LIMITED FEEDBACK DESIGN FOR V-BLAST

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ABSTRACT

We present a quantized precoding scheme for the V-BLAST system. The precoding matrix is decomposed into the multiplication of a beamforming matrix and a power allocation matrix. They are obtained by solving an optimization problem that maximizes the minimum post-detection SINR of the MMSE-SIC receiver under a total power constraint. The beamforming matrix is constrained to be in a predetermined codebook. We show that the precoder of UCD-VBLAST happens to be the optimal solution of the problem. Therefore, the proposed scheme can be regarded as a natural but nontrivial limited feedback version of UCD-VBLAST. Simulation results are presented to demonstrate the effectiveness of the proposed scheme.

***Index Terms*—** V-BLAST, MMSE-SIC, beamforming, power allocation, limited feedback

1. INTRODUCTION

Multiple antennas are being considered for high data rate services on rich scattering wireless channels because they promise high spectral efficiencies. The Vertical Bell Laboratories Layered Space-Time (V-BLAST) architecture was designed to take the spectral efficiency advantage of the multiple-antenna system [1]. It has been demonstrated to be able to exploit the potentially enormous MIMO link capacity.

When perfect channel state information (CSI) is available at the transmitter, the V-BLAST system can be optimized to increase spectral efficiency or enhance the link reliability. Linear precoding is a simple and effective means to accomplish either or both of the two goals. An efficient linear precoding approach based on the uniform channel decomposition (UCD) was proposed in [2]. By combining the UCD channel decomposition and minimum mean square error successive interference cancellation (MMSE-SIC) receiver, the UCD-VBLAST[1] decomposes a MIMO channel into multiple identical parallel subchannels. The UCD-VBLAST has been proved to be optimal in terms of both spectral efficiency and link reliability aspects. However, despite the capability of UCD-VBLAST, it assumes that the transmitter has perfect CSI, which is often unrealistic. In some practical systems, the CSI is sent to the transmitter through a feedback channel whose capacity is limited. In order to overcome this difficulty, we propose a limited feedback version of UCD-VBLAST, which is denoted as QUCD.

Limited feedback design has been explored in [3, 4, 5, 6] for single-stream beamforming and proposed in [7, 8, 9] for multiple-stream precoding. The basic idea behind limited feedback is to select the precoding matrix at receiver from a predetermined finite-element codebook according to some criterion. After that, the index of the selected matrix is sent to the transmitter through the finite rate feedback channel. Previous work on limited feedback has primarily focused on either maximizing the information rate or minimizing the error rate, hence, there is an apparently inevitable tradeoff between the information rate and error rate if the same coding and symbol constellation is used on different data streams. In this paper, we will introduce the QUCD scheme and clarify that there is not necessarily a tradeoff between information rate and error rate. Indeed, the QUCD scheme attempts to achieve the best of both worlds simultaneously. In the proposed QUCD scheme, the index of the selected beamforming matrix is sent to transmitter through the feedback channel. The power allocation matrix is a real diagonal matrix, and hence its elements can be simply scalar quantized and feedback.

This work was supported by the National Natural Science Foundation of China under Grant 60675002 and 60496311, and funded by Basic Research Foundation of Tsinghua National Laboratory for Information Science and Technology and State Key Laboratory on Intelligent Technology and Systems.

[1]For convenience, the nonlinear transceiver proposed in [2] will be referred to as UCD-VBLAST in this paper.

2. BACKGROUND

We consider a communication system with N_T transmit antennas and N_R receive antennas in a block Rayleigh flat fading channel. At the transmitter, the input binary data stream is demultiplexed into M data streams, each of which is coded and modulated independently. Let $\boldsymbol{x} = [x_1, \cdots, x_M]^T$ be the modulated data symbol vector with a covariance matrix $\mathbb{E}\{\boldsymbol{x}\boldsymbol{x}^H\} = \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix of proper dimensions. The complex baseband system model can be represented by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{z} = \boldsymbol{H}\boldsymbol{F}\sqrt{\boldsymbol{Q}}\boldsymbol{x} + \boldsymbol{z} \tag{1}$$

