KERDOCK CODES FOR LIMITED FEEDBACK MIMO SYSTEMS

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ABSTRACT

A codebook based limited feedback strategy is a practical way to obtain partial channel state information at the transmitter in a precoded multiple-input multiple-output (MIMO) wireless systems. Construction of conventional codebooks, such as Grassmannian and Fourier codebooks, relies on nonlinear search and iterative algorithms which often does not exhibit structure to ease storage or online search computation. Furthermore, multiple codebooks are needed to support beamforming and spatial multiplexing. In this paper, we propose a new codebook design based on Kerdock codes and mutually unbiased bases which enjoys performance similar to previously known codebooks. The proposed Kerdock codebook with quaternary alphabet has systematic construction, reduced storage, and reduced online search computation. Special structure in the codebook is used to derive a spatial multiplexing codebook from multiple columns of the beamforming codebook resulting in further storage reduction.

Index Terms— Array signal processing, MIMO systems, codes, feedback communication.

1. INTRODUCTION

Codebook based limited feedback is a practical method to obtain channel state information (CSI) at the transmitter in a multiple-input multiple-output (MIMO) communication system [1]. Conventional codebook design has focused on the achievable capacity and bit error performance without fully addressing practical considerations such as systematic construction, storage, and search complexity [1–5]. In practical systems such as 3GPP long term evolution (3GPP-LTE), multiple codebooks of different sizes are being considered which further increases the storage requirements [6]. Furthermore, up to 300km/h mobility is being considered in 3GPP-LTE which necessitates fast search for the optimum codeword. Thus, there has been increasing interest in developing a codebook with structure to ease the storage and search requirements. The central topic of this paper is to connect Kerdock codes to limited feedback codebook design and address practical considerations.

Conventional codebook designs such as Grassmannian [2], equiangular frames [7], and vector quantization (VQ) based [4] codebooks typically require extensive numerical search or iterative algorithms to design the codebook. While the asymptotic capacity and bit error performance as a function of codebook size have been shown to be near optimal, the codebook usually does not exhibit any structure so that the codebook entries must be stored element by element. Furthermore, these codebooks requires an exhaustive online search with complex-valued matrix computation. One structured codebook is the Fourier based codebook based on unitary space-time codes [8]. The Fourier based codebook only requires a generator matrix and a discrete Fourier transform (DFT) matrix to generate the codebook. Quantization still requires exhaustive search with complex matrix computations. To minimize the storage and online search complexity, a quadrature amplitude modulation (QAM) symbol based codebook design was proposed [5]. This codebook is essentially an element by element quantization of the optimal precoder to the nearest QAM constellation with maximum likelihood lattice decoding. Unfortunately, the codebook size is generally larger than other codebook designs which makes it less attractive for practical use.

In this paper, we propose to use Kerdock codebooks as a solution to the problem of practical limited feedback precoding in MIMO systems. The Kerdock codebook is constructed using mutually unbiased bases which is intimately related to Grassmannian packing and equiangular frames [7, 9]. We consider the practicality of the codebook by jointly considering 1) system performance, 2) construction and storage requirements, and 3) online search efficiency. We show that the Kerdock codebook can achieve performance comparable to previously known codebooks with further benefits of systematic construction, quaternary alphabet, reduced storage, and reduced search complexity. Furthermore, we propose a new precoding codebook for spatial multiplexing derived from the beamforming codebook thereby further reducing the storage requirements.

For notation, we use lower case bold letters, *e.g.* $\mathbf{v}$, to denote vectors and upper case bold letters, *e.g.* $\mathbf{H}$, to denote matrices. The $n \times n$ identity matrix is denoted by $\mathbf{I}_n$. The space of real and complex are denoted by $\mathbb{R}$ and $\mathbb{C}$, respectively with an appropriate superscript to denote the dimension of the respective spaces. We use ^{T} and * to denote the transposition and Hermitian transpose, respectively.

