ON THE PERFORMANCE OF SPATIAL-MULTIPLEXING MIMO-CDMA DOWNLINK RECEIVERS BASED ON CHIP RATE EQUALIZATION AND DESPREADING

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ABSTRACT

This paper is devoted to the performance analysis of downlink MIMO-CDMA receivers based on a chip rate MIMO equalizer followed by despreading. The various transmit antennas send independent streams and the spreading codes are scrambled Walsh-Hadamard sequences, the various transmit antennas being allocated different scrambling codes. The behaviour of the SINR provided by the receiver is first studied when the spreading factor and the number of users per antenna converge to $+\infty$ at the same rate. It is shown that the SINR converges towards a simple expression depending on the particular realization of the frequency selective channel between the base station and the mobile of interest. In order to get more insights on the long term performance, the ergodic sum-capacity delivered by the system is evaluated in the large number of antennas regime in the case the impulse response taps of the channel are Gaussian random matrices. A simple expression is obtained. It appears to be quite reliable for a realistic number of antennas, and is discussed in order to get some insights on the overall performance.

Index Terms— CDMA, MIMO, large system performance

1. INTRODUCTION

It is now well established that using multiple transmit and receive antennas potentially allows to increase the Shannon capacity of digital communications systems. Since the seminal work of Teletar ([9]), the capacity of block fading MIMO systems has been studied extensively and important questions such as the impact of channel correlations on the capacity or the design of optimal precoding schemes have been addressed by several authors (see e.g. [4]). MIMO systems is also considered as a valuable solution to increase the overall performance of CDMA systems. In particular, the use of transmit diversity as well as space-time codes is considered in the 3G UMTS system, and the so-called HSDPA, an evolution of UMTS, is supposed to work with multiple transmit and receive antennas. It is therefore very important to study the capacity of MIMO CDMA systems in order to precise how relevant parameters influence the best achievable performance. Although several works have been devoted to MIMO CDMA systems, the above capacity issues do not seem to have been considered extensively in the past. [6] studied the spectral efficiency of such systems when the number of users and the spreading factor converge to $+\infty$ at the same rate. This work addresses specifically the uplink context: the spreading codes allocated to the various users are independent i.i.d. sequences while in the downlink orthogonal codes are used. Moreover, MMSE receivers considered in [6] can in practice be used if the receiver is aware of the codes allocated to the various users; this is not the case in the downlink context.

In this paper, we consider the downlink case, which as mentioned in the recent work [1], is of special importance. We assume that each transmit antenna sends a standard downlink UMTS signal (see below for more details), that the spreading factor and the number of symbol sequences corresponding to each antenna coincide, and that the symbols transmitted by the various antennas are mutually independent. Moreover, the scrambling codes allocated to transmit antennas are assumed mutually independent. We study the SINR provided by the chip rate MMSE equalizer followed by despreading, and show that when the spreading factor and the number of users converge to $+\infty$, then the SINR converges to a simple expression depending on the current value of the frequency selective channel transfer function. This extends our previous results [7] obtained in the SISO case.

In order to have a better understanding of the average performance provided by this receiver, we study the mathematical expectation w.r.t. the channel distribution of the sum-capacity delivered to mobile stations in the following particular case: the number of physical users (i.e. the number of mobile stations) coincides with the (common) number of symbol sequences sent by each antenna, and each mobile receives one symbol sequence per transmit antenna. In this context, the transmit antennas array is used in order to increase the rate delivered to each mobile. The impulse response taps of the propagation channel are modelled as mutually independent Gaussian random matrices. We also assume that the transmit antennas can be correlated. As the expression of the above ergodic capacity is very complicated, we propose to evaluate its behaviour when the number of transmit and receive antennas converge to $+\infty$. We show that it converges to a rather simple expression, which, as in other contexts (see e.g. [8]) appears to be quite reliable for a realistic number of transmit and receive antennas. We take benefit of this result to discuss on the influence of the loading factor of the system and of the transmit antennas correlation on the capacity. We show that the capacity increases when the loading factor increases. Moreover, we prove that the best result is obtained if the transmit antennas are not correlated. Finally, we prove that if the transmit antennas are not correlated, then the best transmission scheme does not correspond to a uniform power allocation to the transmit antennas, but that an antenna selection strategy can provide better results. Our approximate expression of the sum-capacity can also be used in order to address other topics such as optimum precoding, but these issues are not discussed in the present paper.

