DOWNLINK PRECODING FOR MULTIUSER MISO SYSTEMS WITH IMPERFECT CHANNEL KNOWLEDGE

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ABSTRACT

It is well-known that the downlink beamforming problem of minimizing the total transmit power under users' signal-to-interference-plus-noise ratio (SINR) constraints can be reformulated as a conic quadratic optimization problem and efficiently solved, if the transmitter is provided with the perfect information about the channel. In this work, we study the robust counterpart of the latter, convex problem. By robustness it is meant that the base station knows only uncertainty regions where the exact channels lie, and that it is supposed to satisfy the conic quadratic constraints for all channels that belong to these regions. We provide a direct optimal solution for this problem, based on the ellipsoid method from convex optimization theory. By exploiting the structure of the problem, we define also a virtual robust mean square error optimization problem, that can be solved by semidefinite programming methods in a much more efficient manner, and which presents (at least) a tight conservative approximation of the main problem.

***Index Terms*—** Broadcast channels, antenna arrays, robustness

1. INTRODUCTION

In this paper, we consider a downlink multiuser multiple-input single-output (MISO) flat-fading wireless system, i.e., the base station (BS) is equipped with multiple antennas, while the users have single antennas. The BS simultaneously transmits independent signals to the receivers. In one such setup, transmit beamforming can be used to mitigate the interference and control the signal-to-interference-plus-noise ratios (SINRs) [1]. If the channel state information (CSI) is available at the transmitter, a sort of precoding can be applied, from which the communication process can benefit significantly [2].

However, the assumption of having the perfect CSI at the BS in a downlink scenario is quite unrealistic, due to various harmful effects like noise, outdated channel estimates, limited feedback capacities, etc. (see, e.g., [3] and the references therein) that appear in real-world wireless systems. Therefore, robust designs have to be considered, in order to account for the disturbances in the CSI at the transmitter. The robustness in this paper means that the BS knows only the erroneous channel estimates and the uncertainty regions that contain the accurate channel values, defined by the bounds on the spectral norms of the CSI error vectors.

The problem of interest is a variation of the standard beamforming problem, where the spatial transmit filter (precoder) of the BS is designed to minimize the total transmit power under a set of predefined SINR targets [1]. The solution of this problem with the perfect CSI at the transmitter is known from [1, 4] (see also the references therein). In [1], for the case of rank-one channels, the problem is shown to be equivalent to a conic quadratic problem. In this paper, we consider the robust counterpart of the latter problem, by demanding the fulfillment of constraints for all channels from the uncertainty regions. This problem was solved suboptimally in [5] for the same uncertainty model as the one assumed in this paper. Under somewhat different uncertainty assumptions, a restriction of a similar robust problem was analyzed also in [1].

Our first contribution is in noticing that the problem of interest belongs to a class of uncertain conic quadratic problems with simple ellipsoidal uncertainty, which can be solved optimally by the ellipsoid method from convex optimization theory [6]. Then, we approach the problem by exploiting its specific structure, and derive a suboptimal solution using semidefinite programming (SDP) methods [7]. The idea is based on the previous work [8], where the same robust problem with maximal allowable mean square error (MSE) constraints, instead of the minimal SINR targets, was optimally solved under uncertainty. In this way, a very efficient numerical tool for the robust SINR-constrained precoder design, which outperforms the related results in the literature in terms of the performance/complexity tradeoff, is provided. Furthermore, from the numerical simulations it seems that no performance gap exists in comparison to the computationally much more involved, optimal solution by the ellipsoid method.

We adopt the following notation: Small and large bold fonts are used for vectors and matrices, respectively. The trace of a matrix, the spectral norm and the Frobenius norm are denoted with $\mathrm{Tr}(\cdot)$, $\|(\cdot)\|_2$ and $\|(\cdot)\|_F$, respectively [9]. $\boldsymbol{A} \succeq \boldsymbol{B}$ means that $\boldsymbol{A} - \boldsymbol{B}$ is positive semidefinite. The matrix transpose and the Hermitian transpose are written as $(\cdot)^T$ and $(\cdot)^*$, respectively. $\boldsymbol{A}_{(k,:)}$ $\left(\boldsymbol{A}_{(:,k)}\right)$ denotes the kth row (column) of $\boldsymbol{A}$. $\Re(\cdot)$ and $\Im(\cdot)$ extract real and imaginary parts of the argument, respectively. $\mathbb{L}^n$ is the Lorentz cone (the second-order cone) in $\mathbb{R}^n$ [10].

