TRACTABLE APPROACHES TO FAIR QOS BROADCAST PRECODING UNDER CHANNEL UNCERTAINTY

M. Botros Shenouda and T. N. Davidson

Department of Electrical and Computer Engineering
McMaster University, Hamilton, Ontario, L8S 4K1, Canada

ABSTRACT

We consider the design of linear precoders for broadcast channels with Quality of Service (QoS) constraints for each user, in scenarios with uncertain channel state information at the transmitter. Given a total power constraint on the transmission power, our goal is to design a robust fair precoder that maximizes the minimum QoS over all users that can be guaranteed for every channel within a specified uncertainty region around the estimate of each user's channel. Since this problem is not known to be computationally tractable, we will derive three conservative design approaches that yield quasi-convex and computationally-efficient restrictions of the original design problem. The three approaches yield formulations that offer different trade-offs between the degree of conservatism and the size of the design problem. Our simulations indicate that the proposed approaches can significantly increase the minimum QoS of all users when the available channel knowledge at the transmitter is imperfect.

Index Terms— MIMO Systems, Broadcast Channel, Multiuser Channels, Robustness.

1. INTRODUCTION

Using multiple antennas at the base station of a wireless downlink offers the potential to improve the quality of service (QoS) offered to assigned users. When accurate channel state information (CSI) is available at the base station, improvements can be realized by precoding the users' data symbols so as to mitigate the multiuser interference experienced at the receivers. Assuming perfect CSI, the problem of designing a linear precoder to minimize the transmitted power required to satisfy a set of users' QoS constraints was studied in [1, 2, 3]. On the other hand, the design of a "fair" linear precoder that maximizes the minimum QoS for all users under a total transmission power constraint was studied in [2].

In practice, the CSI that is available at the transmitter is subject to uncertainties that arise from sources such as estimation error, channel quantization, and short channel coherence time. These uncertainties can result in a serious degradation to the quality of the received signals; e.g., [4]. This suggests that one ought to incorporate robustness to channel uncertainty into the formulation of the precoder design problem. For example, robust approaches to the design of linear precoders that minimize the transmitted power required to satisfy a set of users' QoS constraints have been studied in [3, 5, 6, 7]. In this paper, we consider a bounded model for the transmitter's estimate of the users' channels, and we study the design of a robust fair precoder that maximizes the minimum QoS over all users that can be guaranteed for every channel within a specified uncertainty region around the estimate of each user's channel (See [8] for minimum MSE transceiver designs for a similar model.) This design is particularly appropriate for systems in which users feed back quantized channel measurements to the transmitter, as knowledge of the quantization codebooks can be used to bound the quantization error. The exact solution of this design problem is not known to be computationally tractable, and we present three conservative design approaches that yield quasi-convex and computationally-efficient restrictions of the original design problem. These formulations offer different trade-offs between the degree of conservatism and the size of resulting optimization problem. Our simulations indicate that the proposed approaches can significantly increase the minimum QoS that can be provided to all users when the available CSI at the transmitter is imperfect.

This work was supported in part by Natural Sciences and Engineering Research Council of Canada. The work of the second author is also supported by the Canada Research Chairs Program.

2. SYSTEM MODEL

We consider broadcast channels with N_t antennas at the transmitter and K receivers, each with a single antenna. Let $\mathbf{s} \in \mathbb{C}^K$ be the vector of data symbols intended for each receiver. The transmitter generates a vector of transmitted signals, $\mathbf{x} \in \mathbb{C}^{N_t}$, by linearly precoding $\mathbf{s}$:

$$\mathbf{x} = \mathbf{P}\mathbf{s} = \textstyle\sum_{j=1}^{K} \mathbf{p}_j s_j, \tag{1}$$

where $\mathbf{p}_j$ is the j^{th} column of the precoding matrix $\mathbf{P}$, and s_j is the j^{th} element of $\mathbf{s}$. Without loss of generality, we will assume that $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$, and hence, a constraint on the average total transmitted power is $\mathrm{tr}(\mathbf{P}^H\mathbf{P}) = \sum_{k=1}^{K} \|\mathbf{p}_k\|^2 \leq P_{\text{total}}$.