2. SIGNAL MODEL AND UNDERLYING ASSUMPTIONS.

We denote by r and t the number of receive and transmit antennas respectively. We assume that each transmit antenna j sends K symbol sequences $(b_{j,k})_{k=1,\dots,K}$ **that are supposed to be independent across the transmit antennas**. The spreading factor is the same on each transmit antenna, and is denoted by N. The loading factor $\frac{K}{N}$ is denoted α from now on. Motivated by the UMTS specifications, the spreading codes allocated to the various symbol sequences transmitted by antenna j are normalized Walsh-Hadamard codes (i.e. the code sequences terms are equal to $\pm\frac{1}{\sqrt{N}}$ while in the case of Walsh-Hadamard sequences, they are equal to ± 1) scrambled by a long code sequence $(s_j(n))_{n\in\mathbb{Z}}$ which is called the scrambling code of antenna j. We model each sequence s_j as an independent identically distributed QAM4 sequence, and assume that sequences $(s_j)_{j=1,\dots t}$ are independent. The chip sequence generated by antenna j is denoted $(d_j(n))_{n\in\mathbb{Z}}$, and is thus defined by the fact that vector $\mathbf{d}_j(m) = (d_j(mN), \dots, d_j(mN+N-1))^T$ is given by

$$\mathbf{d}_j(m) = \mathbf{S}_j(m)\mathbf{W}_j\mathbf{b}_j(m)$$

where $\mathbf{S}_j(m)$ is the diagonal $N\times N$ matrix with entries $(s_j(mN+n)_{n=0,\dots,N-1}$, $\mathbf{b}_j(m)$ is the K–dimensional vector of transmitted symbols $\mathbf{b}_j(m) = (b_{j,1}(m)), \dots, b_{j,K}(m))^T$, and $\mathbf{W}_j$ is the $N\times K$ matrix corresponding to the normalized Walsh-Hadamard codes of the various users transmitted by antenna j; it satisfies $\mathbf{W}_j^H\mathbf{W}_j = \mathbf{I}_K$. This in particular implies that $\frac{1}{N}\mathbb{E}(\|\mathbf{d}_j(m)\|^2) = \alpha$, and that the average power of sequence $(d_j(n))_{n\in\mathbb{Z}}$ is equal to the loading factor α. Finally, in order to simplify the notations, we assume that the powers allocated to the sequences $(d_j)_{j=1,\dots,t}$ all coincide: in other words, all the antennas transmit the same power.

We now consider a certain mobile of interest equipped with a receiver with r antennas. The r-variate received signal sampled at the chip rate, denoted $\mathbf{y}(n) = (y_1(n), \dots, y_r(n))^T$, is given by

$$\mathbf{y}(n) = \sum_{j=1}^{t}\sum_{l=0}^{L-1}\mathbf{h}_j(l)d_j(n-l) + \mathbf{v}(n) \tag{1}$$

where $\mathbf{h}_j(z) = \sum_{l=0}^{L-1}\mathbf{h}_j(l)z^{-l}$ is the discrete-time equivalent 1 input / r-outputs channel between antenna j and the receive antennas array, and where $\mathbf{v}$ is a white noise such that $\mathbb{E}(\mathbf{v}(n)\mathbf{v}(n)^H) = \sigma^2\mathbf{I}_r$. In the following, we denote by $\mathbf{H}(z) = (\mathbf{h}_1(z), \dots, \mathbf{h}_t(z)) = \sum_{l=0}^{L-1}\mathbf{H}(l)z^{-l}$ the $r\times t$ transfer function of the MIMO channel under consideration.