where $\boldsymbol{y} = [y_1, \cdots, y_{N_R}]^T$ is the received signal vector; $\boldsymbol{z} = [z_1, \cdots, z_{N_R}]^T$ is the additive white Gaussian noise vector, with $E\{\boldsymbol{z}\boldsymbol{z}^H\} = \sigma^2\boldsymbol{I}$; $\boldsymbol{H}$ is the $N_R \times N_T$ channel matrix, whose elements h_{ij} are assumed to be i.i.d zero-mean complex Gaussian variables with unit variance. We assume that the channel can be accurately estimated at the receiver. $\boldsymbol{G}$ is the $N_T \times M$ linear precoding matrix satisfying the total transmit power constraint $\mathrm{trace}(\boldsymbol{G}\boldsymbol{G}^H) \leq P_0$. The linear precoding matrix can be decomposed into the multiplication of a beamforming matrix and a power allocation matrix, i.e., $\boldsymbol{G} = \boldsymbol{F}\sqrt{\boldsymbol{Q}}$, where $\boldsymbol{F} = [\boldsymbol{f}_1, \cdots, \boldsymbol{f}_M]$ is the beamforming matrix; each column vector $\boldsymbol{f}_i$ is the beamforming vector of the i-th data stream and $\|\boldsymbol{f}_i\| = 1$; $\boldsymbol{Q} = \mathrm{diag}\,[q_1, q_2, \cdots, q_M]$ and $\sqrt{\boldsymbol{Q}} = \mathrm{diag}\left[\sqrt{q_1}, \sqrt{q_2}, \cdots, \sqrt{q_M}\right]$ in which $q_i \geq 0$ is the transmission power of the i-th data stream . A noiseless zero-delay feedback channel with limited capacity is used to convey information to the transmitter.

Next we give a brief overview of MMSE-SIC receiver for the V-BLAST system. Suppose that the successive interference cancellation is ideal, i.e., there is no error propagation. Without loss of generality, we also assume that stream M is detected first and stream 1 is detected at last. At detection stage k, the receiver attempts to recover the transmitted data of stream k. The ideal interference cancellation implies that stream k suffers from the interference of later streams $1, \cdots, k-1$ only and the interference of earlier streams $k+1, \cdots, M$ has been cancelled. Since the data symbol vector $\boldsymbol{x}$ and the noise $\boldsymbol{z}$ are independent, the covariance matrix of the interference plus noise suffered by stream k is given as

$$\boldsymbol{R}_k = \sigma^2 \boldsymbol{I} + \sum_{i=1}^{k-1} q_i \boldsymbol{H} \boldsymbol{f}_i \boldsymbol{f}_i^H \boldsymbol{H}^H \tag{2}$$

A linear filter $\boldsymbol{u}_k$ is applied to suppress the interference and noise suffered by stream k. Thus, the post-detection SINR of the k-th detection stage is specified by

$$\mathrm{SINR}_k(\boldsymbol{F}, \boldsymbol{Q}) = \frac{q_k |\boldsymbol{u}_k^H \boldsymbol{H} \boldsymbol{f}_k|^2}{\boldsymbol{u}_k^H \boldsymbol{R}_k \boldsymbol{u}_k} \tag{3}$$

The parentheses with enclosed arguments indicate that the post-detection SINR is a function of both the beamforming matrix $\boldsymbol{F}$ and power allocation matrix $\boldsymbol{Q}$. For a MMSE-SIC receiver, $\boldsymbol{u}_k$ is designed according to the MMSE criterion and is given by

$$\boldsymbol{u}_k(\boldsymbol{F}, \boldsymbol{Q}) = \sqrt{q_k} (\boldsymbol{R}_k + q_k \boldsymbol{H} \boldsymbol{f}_k \boldsymbol{f}_k^H \boldsymbol{H}^H)^{-1} \boldsymbol{H} \boldsymbol{f}_k \tag{4}$$

Substituting this $\boldsymbol{u}_k$ into Eq. (3), followed by some straightforward algebraic manipulations, one gets

$$\mathrm{SINR}_k(\boldsymbol{F}, \boldsymbol{Q}) = q_k \boldsymbol{f}_k^H \boldsymbol{H}^H \boldsymbol{R}_k^{-1} \boldsymbol{H} \boldsymbol{f}_k = q_k \boldsymbol{f}_k^H \boldsymbol{Z}_k \boldsymbol{f}_k \tag{5}$$

where $\boldsymbol{Z}_k \overset{\mathrm{def}}{=} \boldsymbol{H}^H \boldsymbol{R}_k^{-1} \boldsymbol{H}$ and by matrix inversion lemma, we have

$$\boldsymbol{Z}_{k+1} = \boldsymbol{Z}_k - \frac{q_k \boldsymbol{Z}_k \boldsymbol{f}_k \boldsymbol{f}_k^H \boldsymbol{Z}_k^H}{1 + q_k \boldsymbol{f}_k^H \boldsymbol{Z}_k \boldsymbol{f}_k} \tag{6}$$

In a word, the MIMO channel is converted into a set of parallel subchannels and the SNR of the k-th subchannel is SINR_k.