The signal received by the k^{th} receiver is $y_k = \mathbf{h}_k\mathbf{x} + n_k$, where $\mathbf{h}_k \in \mathbb{C}^{1\times N_t}$ is a row vector representing the channel gains from the transmitting antennas to the k^{th} receiver, and n_k is the zero-mean additive white noise at the k^{th} receiver whose variance is $\sigma_{n_k}^2$.

2.1. Fair SINR Maximization: Perfect CSI Case

We consider broadcast scenarios in which each receiver has a QoS requirement that is specified in terms of a lower bound on its signal to interference plus noise ratio SINR_k. This SINR constraint represents a rather general constraint on the minimum QoS of the k^{th} user. Indeed, it can be translated into an equivalent constraint on the symbol error rate or the achievable data rate. Given perfect CSI at the transmitter, the design of a precoder that maximizes the SINR of the "weakest" user subject to a transmitted power constraint can be stated as:

$$\max_{\mathbf{P},\gamma_0} \quad \gamma_0 \tag{2a}$$

$$\text{s. t.} \quad \mathrm{SINR}_k = \frac{|\mathbf{h}_k\mathbf{p}_k|^2}{\sum_{j=1, j\neq k}^{K} |\mathbf{h}_k\mathbf{p}_j|^2 + \sigma_{n_k}^2} \geq \gamma_0, \tag{2b}$$

$$\text{tr}(\mathbf{P}^H\mathbf{P}) \leq P_{\text{total}}, \tag{2c}$$

where we have implicitly assumed that the constraint in (2b) must hold for all $k \in \{1, \ldots, K\}$. The problem in (2) is quasi-convex in $\mathbf{P}$ and γ_0. Indeed, if we define

$$\begin{aligned} \underline{\mathbf{P}} &= \begin{bmatrix} \text{Re}\{\mathbf{P}\} & \text{Im}\{\mathbf{P}\} \\ -\text{Im}\{\mathbf{P}\} & \text{Re}\{\mathbf{P}\} \end{bmatrix}, \ \underline{\mathbf{p}}_k = \begin{bmatrix} \text{Re}\{\mathbf{p}_k\} \\ -\text{Im}\{\mathbf{p}_k\} \end{bmatrix}, \\ \underline{\mathbf{h}}_k &= \begin{bmatrix} \text{Re}\{\mathbf{h}_k\} & \text{Im}\{\mathbf{h}_k\} \end{bmatrix}, \end{aligned} \tag{3}$$

then we can formulate (2) as [2]:

$$\min_{\underline{\mathbf{P}}, \beta_0} \quad \beta_0 \tag{4a}$$

$$\text{s. t.} \quad \left\| [\underline{\mathbf{h}}_k \underline{\mathbf{P}}, \quad \sigma_{n_k}] \right\| \leq \beta_0 \underline{\mathbf{h}}_k \underline{\mathbf{p}}_k, \tag{4b}$$

$$\left\| \text{vec}\big([\underline{\mathbf{p}}_1, \ \ldots, \ \underline{\mathbf{p}}_K]\big) \right\| \leq \sqrt{P_{\text{total}}}. \tag{4c}$$

where $\beta_0 = \sqrt{1 + 1/\gamma_0}$. An optimal solution can be efficiently found using a one-dimensional bisection search on β_0 in which the problem solved at each step is the convex second order cone (SOC) feasibility problem corresponding to (4) with a fixed value for β_0. Our goal is to obtain robust counterparts of this fair SINR maximization design problem in the presence of imperfect CSI.