We now present the statistics of the channel. We assume that the channel is a block fading multi-paths channel (i.e. the matrices $(\mathbf{H}(l))_{l=0,\dots,L-1}$ remain constant on each slot), and that the coefficient taps $(\mathbf{H}(l))_{l=0,\dots,L-1}$ are mutually independent zero mean Gaussian random matrices. Moreover, we model a possible transmit antenna correlation as in [2] by assuming that for each $l = 0, \dots, L-1$, then

$$\mathbf{H}(l) = p^{1/2}\frac{1}{\sqrt{Lt}}\overline{\mathbf{H}}(l)\mathbf{C}(l)^{1/2} \tag{2}$$

where $\overline{\mathbf{H}}(l)$ is a zero mean complex random Gaussian matrix with unit variance independent identically distributed entries. Matrix $\mathbf{C}(l)$ is a positive matrix modelling the impact of transmit antenna correlation on the $l-th$ path. We assume that $\frac{1}{L}\sum_{l=0}^{L-1}\frac{1}{t}\mathrm{Tr}(\mathbf{C}_l) = 1$. This normalization ensures that p represents the total received power per receive antenna. It is independent of L and t; this allows to compare in a fair fashion situations in which the number of paths and the number of transmit antennas differ.

3. BEHAVIOUR THE SINR OF THE CHIP RATE EQUALIZER FOLLOWED BY DESPREADING WHEN K AND N CONVERGE TO $+\infty$

We assume that the symbol sequence $(b_{1,1}(m))_{m\in\mathbb{Z}}$ is transmitted to the mobile station of interest. On each slot, the mobile station is supposed to be aware of the current value of the MIMO transfer function $\mathbf{H}(z)$. In the following, we study the SINR provided by the receiver consisting in a chip-rate MMSE equalizer followed by a despreading. This SINR is denoted $\beta_{1,1}(N)$ in the following. This scheme is an obvious MIMO generalization of the receiver proposed by [3] and [5]. The chip sequence $(d_1(n))_{n\in\mathbb{Z}}$ is first estimated by $\hat{d}_1(n) = \sum_{l=-(N-1)}^{N}\mathbf{g}(l)\mathbf{y}(n-l)$ where $\mathbf{g}(z) = \sum_{l=-(N-1)}^{N}\mathbf{g}(l)z^{-l}$ is the non causal MISO filter that would coincide with the Wiener filter if all sequences $(d_j)_{j=1,\dots,t}$ were mutually independent i.i.d. sequences of variance α. See [7] for more details in the SISO case. Next, symbol $b_{1,1}(m)$ is estimated by $\hat{b}_{1,1}(m) = \mathbf{w}_{1,1}^H\mathbf{S}_1(m)^H\hat{\mathbf{d}}_1(m)$ where $\mathbf{w}_{1,1}$ is the Walsh Hadamard code allocated to symbol sequence $b_{1,1}$ and where
$\hat{\mathbf{d}}_1(m) = (\hat{d}_1(mN), \dots, \hat{d}_1(mN+N-1))^T$. Using the expression of $\hat{b}_{1,1}(m)$, it is possible to evaluate the SINR $\beta_{1,1}(N)$ provided by this receiver. It is a complicated function of the various scrambling codes, Walsh-Hadamard codes, and of the channel coefficients. However, when K and N converge to $+\infty$ in such a way that the loading factor α is kept constant, then, the expression of the SINR converges to a much simpler expression that only depends on the current value $\mathbf{H}(z)$ of the channel transfer function. More precisely, the following result holds.

Theorem 1 *When K and N converge to $+\infty$ in such a way that $\frac{K}{N}$ remains equal to α, the SINR $\beta_{1,1}(N)$ converges in probability towards the term β_1 given*

$$\beta_1 = \frac{1}{\alpha}\frac{\int_{-1/2}^{1/2}\eta_1(e^{2i\pi f})\,df}{1-\int_{-1/2}^{1/2}\eta_1(e^{2i\pi f})\,df} \tag{3}$$

where the function $\eta_1(e^{2i\pi f})$ is defined by

$$\eta_1(e^{2i\pi f}) = \mathbf{h}_1(e^{2i\pi f})^H\left(\mathbf{H}(e^{2i\pi f})\mathbf{H}(e^{2i\pi f})^H + \frac{\sigma^2}{\alpha}\right)^{-1}\mathbf{h}_1(e^{2i\pi f})$$

This result is a an extension of the main result of [7] devoted to the SISO case, and it can be proved similarly.