2. SYSTEM OVERVIEW

In this section, we discuss the limited feedback precoded MIMO system model, codebook selection criteria, and distance criteria considered in this paper.

We consider a limited feedback precoded MIMO wireless system with M_t transmit antennas and M_r receive antennas. The transmit bit stream is sent to the encoder and modulator, which outputs a complex transmit vector, $\mathbf{s}[k] = [s_1[k], s_2[k], \ldots, s_{M_s}[k]]^T$, where k denotes the time index and M_s denotes the number of streams to be sent. Beamforming is the special case for $M_s = 1$, and $1 < M_s \leq$

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M_t for spatial multiplexing. We assume that $E_{\mathbf{s}}[\mathbf{s}\mathbf{s}^*] = \frac{\mathcal{E}_s}{M_s}\mathbf{I}_{M_s}$ in order to constrain the average transmit power, where $E_{\mathbf{s}}$ is used to denote the expectation with respect to the transmit vector and $\mathcal{E}_s$ is used to denote the total transmit power.

The transmit vector $\mathbf{s}[k]$ is then multiplied by the unitary precoder $\mathbf{F}$ ($\mathbf{f}$ for beamforming) of size $M_t \times M_s$ producing a length M_t transmit vector $\mathbf{x}[k] = \sqrt{\mathcal{E}_s/M_s}\mathbf{F}\mathbf{s}[k]$. The unitary precoder $\mathbf{F}$ is selected based on the limited feedback information from the receiver.

Assuming perfect synchronization, sampling, and a linear memoryless channel, the equivalent baseband input-output relationship can be written as

$$\mathbf{y}[k] = \sqrt{\frac{\mathcal{E}_s}{M_s}}\mathbf{H}\mathbf{F}\mathbf{s}[k] + \mathbf{n}[k] \quad (1)$$

where $\mathbf{H}$ is the channel matrix with each entry independent and identically distributed (i.i.d.) according to $\mathcal{CN}(0, 1)$ and $\mathbf{n}[k]$ is the noise vector with each entry i.i.d. with distribution according to $\mathcal{CN}(0, N_0)$. The receive vector $\mathbf{y}[k]$ is then decoded by assuming a perfect knowledge of $\mathbf{HF}$ at the receiver to produce the output vector $\hat{\mathbf{s}}[k]$.

We assume that the receiver has the perfect estimate of the channel matrix $\mathbf{H}$ and uses a linear receiver which applies an $M_s \times M_r$ matrix $\mathbf{G}$ to the receive symbol $\mathbf{y}[k]$. For spatial multiplexing, the zero-forcing (ZF) receiver is used which is given by $\mathbf{G} = (\mathbf{HF})^\dagger$ with $(\cdot)^\dagger$ denoting the Moore-Penrose pseudo inverse. For beamforming, a maximum ratio combining (MRC) receiver is assumed with $\mathbf{G} = (\mathbf{Hf})^*$.

In this paper, the receiver chooses the unitary precoder $\mathbf{F}$ from the codebook, $\mathcal{F} = \{\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_N\}$, of size N shared by the transmitter and the receiver. The index of the selected codeword at the receiver is fed back to the transmitter through a zero-delay limited capacity feedback channel. The index of the codeword, $n = 1, 2, \ldots, N$, is represented by b-bit binary ($N = 2^b$) resulting in b bits of feedback. We say that the codebook is *b-bit codebook* when it has $N = 2^b$ entries.

For beamforming, the receiver finds the beamforming vector that maximizes the effective SNR [2],

$$\mathbf{f} = \arg\max_{\mathbf{w}\in\mathcal{F}} \|\mathbf{H}\mathbf{w}\|_2. \quad (2)$$

We will consider this selection criteria to determine the search complexity associated with beamforming.