2. SYSTEM MODEL AND PROBLEM STATEMENT

The downlink multiuser MISO system with K users is illustrated in Fig. 1. The symbols for transmission by the BS are grouped in the vector $\boldsymbol{s} = [s_1^*, s_2^*, \ldots, s_K^*]^*$, with $\mathrm{E}(\boldsymbol{s}\boldsymbol{s}^*) = \boldsymbol{I}$, w.l.o.g. The BS array consists of M antennas. The complete multiuser flat-fading channel is given as

$$\boldsymbol{H} = \left[\begin{array}{ccc} \boldsymbol{H}^*_{(1,:)} & \cdots & \boldsymbol{H}^*_{(K,:)} \end{array} \right]^*, \tag{1}$$

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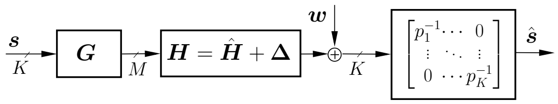

Fig. 1. System model.

where $\boldsymbol{H}_{(k,:)} \in \mathbb{C}^{1\times M}$ is the channel of the user k. The BS is assumed to know only estimates $\hat{\boldsymbol{H}}_{(k,:)}$ of the channels $\boldsymbol{H}_{(k,:)}$. The errors in the CSI are modeled by bounding the spectral norms of the CSI error vectors

$$\boldsymbol{H}_{(k,:)} = \hat{\boldsymbol{H}}_{(k,:)} + \boldsymbol{\Delta}_{(k,:)}, \quad \|\boldsymbol{\Delta}_{(k,:)}\|_2 \le \varepsilon_k, \quad \forall k \in \{1,\dots,K\}. \tag{2}$$

The model (2) naturally corresponds to disturbances that appear as a result of quantization. However, even in the case of unbounded disturbances that typically emerge in the channel estimation process and have Gaussian distribution, the spherically bounded model can serve well in providing the conservative robust solution for a given outage probability [11].

The linear transmit filter of the BS is denoted with $\boldsymbol{G} \in \mathbb{C}^{M\times K}$, while the user k performs a scaling of the received signal with $p_k^{-1} \in \mathbb{R}_{++}$. The additive noise at the reception $\boldsymbol{w} = [w_1^*, w_2^*, \dots, w_K^*]^*$, with $\mathrm{E}(w_k w_k^*) = \sigma_k^2 \boldsymbol{I}$, is uncorrelated with the input signals $\boldsymbol{s}$. The received signal of the user k can be written as

$$\hat{s}_k = \frac{1}{p_k}(\boldsymbol{H}_{(k,:)}\boldsymbol{G}\boldsymbol{s} + w_k), \quad \forall k \in \{1,\dots,K\}, \tag{3}$$

while the transmit power of the BS is

$$P_{\mathrm{Tx}} = \mathrm{E}(\mathrm{Tr}(\boldsymbol{G}\boldsymbol{s}\boldsymbol{s}^*\boldsymbol{G}^*)) = \|\boldsymbol{G}\|_F^2. \tag{4}$$

The SINR of the user k can be expressed as

$$\mathrm{SINR}_k = \frac{|\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)}|^2}{\sum_{l=1,\, l\neq k}^{K} |\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,l)}|^2 + \sigma_k^2}. \tag{5}$$

Clearly, the multiplication with p_k^{-1} at the reception has no effect on the SINR values. However, we will see in the sequel how these equalization coefficients play an important role in defining a virtual MSE problem, whose solution will be of great interest.