4. ANALYSIS OF THE ERGODIC SUM CAPACITY DELIVERED BY THE SYSTEM.

In this section, we study the ergodic sum-capacity per transmit antenna delivered by the system in the following particular situation:

The number of mobile stations of the cell is equal to K

For each transmit antenna j, symbol sequence $(b_{j,k}(n))_{n\in\mathbb{Z}}$ is sent to mobile k

In this particular context, the goal of the MIMO system is thus to increase by a factor t the rate delivered to each mobile station rather than increasing the number of mobile stations communicating with the base station. In order to evaluate the ergodic sum capacity, we first consider the maximum rate than can be provided to mobile station i. This mobile station receives symbol sequences $b_{1,i}, b_{2,i}, \dots, b_{t,i}$. As we now consider K different mobile stations, we have to introduce the corresponding channel transfer functions that we denote $\mathbf{H}_i(z) = \sum_{l=0}^{L_i-1}\mathbf{H}_i(l)z^{-l}$, for $i = 1, \dots, K$, where the matrices $(\mathbf{H}_i(l))_{l=0,\ldots,L_i-1}$ are defined as in Eq. (2) in terms of power p_i and covariance matrices $(\mathbf{C}_i(l))_{l=0,\ldots,L_i-1}$. We denote by $R_i(N)$ the maximum rate delivered to mobile station i, which, expressed in bits/symbol duration/ Hz, is given by

$$R_i(N) = \frac{1}{N}\mathbb{E}\left[\sum_{j=1}^{t}\log_2(1+\beta_{j,i}(N))\right]$$

Theorem 1 implies that when K and N converge to $+\infty$ in such a way that $\frac{K}{N}=\alpha$, then $R_i(N)$, a $O(\frac{1}{N})$ term, can be written as $R_i(N) = R_i + o(\frac{1}{N})$ where the asymptotic rate R_i is given by

$$R_i = \frac{1}{N}\mathbb{E}\left[\sum_{j=1}^{t}\log_2(1+\beta_{j,i}))\right] \tag{4}$$

where $\beta_{j,i}$ is given by (3), but in which function $\eta_1(e^{2i\pi f})$ is replaced by $\eta_{j,i}(e^{2i\pi f})$ given by

$$\mathbf{h}_{j,i}(e^{2i\pi f})^H\left(\mathbf{H}_i(e^{2i\pi f})\mathbf{H}_i(e^{2i\pi f})^H+\frac{\sigma^2}{\alpha}\mathbf{I}_r\right)^{-1}\mathbf{h}_{j,i}(e^{2i\pi f})$$

The exact expression of R_i is of course still quite complicated, and difficult to exploit. As shown below, the terms $(\beta_{j,i})_{j=1,\ldots,t}$ can be approximated by simple deterministic terms (i.e. independent of the particular realization of the channel) when the number of transmit and receive antennas r and t converge to $+\infty$ at the same rate (i.e. in such a way that $\frac{t}{r}$ converges toward a non zero constant). Therefore, R_i can itself be approximated by a simpler expression, in principle valid for a large number of antennas. However, these kind of approximations are known to provide in general very reliable evaluations for realistic number of antennas, see e.g. [8] in other contexts. As our numerical evaluations show (see below), the large antenna numbers approximation turns out to be also quite accurate in our framework.

