# COMPENSATION OF RF IMPAIRMENTS IN MIMO OFDM SYSTEMS

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## ABSTRACT

In this paper we propose a generally applicable frequency domain equalization and radio frequency (RF) impairment compensation technique for orthogonal frequency division multiplexing (OFDM) based multi-input multi-output (MIMO) systems. RF impairments such as in-phase quadrature-phase (IQ) imbalance and carrier frequency offset (CFO) are unavoidable in low-cost analog front-end systems, but can result in a severe performance degradation. In this paper, a digital compensation scheme is developed for joint transmitter and receiver IQ imbalance along with front-end filter mismatch, CFO and frequency selective channel distortions in OFDM based MIMO systems. This scheme can also be extended for the multi-user scenario where each user signal suffers from a different frequency offset along with IQ imbalance.

***Index Terms*—** Multi-input multi-output (MIMO) systems, orthogonal frequency division multiplexing (OFDM), multi-user systems, in-phase quadrature-phase (IQ) imbalance, carrier frequency offset (CFO).

## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) based multi-input multi-output (MIMO) transmission is considered to be an important upcoming broadband wireless technology. MIMO OFDM systems involve multiple front-ends and hence it is extremely important to keep the cost, size and power consumption of these front-ends within an acceptable limit. The direct-conversion based architecture provides a good implementation alternative as it has a small form factor compared to the traditional architecture [1]. However, direct-conversion based systems are very sensitive to component imperfections in the analog front-end leading to Radio Frequency (RF) impairments such as in-phase quadrature-phase (IQ) imbalance and carrier frequency offset (CFO). These RF impairments can result in a severe performance degradation of the OFDM system. Several articles [2]-[7] have been published to study the effects of these impairments and develop their compensation scheme for single-input single-output (SISO) based OFDM systems. In [7] and [8], the authors propose a compensation scheme for IQ imbalance in MIMO systems.

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In this paper we study the joint effect of transmitter and receiver frequency selective IQ imbalance along with CFO and channel distortions for MIMO OFDM systems. A frequency domain based per-tone equalizer (PTEQ) is developed to compensate for these distortions. This scheme can also be extended for the multi-user scenario where each user signal suffers from a different frequency offset along with IQ imbalance.

The paper is organized as follows: We first develop the input-output system model in Section 2. Section 3 describes the IQ imbalance and CFO compensation scheme. Results from a numerical performance evaluation are presented in Section 4 and finally conclusions are given in Section 5.

*Notation:* Vectors are indicated in bold and scalar parameters in normal font. Superscripts $\{\}^*, \{\}^T, \{\}^H$ represent conjugate, transpose and Hermitian respectively. $\mathbf{F}$ and $\mathbf{F}^{-1}$ represent the $N \times N$ discrete Fourier transform and its inverse. $\mathbf{I}_N$ is the $N \times N$ identity matrix and $0_{M \times N}$ is the $M \times N$ all zero matrix. Operators $\otimes$, $\star$ and . denote Kronecker product, convolution and component-wise vector multiplication respectively.

## 2. SYSTEM MODEL

Let $N_t$ and $N_r$ denote the number of transmit and receive antennas in an uncoded MIMO OFDM system. Then $\mathbf{S}_{(k)}$ (for $k = 1 \dots N_t$) is the frequency domain OFDM symbol of size $(N \times 1)$, to be transmitted over the $k^{th}$ transmit antenna. The frequency domain symbols are transformed to the time domain by the inverse discrete Fourier transform (IDFT). A cyclic prefix (CP) of length $\nu$ is then added to the head of each symbol. The resulting time domain baseband signal $\mathbf{s}_{(k)}$ is given as:

$$\mathbf{s}_{(k)} = \mathbf{P}\mathbf{F}^{-1}\mathbf{S}_{(k)} \tag{1}$$

where $\mathbf{P}$ is the cyclic prefix insertion matrix given by:

$$\mathbf{P} = \left[\begin{array}{c|c} 0_{(\nu \times N-\nu)} & \mathbf{I}_\nu \\ \hline \multicolumn{2}{c}{\mathbf{I}_N} \end{array}\right]$$

