GREEDY USER SELECTION FOR ZERO-FORCING AND MMSE MULTIUSER BEAMFORMING WITH CHANNEL ESTIMATION ERRORS

Olof Sjöbergh, Eduard A. Jorswieck

ACCESS Linnaeus Center
KTH Signal Processing Lab
Royal Institute of Technology
olofsj@kth.se, eduard.jorswieck@ee.kth.se

Erik G. Larsson

Linköping University
Department of Electrical Engineering (ISY)
Division of Communication Systems
erik.larsson@isy.liu.se

ABSTRACT

In a multi-user MIMO downlink where the base station has only estimates of the channels of the users, the sum-rate of multi-user beamforming saturates at high SNR. However, this is not the case for single-user beamforming. We propose a low-complexity user scheduling algorithm that selects the number of active users based on a closed-form approximation of the average sum-rate, and in particular does not add multiple users in the single-user optimality range. In order to develop this algorithm we derive the expected value of the rate of zero-forcing beamforming and MMSE beamforming with estimated channels and modify the greedy user selection accordingly. The gain of the proposed method is shown in numerical simulations.

Index Terms— Broadcast channel (BC), downlink scheduling, multiple-input multiple-output (MIMO), imperfect channel state information (CSI), zero-forcing beamforming (ZFBF)

1. INTRODUCTION

Multi-user MIMO systems have the potential to achieve high throughput and to increase the reliability of wireless systems, and have recently attracted much interest. Especially, it has been shown that single-user transmission (TDMA) is suboptimal and that spatial division multiple access (SDMA) should be used to maximize the sum-rate of the system [1].

A capacity-achieving scheme for the MIMO broadcast channel (BC) with perfect channel state information (CSI) at the transmitter and the receivers is the dirty-paper coding [2], which is very complex and impractical to implement. Instead, various suboptimal schemes such as zero-forcing beamforming (ZFBF) [3] and MMSE beamforming (MMSE-BF) [4] show impressive performance while being relatively simple. If there are more users than transmit antennas, user selection or user grouping schemes must be applied to choose the active set of users. Recently, different user selection and linear precoding strategies have been proposed that achieve the same scaling with the number of users as the sum capacity of the MIMO BC [3, 5]. If imperfect CSI is available, it is shown in [6] that a modified opportunistic beamforming algorithm can achieve the same scaling with number of users. Furthermore, user selection algorithms that maximize the sum performance of the system are introduced in [7]. The spatial properties of the selected user sets are analyzed in [8]. User selection under guaranteed performance constraints is studied in [9].

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However, when using ZFBF or MMSE-BF, any imperfection in CSI reduces the achievable sum-rates. In fact, it turns out that the sum-rate is bounded at high SNR, due to the intra-sector interference between users, unless the estimation quality increases with SNR [10, 11]. The number of users and the channel quality may change in a dynamic environment on different time-scales. Therefore, there is a need to adapt the transmit scheme as well as the user selection algorithm to the current scenario.

Transmitting only to a single user does not suffer from the intra-sector interference limit. Therefore, it ought to be possible to design a selection scheme for beamforming which refrains from selecting too many users when this limits the performance. An algorithm that achieves the full multiplexing and diversity gain when the channel estimation allows this and degrades gracefully to the single-user case at high SNR or high uncertainty of CSI should be possible. One such selection scheme would be to use ZFBF or MMSE-BF when these are not limited by the upper bound derived in [10], and switch to a single-user scheme when the upper bound is limiting performance. However, this bound is not tight, and hence such a scheme would be suboptimal.

In this paper we derive an adaptive user selection scheme that maximizes the expected sum-rate given the availability of an estimated channel. The proposed algorithm requires knowledge of the SNR and the estimation error variance. Based on either ZFBF or MMSE-BF it selects a set of active users to maximize the average sum-rate estimate. The proposed algorithm has the advantages that it always selects the optimal number of active users and it has very low computational complexity. The approach does not need the Monte-Carlo evaluation of the average sum-rate because it is closed form. Finally, the scheme can be applied to the quantized feedback scheme proposed in [12] in which the CSI error is usually large and the number of active users must be limited. Numerical simulations show the performance gain and the adaptivity as a function of the SNR and the estimation error variance.