2.2. Bounded Channel Uncertainty Model

We will model the channel uncertainty additively using

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k, \tag{5}$$

where $\hat{\mathbf{h}}_k$ is the transmitter's estimate of the k^{th} user's channel, and $\mathbf{e}_k$ is the corresponding error. In order to avoid making any assumptions on the statistics of $\mathbf{e}_k$, we will merely assume that it lies in the ball $\|\mathbf{e}_k\| \leq \delta_k$, for some given δ_k. This model is a convenient one for systems in which the channel state information is quantized at the receivers and fed back to the transmitter; e.g., [4]. By using the vector $\underline{\mathbf{e}}_k = [\text{Re}\{\mathbf{e}_k\}, \text{Im}\{\mathbf{e}_k\}]$, the uncertainty set of each channel can be described by the following (spherical) region:

$$\mathcal{U}_k(\delta_k) = \{\underline{\mathbf{h}}_k \mid \underline{\mathbf{h}}_k = \underline{\hat{\mathbf{h}}}_k + \underline{\mathbf{e}}_k, \ \|\underline{\mathbf{e}}_k\| \leq \delta_k\}. \tag{6}$$

3. FAIR SINR MAXIMIZATION: UNCERTAIN CSI CASE

Given a constraint on the total transmitted power, our goal is to design a robust precoding matrix that maximizes the minimum SINR of all users that is guaranteed for every channel realizations $\underline{\mathbf{h}}_k$ within the uncertainty region $\mathcal{U}_k(\delta_k)$. That is,

$$\min_{\underline{\mathbf{P}}, \beta_0} \quad \beta_0 \tag{7a}$$

$$\text{s. t.} \quad \left\| [\underline{\mathbf{h}}_k \underline{\mathbf{P}}, \quad \sigma_{n_k}] \right\| \leq \beta_0 \underline{\mathbf{h}}_k \underline{\mathbf{p}}_k, \quad \forall \, \underline{\mathbf{h}}_k \in \mathcal{U}_k(\delta_k), \tag{7b}$$

$$\left\| \text{vec}\big([\underline{\mathbf{p}}_1, \ \ldots, \ \underline{\mathbf{p}}_K]\big) \right\| \leq \sqrt{P_{\text{total}}}. \tag{7c}$$

The problem in (7) has an infinite set of constraints, one for each $\underline{\mathbf{h}}_k \in \mathcal{U}_k(\delta_k)$. Checking the feasibility of (7) for any given β_0 is equivalent to solving a robust second order cone programming problem in which the uncertainty of $\underline{\mathbf{h}}_k$ is present on both left and right hand sides of each SOC constraint in (7b). The structure of this uncertainty is such that the tractability of that robust design problem is still an open problem, e.g., [9]. In order to obtain a robust design technique that is guaranteed to be computationally tractable, we will present three conservative design approaches that yield quasi-convex and computationally-efficient restrictions of (7). These approaches are conservative in the sense that they provide a solution of the robust design problem in (7) for a larger set of channel uncertainties than that described in (6). Therefore these approaches minimize upper bounds on β_0, and hence maximize lower bounds on γ_0.

4. THREE CONSERVATIVE APPROACHES

4.1. First Approach

In the first approach, we will implement the conservative design by assuming independent uncertainties for $\underline{\mathbf{h}}_k$ in each of the left and right hand sides of the constraints in (7b). Relaxing the common uncertainty structure results in a tractable problem. To obtain the tractable formulation we will use the following lemma [9]:

Lemma 1. *Consider the following set of infinite constraints*

$$\|\mathbf{A}\mathbf{x} + \mathbf{b}]\| \leq \mathbf{c}^T\mathbf{x} + d \quad \forall \mathbf{A} \, \in \, \mathcal{Y}, \mathbf{c} \, \in \, \mathcal{W}, \tag{8}$$

where the uncertainty regions $\mathcal{Y}$ and $\mathcal{W}$

$$\begin{aligned} \mathcal{Y} &= \{\mathbf{A} \mid \mathbf{A} = \mathbf{A}^0 + \textstyle\sum_{j=1}^{y} \theta_i \, \mathbf{A}^j, \ \|\boldsymbol{\theta}\| \leq 1\}, \\ \mathcal{W} &= \{\mathbf{c} \mid \mathbf{c} = \mathbf{c}^0 + \textstyle\sum_{j=1}^{w} \phi_i \, \mathbf{c}^j, \ \|\boldsymbol{\phi}\| \leq 1\}, \end{aligned}$$

are independent. Then, there exist positive scalars λ and μ such that (8) is equivalent to the following two constraints:

$$\begin{bmatrix} \lambda - \mu & \mathbf{0} & (\mathbf{A}^0\mathbf{x} + \mathbf{b})^T \\ \mathbf{0} & \mu\mathbf{I} & [\mathbf{A}^1\mathbf{x} \, \ldots \, \mathbf{A}^y\mathbf{x}]^T \\ \mathbf{A}^0\mathbf{x} + \mathbf{b} & [\mathbf{A}^1\mathbf{x} \, \ldots \, \mathbf{A}^y\mathbf{x}] & \lambda\mathbf{I} \end{bmatrix} \geq \mathbf{0},$$

$$\begin{bmatrix} \mathbf{c}^{0\,T}\mathbf{x} + d - \lambda & [\mathbf{c}^{1\,T}\mathbf{x} \, \ldots \, \mathbf{c}^{w\,T}\mathbf{x}] \\ [\mathbf{c}^{1\,T}\mathbf{x} \, \ldots \, \mathbf{c}^{w\,T}\mathbf{x}]^T & (\mathbf{c}^{0\,T}\mathbf{x} + d - \lambda)\mathbf{I} \end{bmatrix} \geq \mathbf{0}. \qquad \square$$

By writing $\underline{\mathbf{h}}_k = \underline{\hat{\mathbf{h}}}_k + \underline{\mathbf{e}}_k = \underline{\hat{\mathbf{h}}}_k + \delta_k \mathbf{u}$, $\|\mathbf{u}\| \leq 1$, and invoking Lemma 1, the conservative design problem can be cast as:

$$\min_{\substack{\underline{\mathbf{P}}, \beta_0, \\ \mu, \lambda}} \quad \beta_0 \tag{10a}$$

$$\text{s. t.} \quad \left\| \text{vec}\big([\underline{\mathbf{p}}_1, \ \ldots, \ \underline{\mathbf{p}}_K]\big) \right\| \leq \sqrt{P_{\text{total}}}, \tag{10b}$$

$$\begin{bmatrix} \beta_0 \underline{\hat{\mathbf{h}}}_k \underline{\mathbf{p}}_k - \lambda_k & \delta_k \beta_0 \underline{\mathbf{p}}_k^{\,T} \\ \delta_k \beta_k \underline{\mathbf{p}}_k & (\beta_0 \underline{\hat{\mathbf{h}}}_k \underline{\mathbf{p}}_k - \lambda_k)\mathbf{I} \end{bmatrix} \geq \mathbf{0}, \tag{10c}$$

$$\begin{bmatrix} \lambda_k - \mu_k & \mathbf{0} & [\underline{\hat{\mathbf{h}}}_k \underline{\mathbf{P}}, \ \sigma_{n_k}] \\ \mathbf{0} & \mu_k \mathbf{I} & \delta_k [\underline{\mathbf{P}}, \ \mathbf{0}] \\ [\underline{\hat{\mathbf{h}}}_k \underline{\mathbf{P}}, \ \sigma_{n_k}]^T & \delta_k [\underline{\mathbf{P}}, \ \mathbf{0}]^T & \lambda_k \mathbf{I} \end{bmatrix} \geq \mathbf{0}. \tag{10d}$$

This problem is a quasi-convex problem, and the optimal robust solution can be efficiently found using a one-dimensional bisection search on β_0 in which the problem solved at each step is the convex semidefinite programming (SDP) feasibility problem corresponding to (10) with a fixed value for β_0.

4.2. Second Approach

In this approach, we will first obtain an equivalent matrix inequality formulation of the fair precoder design problem with perfect CSI in (4). Then, we will incorporate the uncertainty model to obtain a robust counterpart to the perfect CSI formulation.