We now consider a low cost system where all transmit antennas (and also all receive antennas) are supported by a single local oscillator (LO). The IQ imbalance induced by the

LO can be categorized as frequency independent (FI) over the signal bandwidth with amplitude and phase mismatch of $g_{t(k)}$ and $\phi_{t(k)}$ at each $k^{th}$ transmit branch. The filter mismatch caused by digital-to-analog converters (DAC), amplifiers, low pass filters (LPFs) and mixers result in an overall frequency selective (FS) IQ imbalance. We represent this imbalance at the $k^{th}$ transmit antenna by two mismatched filters with frequency responses given as $\mathbf{H}_{ti(k)} = \mathbf{F}\{\mathbf{h}_{ti(k)}\}$ and $\mathbf{H}_{tq(k)} = \mathbf{F}\{\mathbf{h}_{tq(k)}\}$. Following the derivation in [2], the equivalent baseband signal $\mathbf{p}_{(k)}$ can now be specified as:

$$\mathbf{p}_{(k)} = \mathbf{g}_{t1(k)} \star \mathbf{s}_{(k)} + \mathbf{g}_{t2(k)} \star \mathbf{s}^*_{(k)} \tag{2}$$

where

$$\mathbf{g}_{t1(k)} = \mathbf{F}^{-1}\left\{\mathbf{G}_{t1(k)}\right\} = \mathbf{F}^{-1}\left\{\frac{\mathbf{H}_{ti(k)} + g_{t(k)}e^{-j\phi_{t(k)}}\mathbf{H}_{tq(k)}}{2}\right\}$$

$$\mathbf{g}_{t2(k)} = \mathbf{F}^{-1}\left\{\mathbf{G}_{t2(k)}\right\} = \mathbf{F}^{-1}\left\{\frac{\mathbf{H}_{ti(k)} - g_{t(k)}e^{j\phi_{t(k)}}\mathbf{H}_{tq(k)}}{2}\right\}$$

Here $\mathbf{g}_{t1(k)}$ and $\mathbf{g}_{t2(k)}$ are mostly truncated to length $L_t$ and then padded with $N - L_t$ zero elements. They represent the combined FI and FS IQ imbalance for the $k^{th}$ transmit antenna.

When the signal $\mathbf{p}_{(k)}$ is transmitted through a frequency selective time invariant channel $\mathbf{h}_{(r,k)}$ of length $L_c$ (for $r = 1 \ldots N_r$ and $k = 1 \ldots N_t$), then the received baseband signal $\mathbf{u}_{(r)}$ at the $r^{th}$ receive antenna is given as:

$$\mathbf{u}_{(r)} = \sum_{k=1}^{N_t} \mathbf{h}_{(r,k)} \star \mathbf{p}_{(k)} + \mathbf{n}_{(r)} \tag{3}$$

where $\mathbf{n}_{(r)}$ is the additive white Gaussian noise. At the receiver end, we consider a CFO of $\Delta f$ along with the receiver IQ imbalance. Thus the final expression for each received signal $\mathbf{z}_{(r)}$ after front-end receiver distortion can be given as [3]:

$$\mathbf{z}_{(r)} = \mathbf{g}_{z1(r)} \star (\mathbf{u}_{(r)}.e^{j2\pi\Delta f.\mathbf{t}}) + \mathbf{g}_{z2(r)} \star (\mathbf{u}^*_{(r)}.e^{-j2\pi\Delta f.\mathbf{t}}) \tag{4}$$

where $e^{j\mathbf{x}}$ is the element-wise exponential function on the vector $\mathbf{x}$ and $\mathbf{t}$ is a time vector. Both $\mathbf{g}_{z1(r)}$ and $\mathbf{g}_{z2(r)}$ are of length $L_r$ and they are defined similar to $\mathbf{g}_{t1(k)}$ and $\mathbf{g}_{t2(k)}$. The joint effect of both transmitter and receiver IQ imbalance along with CFO results in a severe performance degradation, as will be shown in section 4, and so a digital compensation scheme is needed.