2. SYSTEM MODEL

We consider a single cell MIMO BC with a single base station supporting data traffic to K users. The base station is equipped with n_t antennas, and each user has a single antenna. The signal received by user k is given by

$$y_k = \mathbf{h}_k^{\mathsf{T}}\mathbf{x} + e_k \tag{1}$$

where $\mathbf{h}_k$ is the $n_t \times 1$ channel vector for user k, $\mathbf{x}$ is the $n_t \times 1$ transmitted vector, with power constraint $E[\|\mathbf{x}\|^2] = P$, and e_k is complex Gaussian noise with zero mean and variance σ_n^2.

Now, let

$$\mathbf{h}_k = \sqrt{1-\sigma_e^2}\hat{\mathbf{h}}_k + \sigma_e \tilde{\mathbf{h}}_k \tag{2}$$

where $\hat{\mathbf{h}}_k$ is the estimated channel at the base station, and $\tilde{\mathbf{h}}_k$ is the estimation error. To simplify the analysis, we assume that the channel is zero-mean flat Rayleigh fading, $\hat{\mathbf{h}}_k \sim \mathcal{CN}(0, \mathbf{I})$, and the estimation error is zero-mean Gaussian, $\tilde{\mathbf{h}}_k \sim \mathcal{CN}(0, \mathbf{I})$.

The transmitted vector $\mathbf{x}$ is chosen by downlink beamforming (a.k.a. SDMA), and taken as

$$\mathbf{x} = \sum_{k=1}^{K} \sqrt{P_k}\mathbf{w}_k b_k \tag{3}$$

where $\mathbf{w}_k$ is the beamforming vector, P_k is the allocated power, with $P_k = 0$ for users not scheduled for transmission, and b_k is the modulated symbol intended for user k.

We denote by $\hat{\mathbf{H}}$ the matrix which is built by all estimated channels from the set of active users, i.e. $\mathcal{S} = \{k : P_k > 0\}$ and $\hat{\mathbf{H}}_k = \hat{\mathbf{h}}_{\mathcal{S}_k}^{\mathsf{T}}$ for $1 \le k \le |\mathcal{S}|$, where $\hat{\mathbf{H}}_k$ denotes the kth row of $\hat{\mathbf{H}}$. The problem is to find the set of active users that maximizes the average sum-rate for certain beamforming strategies, namely ZFBF and MMSE-BF.

3. EXPECTED SUM-RATE OF ZFBF WITH CHANNEL ESTIMATION ERRORS

In ZFBF, the beamforming vectors are given by

$$\mathbf{W}(\mathcal{S}) = \hat{\mathbf{H}}^*(\mathcal{S}) \left(\hat{\mathbf{H}}(\mathcal{S})\hat{\mathbf{H}}^*(\mathcal{S})\right)^{-1} \tag{4}$$

where $\hat{\mathbf{H}}(\mathcal{S})$ is the estimated channel matrix of the users $\mathcal{S}$ selected for transmission. To be able to zero-force the interference we must have $|\mathcal{S}| \le n_t$, and we assume this is the case. Hence, we have[1]

$$|\hat{\mathbf{h}}_i^{\mathsf{T}}\mathbf{w}_j| = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \ne j \end{cases} \tag{5}$$

Note, however, that the beamforming vectors are not normalized, i.e. $\|\mathbf{w}_k\|^2 \ne 1$. Hence, the true power transmitted to user k is given by $P_k\|\mathbf{w}_k\|^2$. Under the assumption of perfect CSI, for optimal performance, the powers are assigned by waterfilling [13] over the effective channel powers, i.e. $P_k = \left(\frac{\mu}{\|\mathbf{w}_k\|^2} - \sigma_n^2\right)^+$ where $(x)^+ \triangleq \max\{x, 0\}$ and the water level μ is chosen to satisfy $\sum_{k\in\mathcal{S}} \left(\mu - \sigma_n^2\|\mathbf{w}_k\|^2\right)^+ = P$. Note that it can happen that the power is switched off for some users, so that $P_k = 0$ even if the user has been selected by the user selection algorithm, i.e. $k \in \mathcal{S}$.

