams in downlink is partially lost. Also in the uplink the channel reciprocity is reduced. In order to mitigate the effect, interference suppressing receivers are constructed based on the pilot responses of the receive frame. In the receiver side the pilot reference is timely and correct, so that both the desired signal and interference responses can be estimated and utilized without delay error.

3.5. Receivers

The receivers considered here are the zero-forcing (ZF) and linear minimum mean square error (MMSE) detector. In downlink, due to the limited number of receive antennas, the terminals can effectively resolve only their own data streams. The user-specific ZF and MMSE downlink receivers are

$$\mathbf{W}_{k,ZF}^d = \mathbf{R}_{k,k}^d \left(\mathbf{R}_{k,k}^d \mathbf{R}_{k,k}^d{}^H \right)^{-1} \quad (15)$$

$$\mathbf{W}_{k,MMSE}^d = \left(\sum_{j=1}^K \mathbf{R}_{k,j}^d \mathbf{A}_j^2 \mathbf{R}_{k,j}^d{}^H + N_0 \mathbf{I} \right)^{-1} \mathbf{R}_{k,k}^d \quad (16)$$

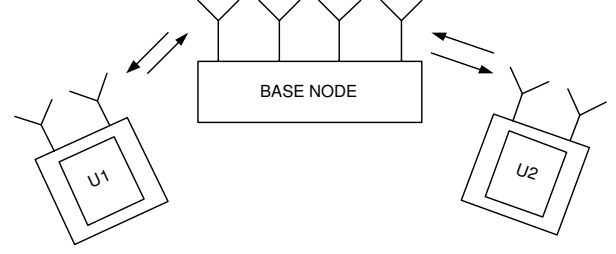


Fig. 1. Two-user MIMO system.

respectively. In the uplink true multiuser detection is possible and required. Let us stack the uplink user responses and receivers into large matrices

$$\begin{aligned} \mathbf{R}^u &= \begin{bmatrix} \mathbf{R}_1^u & \mathbf{R}_2^u & \cdots & \mathbf{R}_K^u \end{bmatrix} \\ \mathbf{W}^u &= \begin{bmatrix} \mathbf{W}_1^u & \mathbf{W}_2^u & \cdots & \mathbf{W}_K^u \end{bmatrix} \end{aligned} \quad (17)$$

and further define the diagonal transmit amplitude matrix $\mathbf{A}^u = \text{diag}(\mathbf{A}_1^u, \dots, \mathbf{A}_K^u)$. The ZF and MMSE uplink multiuser receivers can now be constructed as

$$\mathbf{W}_{ZF}^u = \mathbf{R}^u \left(\mathbf{R}^u \mathbf{R}^u{}^H \right)^{-1} \quad (18)$$

$$\mathbf{W}_{MMSE}^u = \mathbf{R}^u \mathbf{A}^u \left((\mathbf{R}^u \mathbf{A}^u)^H \mathbf{R}^u \mathbf{A}^u + N_0 \mathbf{I} \right)^{-1} \quad (19)$$

respectively.

4. NUMERICAL RESULTS

Different MIMO scenarios were simulated in frequency flat fading with Jakes' Doppler spectrum and uncorrelated channels between antennas. We denote Doppler spread $D_S = 2f_d$ where f_d is the maximum Doppler shift. The equal length UL and DL TDD frames of duration T_{frame} follow each other consecutively. Each simulation comprises 20000 randomly generated, independent channel process bursts of 16 frames. The channel coefficients remain constant over each frame. All the results assume noise-free estimates of the pilot responses both in RX and TX.

In the proposed TX strategy, DL employs BD in conjunction with SVD based on pilot responses, and UL uses SVD also based on pilot responses. In the figures, these are labeled as RX BD SVD and RX SVD, respectively. In addition to the proposed strategy, two alternative UL TX methods were simulated for comparison. The first one is without precoding (No PC), in which $\mathbf{M}_k^u = \mathbf{I}$ and each stream is transmitted with equal power. The second scheme is channel SVD (CH SVD), in which each terminal transmitter performs SVD and WF based on full knowledge of its own individual physical channel \mathbf{H}_k and ignores the multi-user interference.

The effect of Doppler spread to a 4x4 single-user MIMO is shown in Fig. 2. In case of SVD TX, matched filter (MF) receiver is no inferior to MMSE receiver when Doppler spread D_S is zero. As the fading speed increases, the gain against TX without precoding (No PC) is lost.

Furthermore, a single-cell symmetric multiuser MIMO scenario of Fig. 1 was simulated, with two two-antenna user terminals and one four-antenna base node, in which two streams per user were allocated. Fig. 3 and Fig. 4 show the achieved rate, averaged over the two users, in case of ZF and MMSE receiver respectively. Here,

user-specific power constraints were used both in UL and DL. The gain that can be obtained by using WF with common power constraint in this case is minor and thus not shown. As can be seen, DL is much more sensitive to high Doppler due to the loss of orthogonality between users. Uplink capacity seems to be insensitive to Doppler and the differences between different TX schemes are moderate. This is because the four-antenna receiver can always decouple the four streams regardless of the TX strategy.

The reciprocity of the block diagonalized channel can be observed in Fig. 3. In the case of ZF receiver, as the channel speed approaches zero and full orthogonality in DL is reached, the capacities of the DL RX BD SVD with per-user power allocation and UL RX SVD are equal.