Using the Schur Complement Theorem, the formulation in (4) of the design problem with perfect CSI can be written as:

$$\min_{\underline{\mathbf{P}},\,\beta_0}\ \beta_0 \tag{11a}$$

$$\text{s. t. } \left\|\text{vec}\left([\underline{\mathbf{p}}_1,\ \ldots,\ \underline{\mathbf{p}}_K]\right)\right\| \leq \sqrt{P_{\text{total}}}, \tag{11b}$$

$$\mathbf{F}_k(\underline{\mathbf{P}}, \beta_0, \underline{\mathbf{h}}_k) = \begin{bmatrix} \beta_0 \underline{\mathbf{h}}_k \underline{\mathbf{p}}_k & [\underline{\mathbf{h}}_k \underline{\mathbf{P}},\ \sigma_{n_k}] \\ [\underline{\mathbf{h}}_k \underline{\mathbf{P}},\ \sigma_{n_k}]^T & (\beta_0 \underline{\mathbf{h}}_k \underline{\mathbf{p}}_k)\mathbf{I} \end{bmatrix} \geq \mathbf{0}. \tag{11c}$$

The robust counterpart of (11c) takes the form:

$$\mathbf{F}_k(\underline{\mathbf{P}}, \beta_0, \underline{\mathbf{h}}_k) \geq \mathbf{0}, \quad \forall\, \underline{\mathbf{h}}_k \in \mathcal{U}_k(\delta_k). \tag{12}$$

By substituting $\underline{\mathbf{h}}_k = \hat{\underline{\mathbf{h}}}_k + \underline{\mathbf{e}}_k$ in (12), we have that

$$\overline{\mathbf{F}}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k, \mathbf{M}_k) = \mathbf{F}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k) + \mathbf{M}_k \mathbf{R}_k(\underline{\mathbf{P}}, \beta_0) + \mathbf{R}_k^T(\underline{\mathbf{P}}, \beta_0)\mathbf{M}_k^T \geq \mathbf{0}, \tag{13}$$

where the matrices $\mathbf{M}_k$ and $\mathbf{R}_k(\underline{\mathbf{P}}, \beta_0)$ are:

$$\mathbf{M}_k = \mathbf{I} \otimes \underline{\mathbf{e}}_k, \tag{14}$$

$$\mathbf{R}_k(\underline{\mathbf{P}}, \beta_0) = \begin{bmatrix} \frac{1}{2}\beta_k \underline{\mathbf{p}}_k & [\underline{\mathbf{P}},\ \mathbf{0}] \\ \mathbf{0} & (\frac{1}{2}\beta_k)\mathbf{I} \otimes \underline{\mathbf{p}}_k \end{bmatrix}. \tag{15}$$

From (14), we observe that the uncertainty matrix $\mathbf{M}_k$ belongs to a subspace $\mathcal{M}$ of block diagonal matrices with equal blocks:

$$\mathcal{M} = \{\mathbf{M}\,|\mathbf{M} = \mathbf{I} \otimes \underline{\mathbf{e}},\ \mathbf{e} \in \mathbb{R}^{1\times 2N_t}\}. \tag{16}$$

Hence, the spectral norm of $\mathbf{M}_k$ is $\|\mathbf{M}_k\| = \|\underline{\mathbf{e}}_k\| \leq \delta_k$. Given (13), the original problem in (7) can be precisely formulated as:

$$\min_{\underline{\mathbf{P}},\,\beta_0}\ \beta_0 \tag{17a}$$

$$\text{s. t. } \left\|\text{vec}\left([\underline{\mathbf{p}}_1,\ \ldots,\ \underline{\mathbf{p}}_K]\right)\right\| \leq \sqrt{P_{\text{total}}}, \tag{17b}$$

$$\overline{\mathbf{F}}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k, \mathbf{M}_k) \geq \mathbf{0},\ \forall\, \mathbf{M}_k \in \mathcal{M},\ \|\mathbf{M}_k\| \leq \delta_k. \tag{17c}$$

A general instance of (17) is NP-hard for a general subspace $\mathcal{M}$; see [10, 9]. However, the advantage of (17) is that it captures the difficulty of the problem in the particular structure that the matrix $\mathbf{M}$ must possess. If we adopt a conservative approach that drops the block diagonal structure constraint on $\mathbf{M}_k$ by replacing (17c) with

$$\overline{\mathbf{F}}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k, \mathbf{M}_k) \geq \mathbf{0},\ \|\mathbf{M}_k\| \leq \delta_k, \tag{18}$$

an efficiently solvable problem can be obtained. Although (18) is simpler than (17c), it still represents an infinite set of matrix inequalities, one for each admissible $\mathbf{M}_k$. However, this semi-infinite matrix inequality constraint is equivalent to the existence of a positive scalar τ_k, such that [10, Theorem 3.1]

$$\begin{bmatrix} \mathbf{F}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k) - \tau_k\,\mathbf{I} & \mathbf{R}^T(\underline{\mathbf{P}}, \beta_0) \\ \mathbf{R}(\underline{\mathbf{P}}, \beta_0) & \tau_k \delta_k^{-2}\mathbf{I} \end{bmatrix} \geq \mathbf{0}, \tag{19}$$