## 3. IQ IMBALANCE AND CFO COMPENSATION

### 3.1. TEQ based compensation

In order to compensate for the channel and RF distortions, we first design two time domain equalizers (TEQs) $\mathbf{w}_{1(r)}$ and $\mathbf{w}_{2(r)}$ each of length $L' = L_r$. The TEQ $\mathbf{w}_{1(r)}$ is applied to the signal $\mathbf{z}_{(r)}$ and the TEQ $\mathbf{w}_{2(r)}$ to the signal $\mathbf{z}^*_{(r)}$. Here a vector $\mathbf{z}_{(r)}$ is considered of length $(N + L' - 1 \times 1)$. This leads to:

$$\begin{aligned}\mathbf{z}_{t(r)} =& \mathbf{w}_{1(r)} \star \mathbf{z}_{(r)} + \mathbf{w}_{2(r)} \star \mathbf{z}^*_{(r)} \\ =& \underbrace{(\mathbf{w}_{1(r)} \star \mathbf{g}_{z1(r)} + \mathbf{w}_{2(r)} \star \mathbf{g}^*_{z2(r)})}_{\mathbf{f}_{1(r)}} \star (\mathbf{u}_{(r)}.e^{j2\pi\Delta f.\mathbf{t}}) \\ &+ \underbrace{(\mathbf{w}_{1(r)} \star \mathbf{g}_{z2(r)} + \mathbf{w}_{2(r)} \star \mathbf{g}^*_{z1(r)}) \star (\mathbf{u}^*_{(r)}.e^{-j2\pi\Delta f.\mathbf{t}})}_{\mathbf{f}_{2(r)}}\end{aligned} \tag{5}$$

where $\mathbf{z}_{t(r)}$ is of size $(N \times 1)$ and the design target for $\mathbf{w}_{1(r)}$ and $\mathbf{w}_{2(r)}$ is such that the $\mathbf{f}_{2(r)}$ term vanishes. At this point, we can multiply equation (5) with $e^{-j2\pi\Delta f.\mathbf{t}}$, based on any one of the robust CFO estimation algorithm (see e.g. [3] and [4]):

$$\widetilde{\mathbf{z}}_{t(r)} = \mathbf{z}_{t(r)}.e^{-j2\pi\Delta f.\mathbf{t}} = \widetilde{\mathbf{f}}_{1(r)} \star \mathbf{u}_{(r)} \tag{6}$$

where $\widetilde{\mathbf{f}}_{1(r)} = \mathbf{f}_{1(r)}.(e^{-j2\pi\Delta f.(0\ldots(L_r-1))})^{-1}$. Thus, the resulting vector $\widetilde{\mathbf{z}}_{t(r)}$ is free from any time-dependent CFO and $\mathbf{f}_{2(r)}$ interferences. The $\widetilde{\mathbf{z}}_{t(r)}$ now contains only frequency selective transmitter and receiver IQ imbalance along with channel and noise distortions. In conjunction with the TEQ scheme, a DFT is applied to each of the filtered sequences $\widetilde{\mathbf{z}}_{t(r)}$. The output of the DFT is then applied to a two tap frequency domain equalizer (FEQ), combining a sub-carrier output with the conjugate mirror output. Finally, the sum of all the FEQs results in the estimate of the transmitted symbol. We define $\hat{\mathbf{S}}_{(k)}[l]$ as the estimate for the $l$th sub-carrier of the $k$th transmit antenna OFDM symbol. This estimate is then obtained as:

$$\hat{\mathbf{S}}_{(k)}[l] = \sum_{r=1}^{N_r}(\mathbf{v}_{1(r,k)}[l].(\mathbf{F}[l]\widetilde{\mathbf{z}}_{t(r)}) + \mathbf{v}_{2(r,k)}[l].(\mathbf{F}[l_m]\widetilde{\mathbf{z}}_{t(r)})^*) \tag{7}$$

where $\mathbf{v}_{1(r,k)}[l]$ and $\mathbf{v}_{2(r,k)}[l]$ are the two taps of the FEQ for the $l$th sub-carrier of the $k$th transmit antenna OFDM symbol, and $\mathbf{F}[l]$ is the $l$th row of the DFT matrix $\mathbf{F}$. Here $()_m$ denotes the mirroring operation in which the indices are reversed, such that $\mathbf{F}_m[l] = \mathbf{F}[l_m]$ where $l_m = 2 + N - l$ for $l = 2 \ldots N$ and $l_m = l$ for $l = 1$.