The sum-rate is given by

$$R = \sum_{k=1}^{|\mathcal{S}|} \log\left(1 + \frac{P_k|\mathbf{h}_k^{\mathsf{T}}\mathbf{w}_k|^2}{\sum_{j\ne k} P_j|\mathbf{h}_k^{\mathsf{T}}\mathbf{w}_j|^2 + \sigma_n^2}\right) \tag{6}$$

Inserting $\mathbf{h}_k = \sqrt{1-\sigma_e^2}\hat{\mathbf{h}}_k + \sigma_e\tilde{\mathbf{h}}_k$ and using (5) gives

$$R_{\text{ZFBF}} = \sum_{k=1}^{|\mathcal{S}|} \log\left(1 + \frac{P_k|\sqrt{1-\sigma_e^2}\hat{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_k + \sigma_e\tilde{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_k|^2}{\sum_{j\ne k} P_j|\sigma_e\tilde{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_j|^2 + \sigma_n^2}\right) \tag{7}$$

[1] We omit the index $\mathcal{S}$ in the channel vector $\hat{\mathbf{h}}$ for convenience.

The channel estimation error, $\tilde{\mathbf{h}}$, is not known, but we do know the statistics of the error, and therefore seek the expected value of the rate. We have the following

$$\begin{aligned} &P_k|\sqrt{1-\sigma_e^2}\hat{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_k + \sigma_e\tilde{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_k|^2 \\ &\quad = P_k(1-\sigma_e^2) + P_k\sigma_e^2|\tilde{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_k|^2 + c\Re\{\mathbf{w}_k^{\mathsf{T}}\tilde{\mathbf{h}}_k\hat{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_k\} \end{aligned} \tag{8}$$

where $c \triangleq 2P_k\sigma_e\sqrt{1-\sigma_e^2}$. Hence, the sum-rate can be written as

$$\begin{aligned} R_{\text{ZFBF}} = &\sum_{k=1}^{|\mathcal{S}|} \log\left(\sigma_n^2 + P_k(1-\sigma_e^2) + \sum_{j=1}^{|\mathcal{S}|} P_j\sigma_e^2|\tilde{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_j|^2 \right. \\ &\left. + cP_k\Re\{\mathbf{w}_k^{\mathsf{T}}\tilde{\mathbf{h}}_k\hat{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_k\}\right) - \log\left(\sigma_n^2 + \sum_{j\ne k} P_j\sigma_e^2|\tilde{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_j|^2\right) \end{aligned} \tag{9}$$

Now, since $|\tilde{\mathbf{h}}_k^{\mathsf{T}}\mathbf{w}_j|^2$ is the projection of $\tilde{\mathbf{h}}_k$ onto a vector of length $\|\mathbf{w}_j\|$, this is a χ_2^2-distributed random variable with expected value $\|\mathbf{w}_j\|^2$. To find the expected value of the rate above, we need to introduce the following lemma.

Lemma 1: Let $s_1, \ldots, s_K$ be independent standard exponentially distributed random variables. Let $p_1, \ldots, p_K$ be arbitrary positive real-valued constants, and let $a > 0$ be arbitrary. Then

$$\begin{aligned} &E\left[\log\left(a + \sum_{k=1}^{K} p_k s_k\right)\right] \\ &\qquad = \frac{1}{\log 2\prod_{k=1}^{K} p_k} \sum_{l=1}^{K} \frac{p_l \exp\left(\frac{a}{p_l}\right) E_1\left(\frac{a}{p_l}\right)}{\prod_{k=1,k\ne l}^{K}\left(\frac{1}{p_k} - \frac{1}{p_l}\right)} \end{aligned} \tag{10}$$

where $E_1(x)$ is the exponential integral defined as

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t}dt.$$

Proof: The proof follows from (37) in [14].

Unfortunately, the random variables in (9) are not independent. But if the users are selected to be nearly orthogonal, the beamforming vectors will be almost orthogonal, which makes the projections of the channel error $\tilde{\mathbf{h}}_k$ almost independent. Thus, we approximate the average sum-rate by assuming that they are independently distributed, and we expect this approximation to be good for large K. Then, using Lemma 1 it is possible to find the expected value of the sum-rate above. We arrive at