3. UNQUANTIZED PRECODING MATRIX DESIGN

3.1. Problem Formulation

As shown in [10], the MMSE-SIC receiver can achieve the capacity of the MIMO channel. However, in order to achieve this capacity, one has to use a different combination of constellation and coding on each individual stream, since the SINRs of the subchannels are different from each other. As a result, significant complexities to the modulation/demodulation and coding/decoding procedures are introduced. To reduce the complexity, the system can be constrained to use the same constellation and coding scheme on each of the streams. In this alternative scenario, the BER performance is limited by the subchannel with the minimum SINR. In other words, the link reliability is dominated by the worst-case subchannel. Hence, rather than maximizing the mutual information, we design the beamforming matrix $\boldsymbol{F}$ and power allocation matrix $\boldsymbol{Q}$ to maximize the minimum SINR of all subchannels.

The minimum SINR of all subchannels can be maximized by solving the optimization problem

$$\begin{aligned} \max_{\boldsymbol{F}, \boldsymbol{Q}} \min_{k} \quad & \mathrm{SINR}_k(\boldsymbol{F}, \boldsymbol{Q}) \\ \text{s.t.} \quad & \mathrm{trace}(\boldsymbol{Q}) \leq P_0 \text{ and } \boldsymbol{F} \in \mathcal{F} \end{aligned} \tag{7}$$

where $\mathcal{F}$ is a codebook containing N candidate beamforming matrices that is stored at both the transmitter and the receiver. The codebook is denoted as $\mathcal{F} = \{\boldsymbol{F}_1, \cdots, \boldsymbol{F}_N\}$, and each column of $\boldsymbol{F}_i$ is of unit norm.

By introducing a slack variable, we can recast the optimization problem (7) as

$$\begin{aligned} \max_{\boldsymbol{F}, \boldsymbol{Q}, \rho} \quad & \rho \\ \text{s.t.} \quad & \mathrm{SINR}_i(\boldsymbol{F}, \boldsymbol{Q}) \geq \rho, \quad 1 \leq i \leq M \\ & \mathrm{trace}(\boldsymbol{Q}) \leq P_0 \text{ and } \boldsymbol{F} \in \mathcal{F} \end{aligned} \tag{8}$$

It is easy to verify that (7) and (8) are equivalent.

3.2. Characterization of the Optimal Solution

We have formulated the precoder design problem to enhance the link reliability. It happens that the optimal solution of the problem (8) is also optimal in the sense of capacity maximization as stated in Theorem 1 below. Before getting into the theorem, we will first present a lemma which is necessary for the proof of the theorem.

Lemma 1 *Suppose that $(\boldsymbol{F}^*, \boldsymbol{Q}^*, \rho^*)$ is an optimal solution of (8), then the constraints $\mathrm{SINR}_i(\boldsymbol{F}^*, \boldsymbol{Q}^*) \geq \rho^*, i = 1, \cdots M$ are all active, i.e., $\mathrm{SINR}_i(\boldsymbol{F}^*, \boldsymbol{Q}^*) = \rho^*$. Furthermore, ρ^* is continuous and strictly monotonic increasing in P_0.*

Proof: The proof of the first part is straightforward, and hence is omitted here.

To prove the monotonic property of ρ^*, assume that $P_0' > P_0$. The optimal solution under the total power constraints P_0' and P_0 are $(\boldsymbol{F}', \boldsymbol{Q}', \rho')$ and $(\boldsymbol{F}^*, \boldsymbol{Q}^*, \rho^*)$, respectively. There exists $c > 1$ such that $cP_0 < P_0'$ and accordingly

$$\rho' \geq \min_k \mathrm{SINR}_k(\boldsymbol{F}^*, c\boldsymbol{Q}^*) > \rho^* \tag{9}$$

where the second inequality is due to the fact that $\mathrm{SINR}_k(\boldsymbol{F}, c\boldsymbol{Q}^*)$ is a strictly monotonic increasing function of the positive scalar c.