### 3.2. PTEQ based compensation

In order to simplify the entire scheme, we first swap the TEQ filtering operation with the multiplication of the negative CFO estimate. Now the swapped TEQ position can be transferred to the frequency domain resulting in two per-tone equalizers (PTEQs) each employing one DFT and $L_p = L' - 1$ difference terms [9]. Equation (7) is then modified as follows:

$$\begin{aligned}\hat{\mathbf{S}}_{(k)}[l] = \sum_{r=1}^{N_r}(&\mathbf{v}^T_{1(r,k)}[l]\mathbf{F}_i[l]\mathbf{z}_{1(r)} + \mathbf{v}^T_{2(r,k)}[l](\mathbf{F}_i[l_m]\mathbf{z}_{1(r)})^* \\ &+ \mathbf{v}^T_{3(r,k)}[l]\mathbf{F}_i[l]\mathbf{z}_{2(r)} + \mathbf{v}^T_{4(r,k)}[l](\mathbf{F}_i[l_m]\mathbf{z}_{2(r)})^*)\end{aligned} \tag{8}$$

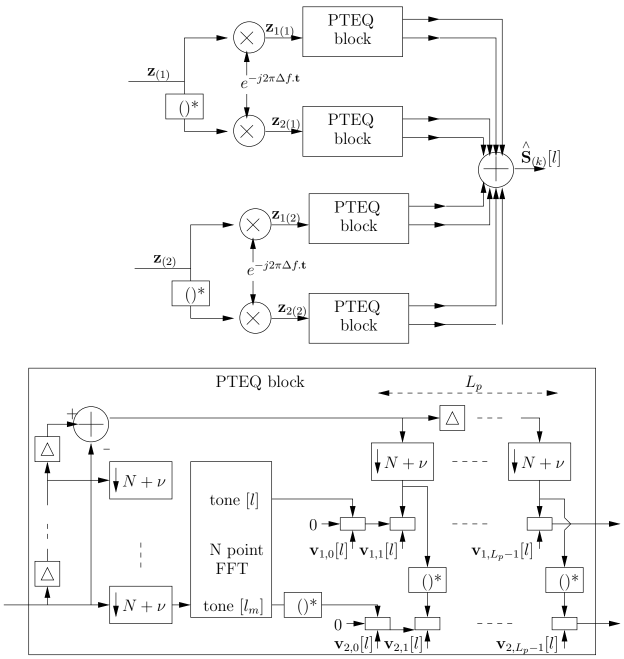



**Fig. 1**. PTEQ compensation for 2x2 MIMO OFDM system

where $\mathbf{z}_{1(r)} = \mathbf{z}_{(r)}.e^{-j2\pi\Delta f.\mathbf{t}}$ and $\mathbf{z}_{2(r)} = \mathbf{z}^*_{(r)}.e^{-j2\pi\Delta f.\mathbf{t}}$. $\mathbf{v}_{d(r,k)}[l]$ (for $d = 1\ldots4$) are PTEQs of size $(L' \times 1)$. $\mathbf{F}_i[l]$ is defined as:

$$\mathbf{F}_i[l] = \left[\begin{array}{c|c|c} \mathbf{I}_{L'-1} & 0_{L'-1\times N-L'+1} & -\mathbf{I}_{L'-1} \\ \hline 0_{1\times L'-1} & \multicolumn{2}{c}{\mathbf{F}[l]} \end{array}\right]$$

where the first block row in $\mathbf{F}_i[l]$ is seen to extract the difference terms, while the last row corresponds to the single DFT. The PTEQ compensation scheme for a 2x2 MIMO system is shown in **Figure 1**. In the case of IQ imbalance only at the receiver side, the coefficients $\mathbf{v}_{2(r,k)}[l]$ and $\mathbf{v}_{4(r,k)}[l]$ are set to null and thus the PTEQ structure is further simplified. Based on equation (8), a maximum-likelihood (ML), least-square (LS) or minimum mean-square-error (MMSE) algorithm can be developed at the receiver side (see e.g. [5]).