We first evaluate the asymptotic behaviour of $\beta_{j,i}$ for $j=1,\ldots,t$ when r and t converge to $+\infty$ at the same rate. For this, we first observe that for each frequency f, matrix $\mathbf{H}_i(e^{2i\pi f})$ is given by

$$\mathbf{H}_i(e^{2i\pi f}) = p_i^{1/2}\frac{1}{\sqrt{L_i t}}\sum_{l=0}^{L_i-1}\overline{\mathbf{H}}_i(l)\mathbf{C}_i(l)^{1/2}e^{-2i\pi l f}$$

As mentioned in e.g. [2], $\mathbf{H}_i(e^{2i\pi f})$ can be written as

$$\mathbf{H}_i(e^{2i\pi f}) = p_i^{1/2}\tilde{\mathbf{H}}_i(e^{2i\pi f})\mathbf{C}_i^{1/2} \tag{5}$$

where matrix $\mathbf{C}_i$ is defined as $\mathbf{C}_i = \frac{1}{L_i}\sum_{l=0}^{L_i-1}\mathbf{C}_i(l)$, and where matrix $\tilde{\mathbf{H}}_i(e^{2i\pi f})$ is a zero mean Gaussian $r\times t$ random matrix with variance $\frac{1}{t}$ i.i.d. entries. This is based on the observation that the covariance matrix of $\mathrm{vec}(\mathbf{H}_i(e^{2i\pi f}))$ coincides with $p_i/t\,\mathbf{C}_i\otimes\mathbf{I}_r$. Next, using the matrix inversion lemma as well as the expression of the diagonal entries of the inverse of a positive matrix, we remark that $\eta_{j,i}(e^{2i\pi f})$ can be written as

$$\eta_{j,i}(e^{2i\pi f}) = 1-\frac{\sigma^2}{\alpha}\left(\mathbf{H}_i(e^{2i\pi f})^H\mathbf{H}_i(e^{2i\pi f})+\frac{\sigma^2}{\alpha}\mathbf{I}_r\right)^{-1}_{j,j}$$

or using (5), $\eta_{j,i}(e^{2i\pi f}) = 1-\frac{\sigma^2}{\alpha p_i}\left(\mathbf{Q}_i(e^{2i\pi f})\right)_{j,j}$ where matrix $\mathbf{Q}_i(e^{2i\pi f})$ is defined as

$$\mathbf{Q}(e^{2i\pi f}) = \left(\mathbf{C_i}^{1/2}\tilde{\mathbf{H}}_i(e^{2i\pi f})^H\tilde{\mathbf{H}}_i(e^{2i\pi f})\mathbf{C}_i^{1/2}+\frac{\sigma^2}{\alpha p_i}\mathbf{I}_t\right)^{-1}$$

It turns out that for each f, the entries of random matrix $\mathbf{Q}_i(e^{2i\pi f})$ converge almost surely towards the entries of a deterministic matrix. The corresponding result is stated in the following theorem whose proof is omitted.

Theorem 2 *For each f, when t and r converge to $+\infty$ at the same rate, then, the entries of matrix $\frac{\sigma^2}{\alpha p_i}\mathbf{Q}(e^{2i\pi f})$ converge in probability towards the entries of the deterministic matrix $(\mathbf{I}_t+\delta_i\mathbf{C}_i)^{-1}$, where δ_i is the unique strictly positive solution of the equation*

$$\delta_i = \frac{r}{t}\,\frac{1}{\sigma^2/\alpha p_i+\frac{1}{t}\mathrm{Tr}\mathbf{C}_i(\mathbf{I}+\delta_i\mathbf{C}_i)^{-1}} \tag{6}$$

This implies that $\eta_{j,i}(e^{2i\pi f})$ can be approximated by the deterministic term $1-((\mathbf{I}_t+\delta_i\mathbf{C}_i)^{-1})_{j,j}$, which, of course, is independant from f. Therefore, if we denote by $\beta^*_{j,i}$ the term defined by

$$\beta^*_{j,i} = \frac{1}{\alpha}\left[\frac{1}{(\mathbf{I}+\delta_i\mathbf{C}_i)_{j,j}}-1\right] \tag{7}$$

then, $\beta_{j,i}-\beta^*_{j,i}$ converges in probability towards 0 when r and t converges to $+\infty$ at the same rate. This in turn implies that R_i has the same asymptotic behaviour than R^*_i defined by

$$R^*_i = \frac{1}{N}\sum_{j=1}^{t}\log_2(1+\beta^*_{j,i}) \tag{8}$$

We now summarize the above discussion informally.