For spatial multiplexing, the minimum singular value selection criteria (MSV-SC)

$$\mathbf{F} \quad = \quad \arg\max_{\mathbf{W}\in\mathcal{F}} \lambda_{\min}\{\mathbf{H}\mathbf{W}\} \quad (3)$$

is used [3]. This selection criteria approximately maximizes the minimum substream SNR. We will consider this selection criteria to determine the search complexity for spatial multiplexing.

We are also interested in deriving the spatial multiplexing codebook from the beamforming codebook. For MSV-SC, the codebook should be designed by maximizing the minimum pairwise projection 2-norm distance [3]. The projection 2-norm is defined as

$$d_{p2}(\mathbf{F}_1, \mathbf{F}_2) \quad = \quad \|\mathbf{F}_1\mathbf{F}_1^* - \mathbf{F}_2\mathbf{F}_2^*\| = \sqrt{1 - \lambda_{\min}\{\mathbf{F}_1^*\mathbf{F}_2\}}. (4)$$

We will utilize this distance between codewords to derive the spatial multiplexing codebook from the beamforming codebook.

3. KERDOCK CODEBOOK

Kerdock codes are known in coding theory community as a nonlinear binary code containing more codewords than any known linear codes [10]. They are constructed as binary images under the Gray map of linear codes over $\mathbb{Z}_4$, the integer modulo 4, resulting in quaternary alphabet. A simplified Kerdock code construction was proposed in connection with CDMA signature sequence and mutually unbiased bases (MUB) [11]. An MUB is a collection of two or more orthonormal bases (ONB) with the property that the members of different ONBs has the same correlation. That is, if $\mathbf{S} = [\mathbf{s}_1 \ldots \mathbf{s}_{M_t}]$ and $\mathbf{U} = [\mathbf{u}_1 \quad \ldots \quad \mathbf{u}_{M_t}]$ are two $M_t \times M_t$ ONBs (*i.e.* $\mathbf{S}^*\mathbf{S} = \mathbf{I}_{M_t}$), the inner product of vectors drawn from each ONB satisfies, $|\langle \mathbf{s}_n, \mathbf{u}_m\rangle| = \frac{1}{\sqrt{M_t}}$, for $n, m = 1, \ldots, M_t$. The Kerdock code is a class of MUB which meets the Welch bound equality (WBE) [9, 12] and it can be linked to equiangular frames [7] and complex projective space [13]. The connection of Kerdock codes with various limited feedback codebook designs were established in [9]. We adopt the simplified construction in [11] and show the utility of Kerdock codes for limited feedback precoded MIMO systems.

3.1. Kerdock Codebook Construction

A Kerdock code consists of L, some power of two, orthonormal matrices, $\mathbf{S}_n$, $n = 0, \ldots, L-1$, where each $\mathbf{S}_n$ is a rotated Sylvester-Hadamard matrix. The key to the construction is the algebraic derivation of the rotating (or generator) matrices, $\mathbf{D}_n$, which does not rely on any search. The general strategy for the Kerdock codebook construction is as follows

1. Construct the diagonal matrices, $\mathbf{D}_n$, for $n = 0, 1, \ldots, L-1$. These are the generator matrices.
2. Each bases is constructed by $\mathbf{S}_n = (1/\sqrt{M_t})\mathbf{D}_n\hat{\mathbf{H}}_L$ where $\hat{\mathbf{H}}_L$ is a size $L \times L$ Sylvester-Hadamard matrix.
3. Let $\hat{\mathbf{F}} = [\mathbf{S}_0 \quad \mathbf{S}_1 \quad \cdots \quad \mathbf{S}_{L-1}]$.
4. For beamforming, each column of $\hat{\mathbf{F}}$ becomes the beamforming vector $\mathbf{f}_1, \ldots, \mathbf{f}_N$.
5. For spatial multiplexing, select specific column combination based on its distance property to form the codebook.