Let γ_k are the predefined users' SINR targets. We formulate the main problem of interest as follows [5, 12]

$$\begin{aligned}
&\min_{\tilde{\boldsymbol{G}}} \quad \frac{1}{2}\|\tilde{\boldsymbol{G}}\|_F^2,\\
&\text{s.t.} \quad \left\| \begin{bmatrix} \tilde{\boldsymbol{H}}_{(k,:)}\tilde{\boldsymbol{G}} & \sigma_k \end{bmatrix} \right\|_2 \le \sqrt{1+\gamma_k^{-1}}\, \tilde{\boldsymbol{H}}_{(k,:)}\tilde{\boldsymbol{G}}_{(:,k)},\\
&\forall \|\tilde{\boldsymbol{\Delta}}_{(k,:)}\|_2 \le \varepsilon_k, \quad \forall k \in \{1,\dots,K\},
\end{aligned} \tag{6}$$

where

$$\tilde{\boldsymbol{H}}_{(k,:)} = \begin{bmatrix} \Re\{\boldsymbol{H}_{(k,:)}\} & \Im\{\boldsymbol{H}_{(k,:)}\} \end{bmatrix}, \tag{7}$$

$$\tilde{\boldsymbol{\Delta}}_{(k,:)} = \begin{bmatrix} \Re\{\boldsymbol{\Delta}_{(k,:)}\} & \Im\{\boldsymbol{\Delta}_{(k,:)}\} \end{bmatrix}, \tag{8}$$

$$\tilde{\boldsymbol{G}} = \begin{bmatrix} \Re\{\boldsymbol{G}\} & \Im\{\boldsymbol{G}\} \\ -\Im\{\boldsymbol{G}\} & \Re\{\boldsymbol{G}\} \end{bmatrix}. \tag{9}$$

The fulfillment of the constraints in (6) guarantees obviously that the SINR targets will be satisfied under uncertainty:

$$\mathrm{SINR}_k \ge \gamma_k, \quad \forall k \in \{1,\dots,K\}, \quad \forall \|\boldsymbol{\Delta}_{(k,:)}\|_2 \le \varepsilon_k. \tag{10}$$

In [1], it is shown that the non-robust instance of (6), with $\varepsilon_k = 0$, $\forall k$, is actually equivalent to the standard beamforming problem of minimizing the total transmit power (4) under the constraints (10) with no uncertainty. The idea for the transformation was in noticing that the kth column of the matrix $\boldsymbol{G}$ can be multiplied with a complex factor $e^{j\omega_k}$ without changing the objective function, so that the term $\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)}$ in the numerator of (5) can be real and positive. However, it is important to remark that, due to the channel uncertainty, the same reasoning cannot be applied for supporting an infinite number of channels contained in the uncertainty regions, with a single set of filters. Therefore, the solution of our main problem of interest (6) yields an achievable performance (an upper bound) for the even more involved problem of minimizing the total transmit power under the constraints (10).

3. SOLUTION BY THE ELLIPSOID METHOD

In this section, we show that the problem (6) belongs to a class of uncertain conic quadratic problems with simple ellipsoidal uncertainty. These problems are known to be tractable, and they can be solved exactly using the ellipsoid method from convex optimization theory [6].

The first necessary requirement for the ellipsoid method is the ability to calculate the subgradient of the objective function. In our case this is trivial. Map the unknown coefficients of the transmit filter in the vector $\boldsymbol{g} = \begin{bmatrix} \tilde{\boldsymbol{G}}_{(:,1)}^* & \cdots & \tilde{\boldsymbol{G}}_{(:,K)}^* \end{bmatrix}^*$. Clearly, the objective function is equal to $\|\boldsymbol{g}\|_2^2$ and it is differentiable, so the subgradient is equal to the usual gradient: $\partial P_{\mathrm{Tx}} = 2\boldsymbol{g}$.