Hence, the second conservative formulation of the robust fair SINR maximization can be written as:

$$\min_{\substack{\underline{\mathbf{P}},\beta_0,\\ \tau_1,\ldots,\tau_K}}\ \beta_0 \tag{20a}$$

$$\text{s. t. } \left\|\text{vec}\left([\underline{\mathbf{p}}_1,\ \ldots,\ \underline{\mathbf{p}}_K]\right)\right\| \leq \sqrt{P_{\text{total}}}, \tag{20b}$$

$$\begin{bmatrix} \mathbf{F}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k) - \tau_k\,\mathbf{I} & \mathbf{R}^T(\underline{\mathbf{P}}, \beta_0) \\ \mathbf{R}(\underline{\mathbf{P}}, \beta_0) & \tau_k \delta_k^{-2}\mathbf{I} \end{bmatrix} \geq \mathbf{0}, \tag{20c}$$

where $\mathbf{F}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k)$ and $\mathbf{R}(\underline{\mathbf{P}}, \beta_0)$ were defined in (11c) and (15), respectively. This optimization problem is quasi-convex and can be efficiently solved by bisection search, in which each iteration solves the semidefinite program in (20) for a given β_0.

4.3. Third Approach

In the previous section we have seen that a conservative formulation of the robust fair precoding problem could be obtained by relaxing the structural constraint $\mathbf{M}_k \in \mathcal{M}$ on $\mathbf{M}_k$ in the precise robust formulation in (17) of the original robust design problem. In this section we will present an alternative conservative formulation of (17) that retains the structural constraint $\mathbf{M}_k \in \mathcal{M}$. In particular, by using [10, Theorem 3.2], we can show that the solution of the following quasi-convex problem generates a conservative solution to the original robust design problem in (17).

$$\min_{\substack{\underline{\mathbf{P}},\beta_0\\ \mathbf{S}_1,\ldots,\mathbf{S}_K}}\ \beta_0 \tag{21a}$$

$$\text{s. t. } \left\|\text{vec}\left([\underline{\mathbf{p}}_1,\ \ldots,\ \underline{\mathbf{p}}_K]\right)\right\| \leq \sqrt{P_{\text{total}}}, \tag{21b}$$

$$\mathbf{S}_k \geq \mathbf{0}, \tag{21c}$$

$$\begin{bmatrix} \mathbf{F}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k) - \mathbf{S}_k & \mathbf{R}^T(\underline{\mathbf{P}}, \beta_0) \\ \mathbf{R}(\underline{\mathbf{P}}, \beta_0) & \delta_k^{-2}\mathbf{S}_k \otimes \mathbf{I} \end{bmatrix} \geq \mathbf{0}, \tag{21d}$$

where $\mathbf{F}_k(\underline{\mathbf{P}}, \beta_0, \hat{\underline{\mathbf{h}}}_k)$ and $\mathbf{R}(\underline{\mathbf{P}}, \beta_0)$ are as defined in the previous section. We point out that the problem in (20) is the special case of that in (21) that is obtained when $\mathbf{S}_k$ takes the value $\tau_k \mathbf{I}$ for $\tau_k \geq 0$. Therefore, the solution of (21) yields a tighter upper bound on β_0, and hence a larger lower bound on γ_0, than the solution of the one in (20). In the Section 5, we will demonstrate that the preservation of the structure of $\mathbf{M}_k$ by allowing each $\mathbf{S}_k$ to be any positive semidefinite matrix results in substantial performance improvement.