Due to space limitations, please refer to [9, 11] for detailed derivation of the generator matrices, $\mathbf{D}_n$. Having computed the set of $\mathbf{S}_n$, we arrange them into a codebook as follows. For beamforming, construct the composite matrix, $\hat{\mathbf{F}} = [\mathbf{S}_0 \quad \mathbf{S}_1 \quad \cdots \quad \mathbf{S}_n]$ and define the codebook as the columns of $\hat{\mathbf{F}}$,

$$\mathcal{F} = \{\mathbf{f}_1 = [\hat{\mathbf{F}}]_1, \mathbf{f}_2 = [\hat{\mathbf{F}}]_2, \ldots, \mathbf{f}_N = [\hat{\mathbf{F}}]_N\}, \quad (5)$$

where $[\cdot]_n$ is used to denote the selection of n-th column of a given matrix. In [9], a construction method is proposed in which identity matrix (non-quaternary) can be included in the codebook.

For spatial multiplexing, a subset of columns are selected from each $\mathbf{S}_n$ to form the codebook. Notice that for a subset of columns drawn from a single $\mathbf{S}_n$, each column is orthogonal to each other because each $\mathbf{S}_n$ is an ONB. Hence, the design problem is to take a column subset from each $\mathbf{S}_n$ so that the minimum of pairwise distance (4) is maximized. Specifically, for M_s stream spatial multiplexing codebook, take all M_s-column subsets from each $\mathbf{S}_n$. There are $\binom{M_t}{M_s}$ column subset combinations in each $\mathbf{S}_n$. We select a unique combinations from each $\mathbf{S}_n$ so that each column is used only once

Table 1. Unitary precoding codebook for 2 streams derived from the beamforming codebook. The column selection in the subscript is chosen to maximize the pairwise projection 2-norm distance.

$[\mathbf{S}_0]_{1,2}$	$[\mathbf{S}_0]_{3,4}$	$[\mathbf{S}_1]_{1,3}$	$[\mathbf{S}_1]_{2,4}$
$[\mathbf{S}_2]_{1,4}$	$[\mathbf{S}_2]_{2,3}$	$[\mathbf{S}_3]_{1,4}$	$[\mathbf{S}_3]_{2,3}$

and that the pairwise distance is maximized using exhaustive search. Using this construction, we can derive a size $N/2$ codebook for $M_s = M_t/2$ and $N = M_t$ codebook for $M_t \geq M_s > M_t/2$ from an N-entry beamforming codebook. We do not, however, claim optimality of the so constructed spatial multiplexing codebook.

3.2. Example: Four Transmit Antenna Construction

In this section, we provide the Kerdock codebook for $M_t = 4$ using the proposed construction. Our simulation results in Section 5 utilizes the codebook constructed in this example. For $M_t = 4$, we have $L = M_t = 4$. Computing $\mathbf{S}_n = (1/\sqrt{M_t})\mathbf{D}_n\hat{\mathbf{H}}_L$ yields

$$\begin{aligned}&\begin{bmatrix}\mathbf{S}_0 & \mathbf{S}_1\end{bmatrix} = \\ &\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1 & j & -j & j & -j\\ 1 & 1 & -1 & -1 & j & j & -j & -j\\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1\end{bmatrix},\\ &\begin{bmatrix}\mathbf{S}_2 & \mathbf{S}_3\end{bmatrix} = \\ &\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1 & j & -j & j & -j\\ j & j & -j & -j & 1 & 1 & -1 & -1\\ -j & j & j & -j & -j & j & j & -j\end{bmatrix}. \quad (6)\end{aligned}$$

For the beamforming codebook, we construct the composite matrix, $\hat{\mathbf{F}} = \begin{bmatrix}\mathbf{S}_0 & \mathbf{S}_1 & \mathbf{S}_2 & \mathbf{S}_3\end{bmatrix}$ and define the codebook as the columns of $\hat{\mathbf{F}}$. That is,

$$\mathcal{F} = \{\mathbf{f}_1 = [\hat{\mathbf{F}}]_1, \mathbf{f}_2 = [\hat{\mathbf{F}}]_2, \ldots, \mathbf{f}_N = [\hat{\mathbf{F}}]_{16}\}, \quad (7)$$

for a $N = 16$, 4-bit codebook.