The continuity can be shown by noting that a small increment ΔP_0 can at most result in an increase of ρ^* by

$$\Delta\rho \leq \Delta P_0 \boldsymbol{f}_1^H \boldsymbol{H}^H (\sigma^2 \boldsymbol{I})^{-1} \boldsymbol{H} \boldsymbol{f}_1 \leq \Delta P_0 \lambda_1$$

where λ_1 is the principal eigenvalue of $\boldsymbol{H}^H (\sigma^2 \boldsymbol{I})^{-1} \boldsymbol{H}$. ■

By Lemma 1, the capacity achieved by the MMSE-SIC receiver is then

$$C_{\mathrm{MMSE-SIC}}(\boldsymbol{F}^*, \boldsymbol{Q}^*) = M \log_2(1 + \rho^*)$$

i.e., the MIMO channel is decomposed into M subchannels with identical capacities. The main result of this section is stated below.

Theorem 1 *If $\mathcal{F} = \{\boldsymbol{F} \,|\, \text{each column of } \boldsymbol{F} \text{ is of unit norm}\}$ and $M \geq \mathrm{rank}(\boldsymbol{H})$, then the optimal solution of (8) is also optimal in the sense of capacity maximization, i.e.,*

$$C_{\mathrm{MMSE-SIC}}(\boldsymbol{F}^*, \boldsymbol{Q}^*) = C_{\mathrm{opt}}$$

where C_{opt} is the capacity of the given channel.

Proof: Denote the precoding matrix of UCD-VBLAST as $\boldsymbol{G}_U$ and it can always be decomposed as $\boldsymbol{G}_U = \boldsymbol{F}_U \sqrt{\boldsymbol{Q}_U}$ so that $(\boldsymbol{F}_U, \boldsymbol{Q}_U, \rho_U)$ is a feasible solution to (8), where ρ_U is the SINR of the identical subchannels in UCD-VBLAST. As a result, we have $\rho^* \geq \rho_U$. On the other hand, it has been shown in [2] that the UCD-VBLAST is strictly capacity lossless, i.e., $C_U = M \log_2(1+\rho_U) = C_{\mathrm{opt}}$. Since $\rho^* \geq \rho_U$, we have

$$C_{\mathrm{opt}} \geq M \log_2(1 + \rho^*) \geq M \log_2(1 + \rho_U) = C_{\mathrm{opt}} \tag{10}$$

which implies that $C_{\mathrm{MMSE-SIC}}(\boldsymbol{F}^*, \boldsymbol{Q}^*) = M \log_2(1 + \rho^*) = C_{\mathrm{opt}}$ and $\rho_U = \rho^*$. ■

The result of Theorem 1 does not necessarily guarantee the capacity optimality of limited feedback. However, as the size of the codebook getting large, the capacity will approach to that of perfect feedback. Actually, a not so large codebook will be suffice to achieve a significant portion of the capacity gain achieved by perfect feedback. In summary, the feedback scheme is proposed to enhance link reliability, and at the same time significant capacity gain can also be obtained.

3.3. The Solution

The optimization problem in (8) maximizes the minimum post-detection SINR jointly with respect to $\boldsymbol{F}$ and $\boldsymbol{Q}$ under the total power constraint. For any given $\boldsymbol{F}$, the problem (8) can be reduced to

$$\begin{aligned} \max_{\boldsymbol{Q},\rho} \quad & \rho \\ \text{s.t.} \quad & \mathrm{SINR}_k(\boldsymbol{F},\boldsymbol{Q}) \geq \rho, \quad 1 \leq k \leq M \\ & \mathrm{trace}(\boldsymbol{Q}) \leq P_0 \end{aligned} \tag{11}$$

It is worthy noting that the properties stated in Lemma 1 still hold for problem (11), i.e., $\mathrm{SINR}_k(\boldsymbol{F},\boldsymbol{Q}^*) = \rho^*$ and ρ^* is continuous and strictly monotonic increasing in P_0, where $(\boldsymbol{Q}^*,\rho^*)$ is an optimal solution of the problem (11). As a result, the optimal solution of (11) can be found by solving

$$\mathrm{SINR}_k(\boldsymbol{F},\boldsymbol{Q}) = q_k \boldsymbol{f}_k^H \boldsymbol{Z}_k \boldsymbol{f}_k = \rho, \quad k = 1, \cdots M \tag{12}$$

for $\boldsymbol{Q}$ at different values of ρ until we get $\mathrm{trace}(\boldsymbol{Q}) = P_0$. The strict monotonicity and continuity guarantee that ρ^* can be found efficiently by a one-dimensional bisection search. In short, the problem (8) can be solved by enumerating all beamforming matrices in the codebook and solving (11). The one that achieves the maximum ρ is the optimal solution to (8). The algorithm is summarized as follows.