5. SIMULATION STUDIES

In this section we will compare the performance of the three robust fair precoding approaches that are proposed in Section 4 (Robust Appr 1, 2, 3). We will also provide performance comparisons with existing approaches, namely the robust autocorrelation matrix approach in [3] (Robust Correl. Appr.), and the robust downlink power loading approach in [7]. The approach in [7] requires the beamforming vectors to be specified, and we will consider two choices: the columns of the pseudo-inverse of $\hat{\mathbf{H}}$ (Robust Power Load. 1); and the beamforming vectors obtained by assuming that $\hat{\mathbf{H}}$ is the actual channel and using the existing methods for QoS precoding with perfect CSI [2] (Robust Power Load. 2). The approaches in [3] and [7] are based on uncertainty models that are different from the one in (6), and from each other. The approach in [3] considers a model in which the spectral norm of the error in the (deterministic) autocorrelation matrix $\mathbf{C}_k = \mathbf{h}_k^H \mathbf{h}_k$ is bounded, and in the approach in [7] the Frobenius norm of the error in $\mathbf{C}_k$ is bounded. However, by bounding these norms of $\mathbf{C}_k$ in terms of the norm of $\mathbf{e}_k$, a comparable uncertainty set can be generated.[1] We will compare these schemes in an environment with $N_t = 3$ transmit antennas and $K = 3$ users. We

[1] A bound on the spectral norm of the error in $\mathbf{C}_k$ can be obtained as follows: $\|(\hat{\mathbf{h}}_k+\mathbf{e}_k)^H(\hat{\mathbf{h}}_k+\mathbf{e}_k) - \mathbf{h}_k^H\mathbf{h}_k\| = \|\hat{\mathbf{h}}_k^H\mathbf{e}_k + \mathbf{e}_k^H\hat{\mathbf{h}}_k + \mathbf{e}_k^H\mathbf{e}_k\| \leq \|\hat{\mathbf{h}}_k^H\mathbf{e}_k\| + \|\mathbf{e}_k^H\hat{\mathbf{h}}_k\| + \|\mathbf{e}_k^H\mathbf{e}_k\| = 2\|\hat{\mathbf{h}}_k\|\|\mathbf{e}_k\| + \|\mathbf{e}_k\|^2$. This bound also holds for the Frobenius norm, since the matrices on the immediate right hand side of the inequality are all rank one. Furthermore, the uncertainty $\mathbf{e}_k = \delta_k\hat{\mathbf{h}}_k/\|\hat{\mathbf{h}}_k\|$ achieves this upper bound with equality for both norms.

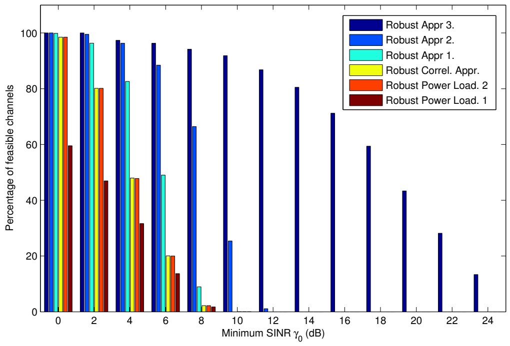

Fig. 1. Percentage of channel realizations for which the considered robust designs are able to achieve the prescribed guaranteed-SINR threshold (Minimum SINR), for a system with $N_t = 3$ and $K = 3$.

will consider the conventional independent Rayleigh fading channel model, and zero-mean additive white Gaussian noise (AWGN) with unit variance.

In the first experiment, we examine the performance of 2000 randomly generated realizations of the set of channel estimates $\{\hat{\mathbf{h}}_k\}_{k=1}^K$ in the presence of equal uncertainty, $\delta_k = \delta = 0.05$, $\forall k$. For each set of channel estimates a robust fair precoder was designed using one of the proposed methods (or one of the existing desings), and we examine the performance of these designs by choosing a fair SINR threshold above which the QoS guarantee is deemed acceptable for all users, and determining the fraction of the sets of channel estimates for which this threshold was satisfied. In Fig. 1 we provide a histogram of the fraction of the 2000 channel realizations for which there existed a feasible robust fair precoder (with any finite power) to satisfy this threshold. From this figure, it is clear that both the second and third approaches in Section 4 are more likely to be able to meet the SINR threshold, especially as the threshold increases, with the third approach that retains the structure of the uncertainty having a significant advantage.