The second ingredient is the construction of a routine for determining whether a particular point $\bar{\boldsymbol{g}} \in \mathbb{R}^{2MK}$ is feasible for (6), and, if not, for calculating the separation vector

$$\boldsymbol{a}^T\bar{\boldsymbol{g}} \ge \sup_{\boldsymbol{g}\in\mathcal{D}} \boldsymbol{a}^T\boldsymbol{g}, \quad \boldsymbol{a} \neq \boldsymbol{0}, \tag{11}$$

where $\mathcal{D}$ is the domain of (6). This task is considerably more complex. We start by noticing that $\mathcal{D}$ is an intersection of sets $\mathcal{D}_k$, $k = 1\dots K$, where $\mathcal{D}_k$ is the feasible region for the constraint related to the user k in (6). From (6), it follows that $\mathcal{D}_k$ presents an intersection of an infinite number of sets that correspond to the individual channels from the uncertainty regions. It can be easily seen that these sets are convex, so $\mathcal{D}_k$ must be convex too, as intersection preserves convexity [10]. In the sequel, we focus on determining the separation hyperplanes related to the kth user constraint and the domain $\mathcal{D}_k$. Clearly, the global separation oracle consists then of examining each of the user constraints, and, if any one of them is infeasible, the separation hyperplane obtained for $\mathcal{D}_k$ will be an appropriate separation hyperplane for $\mathcal{D}$, as well.

We proceed by noticing that the kth user constraint in (6) can be easily rewritten in the form

$$\left\|\boldsymbol{\Phi}(\boldsymbol{g})\boldsymbol{\delta} + \boldsymbol{\psi}(\boldsymbol{g})\right\|_2 \le \boldsymbol{\alpha}^T(\boldsymbol{g})\boldsymbol{\delta} + \beta(\boldsymbol{g}), \quad \|\boldsymbol{\delta}\|_2 \le \varepsilon, \tag{12}$$

where $\boldsymbol{\delta} = \tilde{\boldsymbol{\Delta}}_{(k,:)}^T$, $\varepsilon = \varepsilon_k$, and $\boldsymbol{\Phi}$, $\boldsymbol{\psi}$, $\boldsymbol{\alpha}$ and β are affine in the unknown precoder coefficients $\boldsymbol{g}$. Due to the lack of space, in this paper, we only give a sketch of the algorithm for the analysis of (12), applying the robust optimization methodology from [13]. The user index k and the dependence on $\boldsymbol{g}$ will be occasionally omitted in rest of this section, in order to simplify the expressions.

Using the Cauchy-Schwarz inequality [9] and the $\mathcal{S}$-Lemma [10], it can be proved that (12) is valid, if and only if, the following two inequalities are fulfilled

$$\varepsilon\|\boldsymbol{\alpha}\|_2 \leq \beta, \tag{13}$$

$$\begin{bmatrix} \lambda \boldsymbol{I} + \boldsymbol{\alpha}\boldsymbol{\alpha}^T - \boldsymbol{\Phi}^T\boldsymbol{\Phi} & \beta\boldsymbol{\alpha} - \boldsymbol{\Phi}^T\boldsymbol{\psi} \\ \beta\boldsymbol{\alpha}^T - \boldsymbol{\psi}^T\boldsymbol{\Phi} & \beta^2 - \boldsymbol{\psi}^T\boldsymbol{\psi} - \lambda\varepsilon^2 \end{bmatrix} \succeq \mathbf{0}, \tag{14}$$

for some $\lambda \geq 0$. Examining the validity of (13) is trivial for a given vector $\boldsymbol{g} = \bar{\boldsymbol{g}}$, and, in the case when (13) is not valid, the separation vector is easily obtained using the Cauchy-Schwarz inequality. It can be also noticed that for a fixed $\boldsymbol{g}$, (14) is a linear matrix inequality (LMI) in λ. Therfore, its feasibility can be checked using some of the well-developed methods from convex optimization theory. In the case when (14) is infeasible, a direct, efficient procedure suggested in [13] yields as a byproduct a vector $\bar{\boldsymbol{\delta}}$ for whom (12) is not valid. Having found one such vector, the separation hyperplane can be constructed as follows.