For the 2-stream unitary precoding spatial multiplexing codebook, we shall construct a 3-bit codebook. By selecting two columns from each $\mathbf{S}_n$ and maximizing the distance (4) between every possible codeword pair, we find the column selections in Table 1, where $[\cdot]_{x,y}$ is used to denote the selection of the x-th and y-th column of a given matrix. Similar procedure can be applied to obtain 2-bit codebook for 3-stream spatial multiplexing. We have thus obtained a quaternary alphabet codebook which can be used for both beamforming and spatial multiplexing.

4. CODEBOOK STORAGE AND SEARCH COMPLEXITY

In this section, we quantify the storage and search complexity of the proposed Kerdock codebook and compare it with Grassmannian and Fourier based designs.To estimate the storage requirements, we consider the number of real elements (*i.e.* two real elements for one complex value) to store a codebook. Let N_b denote the number of bits available in the system to represent a real number.

The Grassmannian codebook [2,3] does not yield any systematic construction so the entire codebook, element by element, must be stored. Thus, the storage requirement is $2N_bNM_tM_s$-bits for each codebook. The Fourier based codebook [8] requires the storage of one diagonal generator matrix (*i.e.* M_t complex entries) and $M_t \times M_s$ entries of a DFT matrix. The storage requirement for Fourier based codebook is $2N_b(M_t + M_tM_s)$-bits. Note that the storage requirement is independent of the codebook size, N, because the generator matrix is designed for a given codebook size.

The proposed Kerdock codebook requires storage of $M_t \times M_t$ generator matrix elements, each represented by 2-bits, and 2×2 Sylvester-Hadamard matrix with binary entries. The total storage requirement is thus $2M_t^2 + 4$. Note that the storage for Kerdock codebook is independent of the N_b used in the system. Furthermore, the same codebook can be used for different modes of transmission to further reduce the storage requirements.

For a fair comparison, Table 2 shows the number of bits required to store the Kerdock, Fourier, and Grassmannian codebook for $M_t = 4$ using $N = 16$ for beamforming and $N = 8$ for 2-stream unitary precoded spatial multiplexing. The Kerdock codebook results in a significant storage savings.

Table 2. Number of bits required for storing Kerdock, Fourier, and Grassmannian codebooks for $M_t = 4$ and using $N = 16$ for beamforming and $N = 8$ for 2-stream spatial multiplexing. A system dependent number of bits used to represent a real number is denoted by N_b.

Kerdock	Fourier	Grassmannian
36	$40N_b$	$256N_b$

For search complexity, we will consider the number of arithmetic computation required to arrive at the desired codeword. Since the norm and singular value computations are common for all codebook entries, we compare the computation required to compute $\mathbf{Hf}$ for (2) and $\mathbf{HF}$ for (3) for each codeword in the codebook. For the Grassmannian and Fourier codebooks, the entries of $\mathbf{f}$ and $\mathbf{F}$ are complex valued thus requiring complex multiplies and additions. The Kerdock codebook, though, has entries $\{\pm 1, \pm j\}$ which reduces the complex multiplications to a sign change or swapping the real and imaginary parts. Therefore, the Kerdock codebook effectively achieves multiplier-less computation of $\mathbf{Hf}$ and $\mathbf{HF}$.