### 3.3. Multi-user Scenario

In a multi-user (MU) scenario, we consider $N_u$ users, each with their individual LO supporting $N_c$ antennas. These LOs may not be perfectly synchronized with the base station (BS) leading to individual CFOs $\Delta f_{(u)}$ (for $u = 1\ldots N_u$). The BS has $N_b$ multiple antennas. In the down-link case (from BS to MUs), the expression for the received signal in equation (4) remains the same other than the subscript $(r)$ being replaced by the subscript $(c, u)$, where the subscript denotes the respective antenna and the user, and CFO $\Delta f_{(u)}$ replacing the single-user (SU) CFO $\Delta f$. The rest of the compensation scheme remains unchanged.

For the MU up-link case (from MUs to BS), the equation (4) is modified as:

$$\begin{aligned}\mathbf{z}_{(b)} &= \mathbf{g}_{z1(b)} \star \Big(\sum_{u=1}^{N_u}\sum_{c=1}^{N_c}\mathbf{h}_{(b,c,u)} \star \mathbf{p}_{(c,u)}.e^{j2\pi\Delta f_{(u)}.\mathbf{t}} + \mathbf{n}_{(b)}\Big) \\ &+ \mathbf{g}_{z2(b)} \star \Big(\sum_{u=1}^{N_u}\sum_{c=1}^{N_c}\mathbf{h}^*_{(b,c,u)} \star \mathbf{p}^*_{(c,u)}.e^{-j2\pi\Delta f_{(u)}.\mathbf{t}} + \mathbf{n}^*_{(b)}\Big)\end{aligned} \tag{9}$$

Here we apply the same compensation scheme as in §**3.1** until equation (5) to obtain $\mathbf{z}_{t(b)}$. We then have to design a set of TEQs depending on the number of users, antennas and different frequency offsets. Here we consider a simple 2x2 MU MIMO system where each user has one transmit antenna and a different CFO and the base station has two receive antennas. It is possible to design TEQs for more users, but for each case the compensation structure may differ. In the present case, we design two TEQs $\mathbf{w}_a$ and $\mathbf{w}_b$ each of length $L'' = L_c$. The two TEQs along with the negative of the MU CFO estimate are applied to $\mathbf{z}_{t(1)}$ and $\mathbf{z}_{t(2)}$ obtained from equation (5):

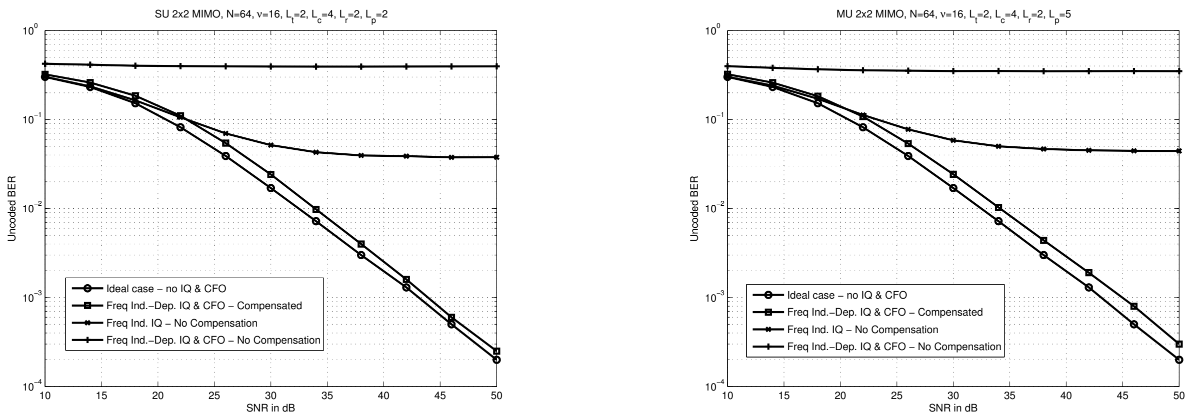
$$\widetilde{\mathbf{z}}_{s(u)} = (\mathbf{w}_a \star \mathbf{z}_{t(1)}).e^{-j2\pi\Delta f_{(u)}.\mathbf{t}} + (\mathbf{w}_b \star \mathbf{z}_{t(2)}).e^{-j2\pi\Delta f_{(u)}.\mathbf{t}}$$