In the second experiment, we selected all the sets of channel estimates from the 2000 sets used in the previous experiment for which all design methods were able to satisfy a guaranteed SINR threshold of 6 dB. We used these 264 sets of channel realizations to plot the achieved fair guaranteed-SINR against the average transmitted power in Fig. 2. We have also included the corresponding curve for the case of perfect CSI at the transmitter; cf. [2] and (4). This figure illustrates the saturation effect that channel uncertainty imposes on the growth of the fair guaranteed-SINR with the transmitted power. This effect was observed in [4] for non-robust linear precoding on the MISO downlink with quantized CSI. Fig. 2 also illustrates the role that robust fair precoding can play in extending the interval over which linear growth of the minimum SINR, γ_0, with the transmitted power can be achieved. This is particularly evident for the second and third robust approaches.

6. CONCLUSION

We have studied the design of a robust fair precoder that maximizes a minimum QoS requirement over all users that can be guaranteed for every channel within a specified uncertainty region around the estimate of each user's channel. Although that problem's tractability is not known, we presented three conservative design approaches that yield quasi-convex and computationally-efficient restrictions of the original problem. As illustrated by the simulations, the proposed approaches can significantly increase the minimum guaranteed QoS of all users when the available CSI at the transmitter is imperfect.

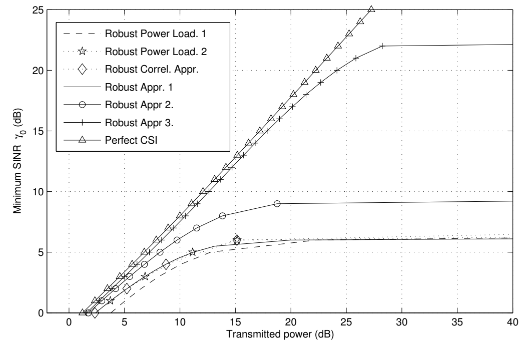

Fig. 2. Achieved fair guaranteed-SINR (Minimum SINR) against the average transmitted power, for a system with $N_t = 3$ and $K = 3$.

7. REFERENCES

[1] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, pp. 18–28, Jan. 2004.

[2] A. Wiesel, Y.C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Processing*, vol. 54, no. 1, pp. 161–176, Jan. 2006.

[3] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using semidefinite optimization," in *Proc. 37th Allerton Conf. Comm., Control, Computing*, Monticello, IL, 1999, pp. 987–996.

[4] N. Jindal, "MIMO broadcast channels with finite rate feedback," *IEEE Trans. Inform. Theory*, vol. 52, no. 11, pp. 5045–5059, Nov. 2006.

[5] M. Botros Shenouda and T. N. Davidson, "Convex conic formulations of robust downlink precoder designs with quality of service constraints," *IEEE J. Select. Topics Signal Processing*, vol. 1, no. 4, pp. 714–724, Dec. 2007.

[6] M. Botros Shenouda and T. N. Davidson, "Non-linear and linear broadcasting with QoS requirements: Tractable approaches for bounded channel uncertainties," Submitted to *IEEE Trans. Signal Processing* Dec. 2007. See also http://arxiv.org/abs/0712.1659v1.

[7] M. Biguesh, S. Shahbazpanahi, and A.B. Gershman, "Robust downlink power control in wireless cellular systems," *EURASIP J. Wireless Commun. Networking*, vol. 2, pp. 261–272, 2004.

[8] M. Botros Shenouda and T. N. Davidson, "Linear multiuser transceivers: Robustness via worst scenario MSE approach," To appear in the *Proc. IEEE WCNC*, Las Vegas, April 2008.

[9] A. Ben-Tal and A. Nemirovski, "Robust convex optimization," *Math. Oper. Res.*, vol. 23, no. 4, pp. 769–805, 1998.

[10] L. El-Ghaoui, F. Oustry, and H. Lebret, "Robust solutions to uncertain semidefinite programs," *SIAM J. Optim.*, vol. 9, pp. 33–52, 1998.