Let $\bar{\boldsymbol{y}} = \left[\left(\boldsymbol{\Phi}(\bar{\boldsymbol{g}})\bar{\boldsymbol{\delta}} + \boldsymbol{\psi}(\bar{\boldsymbol{g}})\right)^T, \boldsymbol{\alpha}^T(\bar{\boldsymbol{g}})\bar{\boldsymbol{\delta}} + \beta(\bar{\boldsymbol{g}})\right]^T \notin \mathbb{L}^{2K+2}$. Set $\boldsymbol{v} = \left[\left(\boldsymbol{\Phi}(\bar{\boldsymbol{g}})\bar{\boldsymbol{\delta}} + \boldsymbol{\psi}(\bar{\boldsymbol{g}})\right)^T \|\boldsymbol{\Phi}(\bar{\boldsymbol{g}})\bar{\boldsymbol{\delta}} + \boldsymbol{\psi}(\bar{\boldsymbol{g}})\|_2^{-1}, -1\right]^T$ and notice that $\boldsymbol{v}^T\bar{\boldsymbol{y}} \geq \boldsymbol{v}^T\boldsymbol{y}$, for all $\boldsymbol{y} \in \mathbb{L}^{2K+2}$. The homogenous part of $\boldsymbol{v}^T\left[\left(\boldsymbol{\Phi}(\boldsymbol{g})\bar{\boldsymbol{\delta}} + \boldsymbol{\psi}(\boldsymbol{g})\right)^T, \boldsymbol{\alpha}^T(\boldsymbol{g})\bar{\boldsymbol{\delta}} + \beta(\boldsymbol{g})\right]^T$, which can be immediately computed because of the affinity of the respective terms in $\boldsymbol{g}$, gives the required separation vector $\boldsymbol{a}$.

The solution, based on the ellipsoid method, is summarized in Table 1, where $d = 2KM$ is the dimension of the unknown vector $\boldsymbol{g}$. We remark that the initialization and the convergence condition can be formalized, which we omit due to the lack of space.

Table 1 Algorithmic solution for the robust equalizer.

1: Initialize the first search ellipsoid $\{\bar{\boldsymbol{g}}_0 + \boldsymbol{B}_0\boldsymbol{u}, \|\boldsymbol{u}\|_2 \leq 1\}$. Set $t = 0$ (step number).
2: **repeat**
3: $t \leftarrow t + 1$
4: $\bar{\boldsymbol{g}}_{t-1} \notin \mathcal{D}$: Calculate $\boldsymbol{a}_t$ by examining the uncertain constraints in (6) for $k = 1, \ldots, K$. Skip the following step.
5: $\bar{\boldsymbol{g}}_{t-1} \in \mathcal{D}$: Calculate the vector $\boldsymbol{a}_t$ as the subgradient of the objective function in $\bar{\boldsymbol{g}}_{t-1}$. If $\boldsymbol{a}_t = \mathbf{0}$ the optimal solution is $\bar{\boldsymbol{g}}_{t-1}$.
6: Update the search ellipsoid:

$$\boldsymbol{x}_t = \frac{\boldsymbol{B}_{t-1}^T\boldsymbol{a}_t}{\sqrt{\boldsymbol{a}_t^T\boldsymbol{B}_{t-1}\boldsymbol{B}_{t-1}^T\boldsymbol{a}_t}}, \quad \bar{\boldsymbol{g}}_t = \bar{\boldsymbol{g}}_{t-1} - \frac{1}{d+1}\boldsymbol{B}_{t-1}\boldsymbol{x}_t,$$

$$\boldsymbol{B}_t = \frac{d}{\sqrt{d^2-1}}\boldsymbol{B}_{t-1} + \left(\frac{d}{d+1} - \frac{d}{\sqrt{d^2-1}}\right)\boldsymbol{B}_{t-1}\boldsymbol{x}_t\boldsymbol{x}_t^T,$$

7: **until** : convergence is reached

Finally, we formulate the main conclusion of this section as a theorem:

Theorem 1 *The algorithm from Table 1 converges to the optimal solution of the problem (6).*