$$\begin{aligned} &E[R_{\text{ZFBF}}] \\ \approx\ &\sum_{k=1}^{|\mathcal{S}|} \log\left(1 + \frac{P_k(1-\sigma_e^2)}{\sigma_n^2}\right) \\ &+ \frac{1}{\log 2\prod_{j=1}^{|\mathcal{S}|} P_j\|\mathbf{w}_j\|^2} \\ &\cdot\sum_{l=1}^{|\mathcal{S}|} \frac{P_l\|\mathbf{w}_l\|^2 \exp\left(\frac{\sigma_n^2 + P_k(1-\sigma_e^2)}{\sigma_e^2 P_l\|\mathbf{w}_l\|^2}\right) E_1\left(\frac{\sigma_n^2 + P_k(1-\sigma_e^2)}{\sigma_e^2 P_l\|\mathbf{w}_l\|^2}\right)}{\prod_{j\ne l}\left(\frac{1}{P_j\|\mathbf{w}_j\|^2} - \frac{1}{P_l\|\mathbf{w}_l\|^2}\right)} \\ &- \frac{1}{\log 2\prod_{j\ne k} P_j\|\mathbf{w}_j\|^2} \\ &\cdot\sum_{l\ne k} \frac{P_l\|\mathbf{w}_l\|^2 \exp\left(\frac{\sigma_n^2}{\sigma_e^2 P_l\|\mathbf{w}_l\|^2}\right) E_1\left(\frac{\sigma_n^2}{\sigma_e^2 P_l\|\mathbf{w}_l\|^2}\right)}{\prod_{j\ne l, j\ne k}\left(\frac{1}{P_j\|\mathbf{w}_j\|^2} - \frac{1}{P_l\|\mathbf{w}_l\|^2}\right)}. \end{aligned} \tag{11}$$

The average sum-rate in (11) can be evaluated for a certain set of beamforming vectors $\mathbf{w}_1, ..., \mathbf{w}_{|\mathcal{S}|}$ and power allocations $P_1, ..., P_{|\mathcal{S}|}$. This is a closed-from approximation which can be efficiently computed. It is used in Section 5 for user selection.

4. EXPECTED SUM-RATE OF MMSE-BF WITH CHANNEL ESTIMATION ERRORS

It is now straightforward to derive the same expected rate for MMSE-BF. However, since the interference is not zero-forced, the expression is slightly more complicated. In MMSE-BF [4] the beamforming vectors are taken as

$$\mathbf{W}(\mathcal{S}) = \hat{\mathbf{H}}^*(\mathcal{S}) \left(\hat{\mathbf{H}}(\mathcal{S})\hat{\mathbf{H}}^*(\mathcal{S}) + \left(\frac{\sigma_n^2}{P} + \sigma_e^2 \right) \mathbf{I} \right)^{-1} \tag{12}$$

where the channel estimation error is considered as additional noise. The ZF condition from (5) does not hold in this case. The sum-rate is given by (6), and analogously with (9) we get

$$\begin{aligned} R_{\text{MMSE}} = \sum_{k=1}^{|\mathcal{S}|} \log \Bigg(\sigma_n^2 + C_k + \sum_{j=1}^{|\mathcal{S}|} P_j \sigma_e^2 |\tilde{\mathbf{h}}_k^{\mathsf{T}} \mathbf{w}_j|^2 \\ + \sum_{j=1}^{|\mathcal{S}|} c P_j \Re\{\mathbf{w}_j^{\mathsf{T}} \tilde{\mathbf{h}}_k \hat{\mathbf{h}}_k^{\mathsf{T}} \mathbf{w}_j\} \Bigg) \\ - \sum_{k=1}^{|\mathcal{S}|} \log \Bigg(\sigma_n^2 + D_k + \sum_{j \neq k} P_j \sigma_e^2 |\tilde{\mathbf{h}}_k^{\mathsf{T}} \mathbf{w}_j|^2 \\ + \sum_{j \neq k} c P_j \Re\{\mathbf{w}_j^{\mathsf{T}} \tilde{\mathbf{h}}_k \hat{\mathbf{h}}_k^{\mathsf{T}} \mathbf{w}_j\} \Bigg) \end{aligned} \tag{13}$$

where

$$C_k \triangleq \sum_{j=1}^{|\mathcal{S}|} P_j (1 - \sigma_e^2) |\hat{\mathbf{h}}_k^{\mathsf{T}} \mathbf{w}_j|^2 \tag{14}$$

$$D_k \triangleq \sum_{j \neq k} P_j (1 - \sigma_e^2) |\hat{\mathbf{h}}_k^{\mathsf{T}} \mathbf{w}_j|^2 \tag{15}$$