Algorithm I:

- `initialization:` $\rho^* = 0$, $\boldsymbol{F}^*$ `and` $\boldsymbol{Q}^*$
- **for** `each` $\boldsymbol{F} \in \mathcal{F}$ **do**
  - `initialization:` $\rho_{\min}$, $\rho_{\max}$, ϵ, $\hat{P}_0 = 0$
  - **while** $(|P_0 - \hat{P}_0| > \epsilon)$ **do**
    - $\rho = (\rho_{\min} + \rho_{\max})/2$, $\boldsymbol{Z}_1 = \boldsymbol{H}^H(\sigma^2\boldsymbol{I})^{-1}\boldsymbol{H}$
    - **for** `data stream` $k = 1$ `to` M **do**

$$\begin{aligned} q_k &= \rho/(\boldsymbol{f}_k^H \boldsymbol{Z}_k \boldsymbol{f}_k) \\ \boldsymbol{Z}_{k+1} &= \boldsymbol{Z}_k - \frac{q_k \boldsymbol{Z}_k \boldsymbol{f}_k \boldsymbol{f}_k^H \boldsymbol{Z}_k^H}{1 + q_k \boldsymbol{f}_k^H \boldsymbol{Z}_k \boldsymbol{f}_k} \end{aligned}$$

    - **end for**
    - $\hat{P}_0 = \sum_{i=1}^{M} q_i$; **if** $\hat{P}_0 < P_0$, $\rho_{\min} = \rho$, **otherwise** $\rho_{\max} = \rho$;
  - **end while**
  - **if** $\rho > \rho^*$, **then** $\boldsymbol{F}^* = \boldsymbol{F}, \boldsymbol{Q}^* = \boldsymbol{Q}, \rho^* = \rho$.
- **end for**

The innermost loop solves equations (12) and ρ is given by the outer "while" loop. The outer "while" loop is a one-dimensional bisection search to find the ρ^* such that $\mathrm{trace}(\boldsymbol{Q}) = P_0$. $\rho_{\min}$ and $\rho_{\max}$ define the range in the search of ρ. The outermost loop enumerates all beamforming matrices in $\mathcal{F}$.

4. SIMULATION AND DISCUSSION

In this section we present results from extensive simulations to evaluate the performance of the proposed limited feedback scheme. QPSK modulation with Gray mapping is employed during the simulation. The channel is assumed to be Rayleigh flat fading channel. We consider a MIMO configuration with four transmit antennas and four receive antennas, i.e., $N_T = N_R = 4$. The number of data streams is $M = N_T = 4$. Each data stream is coded independently by a $1/3$ rate turbo encoder(for the details of the turbo encoder please refer to [11]). The use of turbo encoding in the simulation can reduce the effect of error propagation so that we can focus on the effect of limited feedback. A turbo encoding block consists of 320 uncoded bits. Suppose that the channel is quasi-static, i.e., it remains unchanged during the transmission of a single block but varies independently from block to block.

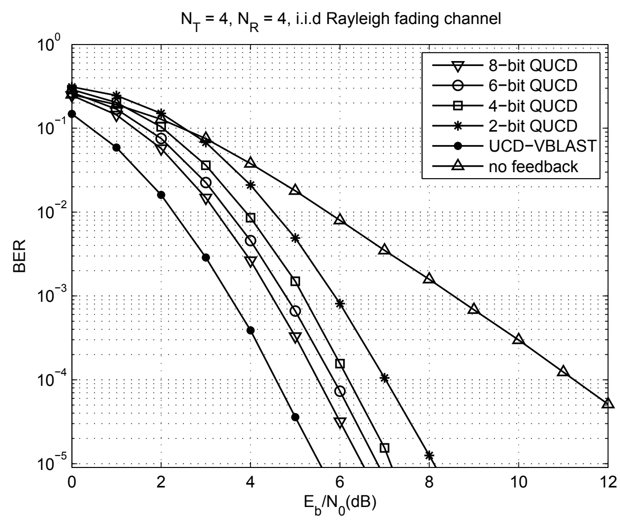

Fig. 1. BER performance of the proposed feedback scheme and UCD-VBLAST. Power allocation information is fed back perfectly.