4. VIRTUAL MSE-OPTIMIZATION PROBLEM

Consider now a variation of the main problem, where instead of the uncertain minimum SINR constraints, maximum MSEs under uncertainty must be satisfied:

$$\begin{aligned} &\min_{\boldsymbol{G}, p_1, \ldots, p_K} \quad \|\boldsymbol{G}\|_F^2, \\ &\mathrm{MSE}_k \leq q_k, \quad \forall \|\boldsymbol{\Delta}_{(k,:)}\|_2 \leq \varepsilon_k \quad \forall k \in \{1, \ldots, K\}, \end{aligned} \tag{15}$$

where $\mathrm{MSE}_k = \mathrm{E}(|s_k - \hat{s}_k|^2)$ and $q_k \in (0,1)$ (notice that other values for q_k would not make sense). Clearly, for calculating MSEs, the scaling factors p_k are of interest [14, 8]. It is shown in [8] that the following claim holds:

Theorem 2 *The problem (15) is equivalent to the SDP problem*

$$\begin{aligned} &\min_{\boldsymbol{G}, p_1, \ldots, p_k, \lambda_1, \ldots, \lambda_K} \|\boldsymbol{G}\|_F^2, \\ &\begin{bmatrix} p_k\sqrt{q_k} - \lambda_k & \hat{\boldsymbol{H}}_{(k,:)}\boldsymbol{G} - p_k\boldsymbol{e}_k^* & \sigma_k & \mathbf{0} \\ \boldsymbol{G}^*\hat{\boldsymbol{H}}_{(k,:)}^* - p_k\boldsymbol{e}_k & p_k\sqrt{q_k}\boldsymbol{I} & \mathbf{0} & -\varepsilon_k\boldsymbol{G}^* \\ \sigma_k & \mathbf{0} & p_k\sqrt{q_k} & \mathbf{0} \\ \mathbf{0} & -\varepsilon_k\boldsymbol{G} & \mathbf{0} & \lambda_k\boldsymbol{I} \end{bmatrix} \\ &\succeq \mathbf{0}, \quad \forall k \in \{1, \ldots, K\}, \end{aligned} \tag{16}$$

where λ_k are slack variables and $\boldsymbol{e}_k$ make the standard basis of $\mathbb{R}^K$.

We are now in position to approach the problem (6) indirectly.

Theorem 3 *If the MSE problem (15) is feasible with the targets $q_k \in (0,1)$, $k = 1 \ldots K$, and if $\boldsymbol{G}_{\mathrm{opt}}$ is the resulting optimal transmit filter, the SINR constraints in (6) are satisfied for $\gamma_k = \frac{1}{q_k} - 1$, $k = 1 \ldots K$, with the same transmit filter $\boldsymbol{G}_{\mathrm{opt}}$.*

Proof: It can be seen that the condition $\mathrm{MSE}_k \leq q_k$ is equivalent to

$$\sum_{l=1,\, l\neq k}^{K} \left|\frac{1}{p_k}\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,l)}\right|^2 + \left|\frac{1}{p_k}\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)} - 1\right|^2 + \frac{\sigma_k^2}{p_k^2} \leq q_k. \tag{17}$$

Since $p_k > 0$ and $q_k \in (0,1)$, from the term in the middle of the left hand-side of (17) it follows that $\Re\{\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)}\} \geq 0$. Therefore, it is sufficient to prove that $\mathrm{MSE}_k \leq q_k$ implies (notice that $1 + \gamma_k^{-1} = (1-q_k)^{-1}$)

$$\left\| \begin{bmatrix} \boldsymbol{H}_{(k,:)}\boldsymbol{G} & \sigma_k \end{bmatrix} \right\|_2^2 \leq \frac{1}{1-q_k}\Big(\Re\{\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)}\}\Big)^2. \tag{18}$$

We proceed by rewriting (17) in an equivalent form

$$\begin{aligned} &\left\| \begin{bmatrix} \boldsymbol{H}_{(k,:)}\boldsymbol{G} & \sigma_k \end{bmatrix} \right\|_2^2 \\ &\leq p_k^2 q_k + \left|\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)}\right|^2 - \left|\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)} - p_k\right|^2. \end{aligned} \tag{19}$$

Denote with D the right hand-side of (19). Finally, the theorem is proved by the fact that

$$D \leq \frac{1}{1-q_k}\Big(\Re\{\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)}\}\Big)^2 \tag{20}$$

is always true, because (20) is equivalent to

$$\Big(\Re\{\boldsymbol{H}_{(k,:)}\boldsymbol{G}_{(:,k)}\} - p_k(1-q_k)\Big)^2 \geq 0. \tag{21}$$