Claim 1 *In the large system limit, the sum capacity $R(N)=\sum_{i=1}^{K}R_i(N)$ has the same behaviour than R^* defined by*

$$R^* = \frac{1}{N}\sum_{i=1}^{K}\sum_{j=1}^{t}\log_2(1+\beta^*_{j,i}) \tag{9}$$

where for each i,j, $\beta^*_{j,i}$ is defined by (7).

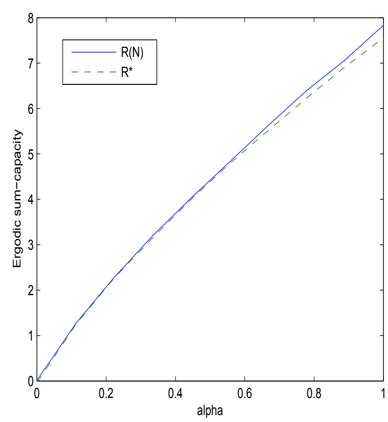

Fig. 1. Accuracy of the approximation.

The sum capacity thus appears as a relatively simple term depending from covariance matrices $(\mathbf{C}_i)_{i=1,\ldots,K}$ and from the received powers $(p_i)_{i=1,\ldots,K}$. We note that if all the matrices $\mathbf{C}_i$ are reduced to $\mathbf{I}$, then terms $(\delta_i)_{i=1,\ldots,K}$ can be expressed in closed form because the equation (6) reduces to a degree 2 polynomial equation. In order to verify the accuracy of R^*, we consider the case where the $(p_i)_{i=1,\dots,K}$ coincide, and where the matrices $\mathbf{C}_i$ are reduced to $\mathbf{I}_t$. For $L = 10$, $N = 256$ and various values of K, Fig 1 represents $R(N)$ and R^* in terms of α for a signal to noise ratio equal to 10 dB for $r = t = 4$. The mathematical expectation introduced in the definition of $R(N)$ is evaluated by Monte-Carlo simulations. It is seen that the fit between $R(N)$ and R^* is very good. Similar results have been obtained in correlated contexts. We also observe that the sum capacity is an increasing function of the loading factor α. Therefore, it is optimum to use a high loading factor in our context.

The expressions of R_i^* and of R^* can be used in various ways. In this paper, due to the lack of space, we just concentrate on two items. We first study the influence of correlation matrices $(\mathbf{C}_i)_{i=1,\dots,K}$, and show that diagonal correlation matrices provide the best results if the loading factor verifies $\alpha \geq 1/2$. More precisely, the following result holds. In order to mention that R_i^* and δ_i depend on $\mathbf{C}_i$, we use the notation $R_i^*(\mathbf{C}_i)$ and $\delta_i(\mathbf{C}_i)$ in the statement of the Theorem.

Theorem 3 *Let $\mathbf{C}_i = \mathbf{U}_i^H \mathbf{\Lambda}_i \mathbf{U}_i$ be the eigenvalue/eigenvector decomposition of matrix $\mathbf{C}_i$. If $\alpha \geq 1/2$, then $R_i^*(\mathbf{C}_i) \leq R_i^*(\mathbf{\Lambda}_i)$, and the equality holds if and only if $\mathbf{C}_i = \mathbf{\Lambda}_i$. In this case, $R_i^*(\mathbf{C}_i) = R_i^*(\mathbf{\Lambda}_i)$ is reduced to*

$$R_i^*(\mathbf{\Lambda}_i) = \frac{1}{N} \log_2 \det \left(\mathbf{I}_t + \frac{\delta_i(\mathbf{\Lambda}_i)}{\alpha} \mathbf{\Lambda}_i \right)$$

Moreover, the entries of the positive diagonal matrix(ces) $\mathbf{\Lambda}_i$ satisfying $\frac{1}{t}Tr(\mathbf{\Lambda}_i) = 1$ and maximizing $R_i^*(\mathbf{\Lambda}_i)$ are as follows: the greatest s entries are all equal to $\frac{t}{s}$ where $s \leq t$, and the smallest $t - s$ entries are zero, where the value of s depends on the various parameters.