5. CONCLUSIONS

We have presented a practical linear uplink multiuser MIMO strategy to co-exist with downlink block diagonalization in cellular TDD systems. The strategy adopts the CSI of the BD channels to be used also in the uplink. Only the precoded pilot symbols are needed in both uplink and downlink to satisfy the needs of both transmission and reception. The strategy lends itself to straightforward power and rate allocation, and works well with suboptimal linear receivers. We also analyzed the performance of the strategy in time-varying channels. From the results we conclude that SVD based transmit precoding in the uplink of multiuser MIMO systems is feasible and beneficial.

6. REFERENCES

- [1] P. Viswanath and D. N. C. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Trans. Inform. Theory*, vol. 49, no. 8, pp. 1912–1921, Aug. 2003.
- [2] N. Jindal, S. Vishwanath, and A. Goldsmith, "On the duality of Gaussian multiple-access and broadcast channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 5, pp. 768–783, May 2004.
- [3] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 461–471, Feb. 2004.
- [4] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, UK: Cambridge University Press, 2005.
- [5] J. G. G. Lebrun and M. Faulkner, "MIMO transmission over a time-varying channel using SVD," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 757–764, Mar. 2005.
- [6] K. Zhang and Z. Niu, "MIMO broadcast transmission with outdated channel state information," in *Proc. Asia-Pacific Conf. Commun.*, Busan, Korea, Aug. 2006, pp. 1–5.
- [7] D. Samardzija and N. Mandayam, "Impact of pilot design on achievable data rates in multiple antenna multiuser TDD systems," *IEEE J. Select. Areas Commun.*, vol. 25, no. 7, pp. 1370–1379, Sept. 2007.

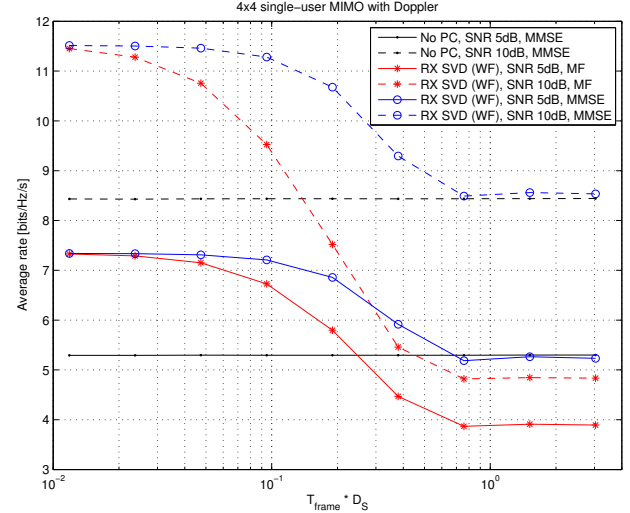


Fig. 2. Point-to-point MIMO.

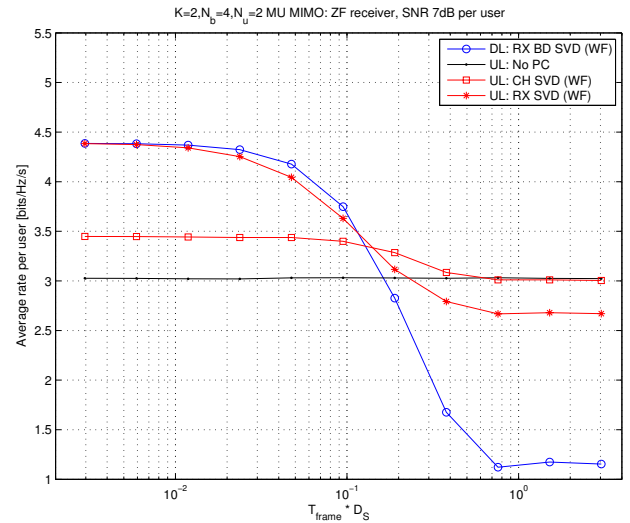


Fig. 3. Two-user MIMO with zero-forcing receiver.

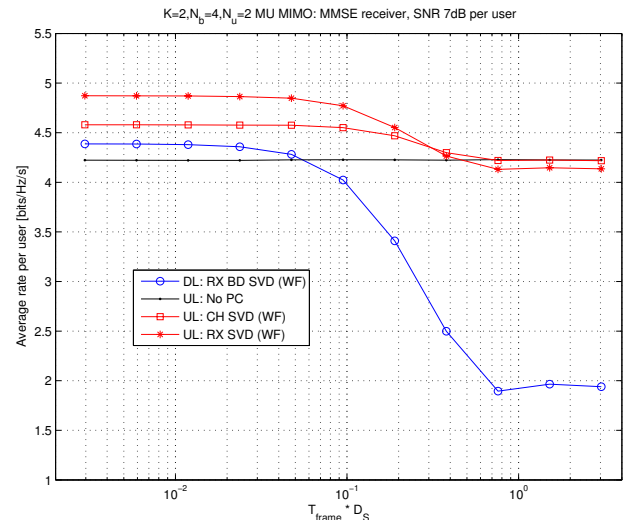


Fig. 4. Two-user MIMO with linear MMSE receiver.