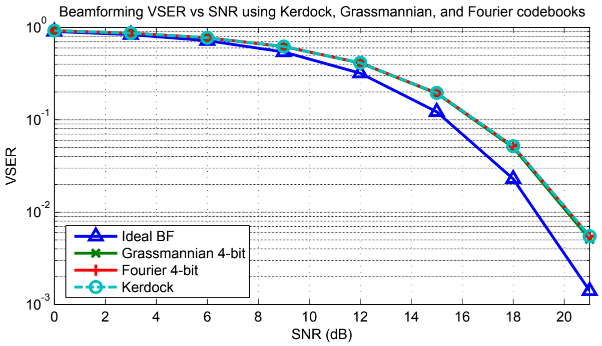
Table 3 shows the required number of arithmetic computation at the receiver for codeword selection. The reduced computational cost and compact storage properties of Kerdock codebook makes it an attractive solution for implementation.

Table 3. Comparison of codeword selection computational requirement for the proposed design and equivalent Grassmannian and Fourier codebooks

Beamforming Selection		
	Grassmannian or Fourier	Kerdock
Multiply	NM_tM_r	0
Addition	$NM_r(M_t - 1)$	$NM_r(M_t - 1)$
Spatial Multiplexing: MSV-SC		
	Grassmannian or Fourier	Kerdock
Multiply	$NM_tM_rM_s$	0
Addition	$NM_sM_r(M_t - 1)$	$NM_sM_r(M_t - 1)$

5. SIMULATION RESULT

In this section, we give numerical simulation results comparing 1) vector symbol error rate (VSER) performance of limited feedback



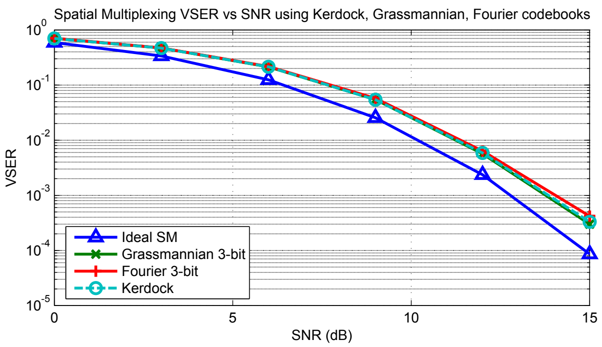


Fig. 1. VSER performance of $M_t = M_r = 4$ beamforming system using 256-QAM with all codebooks of size $N = 16$ and VSER performance of $M_t = M_r = 4$ unitary precoded spatial multiplexing system using 64-QAM with all codebooks of size $N = 8$.

beamforming system, and 2) VSER performance of two stream unitary precoded spatial multiplexing system for equal size Grassmannian, Fourier, Kerdock codebooks, and ideal beamforming and spatial multiplexing using perfect CSI. All simulations are performed for $M_t = M_r = 4$ assuming delay-free feedback. No forward error correction is used.

The VSER performances are shown in Fig. 1. The case for perfect CSIT are also shown for reference. For beamforming, the Kerdock codebook yields performance indistinguishable from Grassmannian and Fourier codebooks. For spatial multiplexing, the Kerdock codebook outperforms the Fourier codebook, but is marginally worse than the Grassmannian codebook. Therefore, the Kerdock codebook provides comparable performance with same size Grassmannian and Fourier codebooks. This is remarkable considering the fact that the codebook only has quaternary alphabet.

6. CONCLUSION

In this paper, we proposed to use Kerdock codes for limited feedback precoded MIMO systems. The Kerdock codebook has quaternary alphabet, systematic construction, reduced storage requirements, and results in a reduced computational search. We proposed a method to derive spatial multiplexing precoders from the beamforming codebook. The mutually unbiased bases structure of the Kerdock codebook enabled the codebook derivation leading to an extra storage savings. We presented numerical simulation results which showed that the Kerdock codebook provides performance comparable or better than previously known codebooks. We also showed that Kerdock codebook results in significant storage and search savings. More results and proofs are provided in [9]. One limitation of the proposed design is the limited number of codewords in a given codebook. Future work will consider possible extension to multiuser MIMO systems, MIMO-OFDM systems, and applicability to correlated channel scenarios.

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