The proof is omitted due the lack of space.

We can use this result in order to study the best transmission scheme in the case where the channel taps $\mathbf{H}_i(l)$ are i.i.d. matrices ($\mathbf{C}_i = \mathbf{I}_t$ for each i) , and where the receive powers $(p_i)_{i=1,\dots,K}$ all coincide. Our results allow to consider the case where different powers are allocated to the transmit antennas. Let $\mathbf{\Lambda} = \mathrm{Diag}(\lambda_1, \lambda_2, \dots, \lambda_t)$ be a diagonal matrix with positive entries, and satisfying $\frac{1}{t}\mathrm{Tr}(\mathbf{\Lambda}) = 1$. If transmit antenna j sends sequence $(\lambda_j d_j(n))_{n\in\mathbb{Z}}$ instead of sequence $(d_j(n))_{n\in\mathbb{Z}}$, then it can easily be shown that R^* is given by

$$R^* = \alpha \log_2 \det(\mathbf{I}_t + \frac{\delta(\mathbf{\Lambda})}{\alpha}\mathbf{\Lambda})$$

where $\delta(\mathbf{\Lambda})$ is the solution of the equation (6) in which $p_i = 1$ and $\mathbf{C}_i = \mathbf{\Lambda}$ (just use the above results in the case $p_i = 1, \mathbf{C}_i = \mathbf{\Lambda}$ for each $i = 1, \dots, K$). Theorem 3 implies that the equal power transmission strategy ($\mathbf{\Lambda} = \mathbf{I}_t$) is not necessarily optimum, and that a selection antenna scheme can provide better results. In other words, it is not always optimum to use all the transmit antennas. This is in contrast with the context of MIMO systems in non frequency selective i.i.d. fading channels equiped with maximum likelihood decoders: the transmission scheme that maximizes the ergodic Shannon capacity consists in sending equal power independent symbols on each transmit antenna (see [9]). Note also that Theorem 3 implies that, as a function of $\mathbf{\Lambda}$, $\alpha \log_2 \det(\mathbf{I}_t + \frac{\delta(\mathbf{\Lambda})}{\alpha}\mathbf{\Lambda})$ is not a concave function of $\mathbf{\Lambda}$. Otherwise, the argument of its maximum would be equal to $\mathbf{I}_t$. In other words, the asymptotic sum capacity R^*, and therefore the non asymptotic capacity $R(N) = \sum_{i=1}^K R_i(N)$, are not always concave functions of the correlation matrices $(\mathbf{C}_i)_{i=1,\dots,K}$.

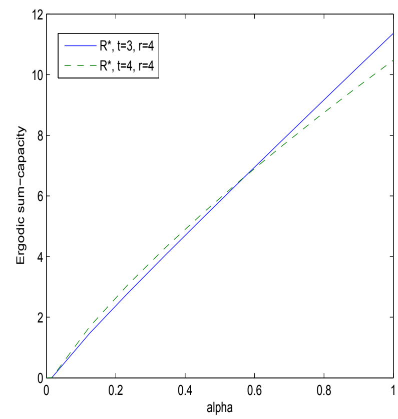

Fig. 2. Sum capacity for different numbers of transmit antennas

In order to illustrate this point, Fig. 2 represents R^* in the above i.i.d. context, for $r = t = 4$, and for $r = 4, t = 3$. The SNR is equal to $15\mathrm{dB}$. It is seen that the sum capacity is better if $t = 3$ than if $t = 4$ if the loading factor is greater than 0.6, thus confirming that a selection antenna scheme is able to provide better results.

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