Using Lemma 1, we finally arrive at

$$\begin{aligned} E[R_{\text{MMSE}}] \approx \sum_{k=1}^{|\mathcal{S}|} \log(\sigma_n^2 + C_k) - \log(\sigma_n^2 + D_k) \\ + \frac{1}{\log 2 \prod_{j=1}^{|\mathcal{S}|} P_j \|\mathbf{w}_j\|^2} \\ \cdot \sum_{l=1}^{|\mathcal{S}|} \frac{P_l \|\mathbf{w}_l\|^2 \exp\left(\frac{\sigma_n^2 + C_k}{\sigma_e^2 P_l \|\mathbf{w}_l\|^2} \right) E_1\left(\frac{\sigma_n^2 + C_k}{\sigma_e^2 P_l \|\mathbf{w}_l\|^2} \right)}{\prod_{j \neq l} \left(\frac{1}{P_j \|\mathbf{w}_j\|^2} - \frac{1}{P_l \|\mathbf{w}_l\|^2} \right)} \\ - \frac{1}{\log 2 \prod_{j \neq k} P_j \|\mathbf{w}_j\|^2} \\ \cdot \sum_{l \neq k} \frac{P_l \|\mathbf{w}_l\|^2 \exp\left(\frac{\sigma_n^2 + D_k}{\sigma_e^2 P_l \|\mathbf{w}_l\|^2} \right) E_1\left(\frac{\sigma_n^2 + D_k}{\sigma_e^2 P_l \|\mathbf{w}_l\|^2} \right)}{\prod_{j \neq l, j \neq k} \left(\frac{1}{P_j \|\mathbf{w}_j\|^2} - \frac{1}{P_l \|\mathbf{w}_l\|^2} \right)} \end{aligned} \tag{16}$$

The average sum-rate in (16) can also be evaluated for a certain set of beamforming vectors $\mathbf{w}_1, ..., \mathbf{w}_{|\mathcal{S}|}$ and power allocations $P_1, ..., P_{|\mathcal{S}|}$. This is a closed-from approximation which can be efficiently computed. We will use it in the next section for user selection.

5. GREEDY SELECTION SCHEME USING THE EXPECTED RATE

To achieve a more robust selection scheme, we use a greedy selection algorithm, as in [5]. At each step we add the user that maximizes the expected sum-rate $E[R]$ above, terminating if the rate does not increase. This will take into account the interference between users, and will stop adding users when the interference negatively affects performance. Hence, we expect this scheme to be the same as the single-user selection at worst, and to use the extra degrees of freedom of multi-user beamforming when possible.

The algorithm is detailed in Algorithm 1. The function $R_{\text{ZF}}(\mathcal{S})$ denotes the expected rate $E[R_{\text{ZFBF}}]$ in (11) given a set of selected users $\mathcal{S}$. For the MMSE case the algorithm is the same, just change $R_{\text{ZF}}(\mathcal{S})$ to $R_{\text{MMSE}}(\mathcal{S})$ in (16).

To improve the speed of the algorithm we first check if we are in the single-user optimality range by checking whether the rate achieved by only transmitting to the strongest user is greater than the limit of ZFBF derived in [10]. This limit is $L = n_t \left(\log\left(1 + \frac{1}{\sigma_e^2}\right) + \frac{\gamma}{\log(2)} \right)$ where γ is Euler's constant. Note however that even without this check the result will be the same, since adding more users will not increase the expected rate, but it does give a small speed-up since we exit earlier in the algorithm.

Algorithm 1 Robust greedy selection for ZFBF

$\pi(1) = \arg\max_{k \in \mathcal{T}} \|\mathbf{h}_k\|$
$\mathcal{S} = \{\pi(1)\}$
$\mathcal{T} = \{1, \ldots, K\} \setminus \pi(1)$
$R_0 = R_{\text{ZF}}(\mathcal{S})$
if $R_0 \geq L$ **then**
  single-user optimality range, exit.
else
  for $i = 2$ to n_t **do**
    $\pi(i) = \arg\max_{k \in \mathcal{T}} R_{\text{ZF}}(\mathcal{S} \cup \{k\})$
    if $R_{\text{ZF}}(\mathcal{S} \cup \{\pi(i)\}) > R_0$ **then**
      $\mathcal{S} \leftarrow \mathcal{S} \cup \{\pi(i)\}$, $\mathcal{T} \leftarrow \mathcal{T} \setminus \pi(i)$, $R_0 \leftarrow R_{\text{ZF}}(\mathcal{S})$
    else
      No increase is achieved, exit.
    end if
  end for
end if

6. NUMERICAL SIMULATIONS

Here we compare the throughput of ZFBF and MMSE-BF with the new robust greedy selection scheme to the usual greedy selection that assumes perfect CSI, as well as to the single-user beamforming scheme that transmits only to the best user (TDMA). A multi-user MIMO system with $K = 40$ users and $n_t = 4$ transmit antennas is simulated. The sum-rate is plotted against $1/\sigma_n^2$.