■

Therefore, by exploiting the specific structure of the considered problem, an involved uncertain conic problem with ellipsoidal uncertainty can be conservatively solved by SDP methods (16), which are known to exhibit significantly better convergence properties [6]. Notice that the well-known relation between MSE and SINR used in Theorem 3 yields equivalent problems in the sense that the same transmit filter is obtained, if there is no uncertainty [14]. Theorem 3 implies that this relation connects, at least to a certain extent, the robust counterparts of the standard beamforming problems, as well.

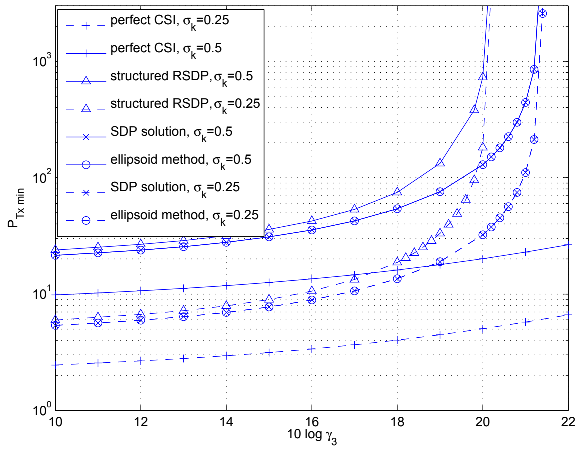

Fig. 2. Minimal transmit power for various SINR targets γ_3.

5. NUMERICAL EXAMPLE

A numerical simulation of a system with $M = K = 3$ is obtained using SeDuMi [15]. One randomly chosen, estimated (erroneous) channel is kept constant for all simulations to illustrate the typical performance and avoid requirements on the feasibility:

$$\hat{\boldsymbol{H}} = \begin{bmatrix} 0.12 + 0.72j & 1.19 - 0.13j & 0.32 + 0.05j \\ 0.28 - 0.58j & 1.18 + 0.11j & 0.17 - 0.09j \\ -1.14 + 2.18j & -0.03 + 1.06j & -0.18 - 0.83j \end{bmatrix}.$$

The SINR targets $\gamma 1$ and γ_2 of the first and the second user are set to 10. In Fig. 2, the minimal transmit power for various SINR targets γ_3 of the third user and two noise powers (assumed fixed for all users) is shown. The bounds on the uncertainty for all three channels are assumed to be $\varepsilon_k = 0.1$. It can be seen that our SDP-based solution outperforms in performance the robust SDP-based method that accounts for structured uncertainty (structured RSDP) from [5], which solves the problem (6) suboptimally. Furthermore, the size of the LMI and the number of additional slack variables corresponding to the kth user constraint are $2(K+1)(2M+1)$ and $(K+1)(2K+3)$ in [5], respectively, which is significantly larger comparing to $2(K+M+2)$ (real-valued SDP representation) and 2 for the same parameters in (16). Finally, the conservative solution from Section 4 seems to match perfectly with the performance of the optimal ellipsoid method, both in the minimal transmit power (the obtained transmit filter) and the feasibility region where the SINR target γ_3 can be supported. However, though we can report the same conclusions for a large number of channels/scenarios we have examined, we still miss the analytical proof of the eventual global optimality of the proposed SDP method.

6. CONCLUSION

Robust precoder design in a SINR-constrained downlink multiuser MISO system has been studied. The problem of interest was firstly solved optimally by noticing that it belonged to a class of uncertain conic problems for whom the ellipsoid method could be applied. Then, a conservative, SDP-based solution was derived by exploiting the structure of the problem. This enables the application of much more efficient numerical tools, while the performance seems not to be degraded. A proof that the virtual MSE problem from Section 4 and the problem (6) are equivalent in the sense that the same transmit filter is obtained, and a relation to the intricate problem of minimizing the total transmit power subject to the constraints (10) remain as interesting topics for the future work.

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