In the first simulation scenario, we assume that the power allocation matrix $\boldsymbol{Q}$ is fed back perfectly and we do not take into account the feedback amount needed to feed back $\boldsymbol{Q}$. Fig. 1 shows the BER performance of a V-BLAST system with the proposed feedback scheme. The performance of the UCD-VBLAST and V-BLAST without feedback is also given for comparison. Receiver for the latter is MMSE-SIC with optimal ordering [1]. The codebooks are generated randomly. As can be seen in the figure, the performance of 8-bit QUCD is within 0.9 dB compared with that of the UCD-VBLAST. Using 8-bit QUCD instead of 4-bit QUCD provides about 0.6 dB gain. Also, we note that even 2-bit QUCD provides more than 4.0 dB gain over V-BLAST without feedback at BER of 10^{-5}. Fig. 2 plots the average channel capacity versus SNR for various feedback schemes. For a low SNR, there is a significant SNR gain of around 3.0 dB between UCD-VBLAST and no feedback. A large portion of feedback gain is achieved by 8-bit QUCD. The feedback gain diminishes at a high SNR, which is also true for UCD-VBLAST.

The effect of the quantized power information feedback is evaluated in the second simulation scenario. The power of each data stream is compressed by the μ-law non-linear quantizer individually. The performance of 12-bit, 16-bit and 20-bit power quantization is

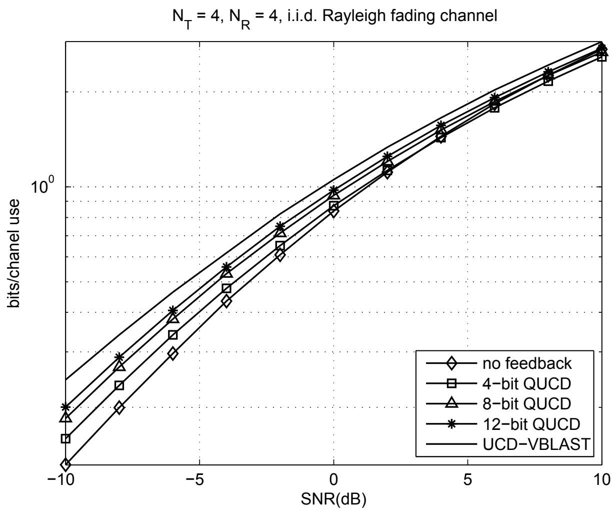

Fig. 2. Average capacity of the proposed feedback scheme and UCD-VBLAST. Power allocation information is fed back perfectly.

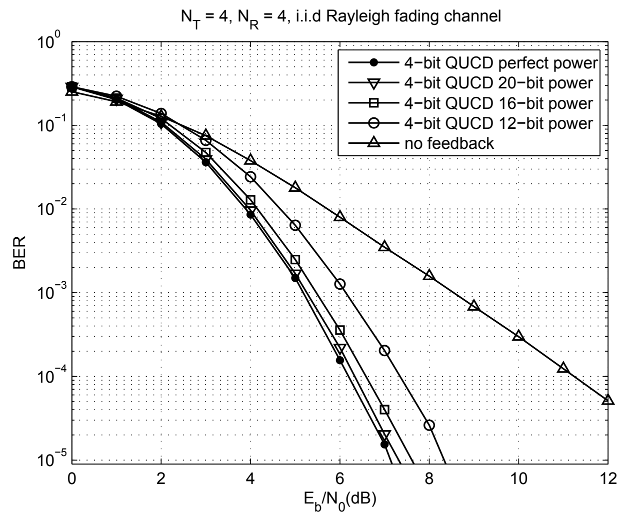

Fig. 3. Effect of power quantization for 4-bit QUCD.

evaluated. Fig. 3 shows the BER of 4-bit QUCD along with quantized power feedback. It can be seen that 20-bit power feedback incurs almost no performance loss compared to the perfect power feedback. It is striking that 4-bit QUCD along with the 12-bit power feedback provides more than 4.0 dB gain at BER of 10^{-5} over V-BLAST without feedback. The total feedback amount is 16 bits (4 bits/stream), which is affordable by future wireless systems [12].

5. CONCLUSION

In this paper, quantized linear precoding for the V-BLAST system with a MMSE-SIC receiver is studied. The beamforming and power allocation matrices are designed to maximize the minimum SINR of the MMSE-SIC receiver. We show that if the matrices can be feedback perfectly, the precoder is optimal in terms of both spectral efficiency and link reliability aspects, i.e., it is equivalent to UCD-VBLAST. Consequently, the proposed limited feedback scheme is a natural but nontrivial extension of UCD-VBLAST. The simulation results show that limited feedback can achieve a large portion of the capacity gain and significantly decrease the BER at the same time.

6. REFERENCES

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