In Figure 1 we have the results for $\sigma_e^2 = 0.2$. As can be seen, the sum-rate is limited at high SNR for both ZFBF and MMSE-BF, unlike the single-user scheme. The robust greedy selection scheme derived here can be seen to perform as the normal greedy selection at low SNR where this gives a higher rate, and reverts to the single-user scheme at high SNR. In Figure 2 the same simulation is run for $\sigma_e^2 = 0.4$. We observe that again the derived selection scheme works as expected. However, for some values of n_t, σ_e^2, and K the proposed selection scheme switches to the single-user case a little too early. This behavior likely stems from the assumption that the

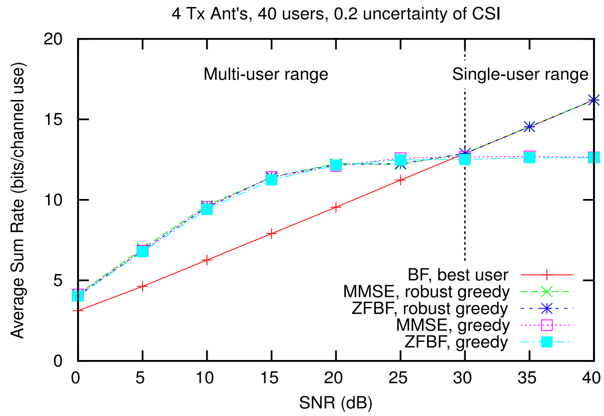

Fig. 1. Throughput of the proposed user selection algorithm with ZFBF and MMSE-BF with $\sigma_e^2 = 0.2$

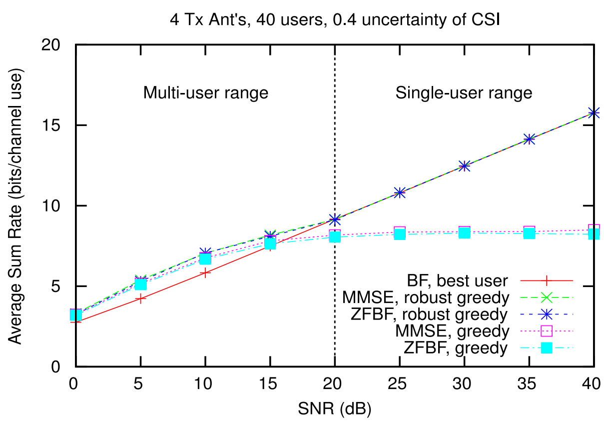

Fig. 2. Throughput of the proposed user selection algorithm with ZFBF and MMSE-BF with $\sigma_e^2 = 0.4$

coefficients in (9) are independently distributed, and it seems to be more pronounced when there are more transmit antennas and when the estimation error is small.

Surprisingly, the sum-rate of the new selection scheme is almost identical to that of the standard greedy selection in the multi-user range with moderate uncertainty of CSI, as can be seen in Figure 1. However, at high CSI uncertainty the achieved sum-rate is higher around the boundary between the single-user and the multi-user range, as can be seen in Figure 2.

From this observation it follows that a very simple scheme could be applied to obtain the performance gain: switch from greedy MMSE or ZF to single-user transmission at the intersection point between the single-user curve and the upper bound on MMSE and ZF beamforming, e.g. $\lim_{\sigma_n^2 \to 0} E[R_{\text{MMSE}}]$. Another observation is that the performance of ZFBF and MMSE-BF is very similar in the studied scenarios.

7. CONCLUSIONS

We have proposed a user selection scheme for beamforming that is robust in the presence of channel estimation errors. The user selection is modified to include a closed-form expression for the average sum-rate under ZF-BF or MMSE-BF. At worst it performs as the single-user beamforming scheme, and when the channel estimation is good enough, the extra degrees of freedom of multi-user beamforming are used. The performance gain was illustrated by numerical simulations.

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