The design target is such that $\widetilde{\mathbf{z}}_{s(u)}$ may not contain any CFO term. Finally, a DFT is applied to each of the filtered sequences $\widetilde{\mathbf{z}}_{s(u)}$. As in §**3.1**, a two tap FEQ is then applied to every sub-carrier output and the conjugate mirror output of the DFT and the sum of these FEQ output results in the estimate of the transmitted symbol $\hat{\mathbf{S}}_{(u)}[l]$. Finally, the entire equalizer scheme can be simplified by first swapping the first set of TEQs $\mathbf{w}_{1(b)}$ and $\mathbf{w}_{2(b)}$ with the multiplication of the negative CFO estimate. Then the two sets of TEQs can be combined by a simple linear operation, giving an overall TEQ of length $L_w = L' + L'' - 1$. The combined TEQ can finally be transferred to the frequency domain resulting in two per-tone equalizers (PTEQ) each employing one DFT and $L_p = L_w - 1$ difference terms. The PTEQ compensation scheme for a MU 2x2 OFDM system in the up-link case is eventually the same as that of the single-user (SU) 2x2 system, shown in Figure 1, with only CFO $\Delta f_{(u)}$ replacing the SU CFO $\Delta f$. The PTEQ tap length per tone in this case is increased from $L'$ to $L' + L'' - 1$. For the down-link case, the PTEQ tap length can be kept at $L'$.

## 4. SIMULATION

To evaluate the performance of the compensation scheme, we consider a system very similar to the MIMO extension of the IEEE 802.11a standard. The performance comparison is made with an ideal system with no front-end distortion and with a system with no compensation algorithm included.

The parameters used in the simulation are as follows: OFDM symbol length $N = 64$, CP length $\nu = 16$. In SU MIMO system, the filter impulse responses are $\mathbf{h}_{ti(k)} = \mathbf{h}_{zi(r)} = [0.1,\ 0.9]$ and $\mathbf{h}_{tq(k)} = \mathbf{h}_{zq(r)} = [0.9,\ 0.1]$, FI amplitude imbalance $g_{t(k)} = g_{z(r)} = 5\%$ and phase imbalance $\phi_{t(k)} = \phi_{z(r)} = 5°$. We have kept the same IQ imbalance values across all the antenna branches in both the SU and MU




(a) SU-MIMO PTEQ compensation

(b) MU-MIMO PTEQ compensation (up-link case)

**Fig. 2**. BER vs SNR for uncoded 64QAM 2x2 MIMO OFDM system.

case so that the simulation process remains simple. For the SU case, we consider CFO $\zeta = 0.32$, where $\zeta$ is the ratio of the actual CFO $\Delta f$ to the sub-carrier spacing $1/T.N$, where T is the sampling period. In MU case, we consider $\zeta_{(1)} = 0.32$ and $\zeta_{(2)} = 0.2$. The multipath channel is of length $L_c = 4$. The taps of the multipath channel are chosen independently with complex Gaussian distribution. We have considered a training based RLS algorithm to initialize the PTEQ scheme [6] as this provides optimal convergence and achieves initialization with an acceptably small number of training symbols.

**Figure 2 (a) and (b)** show the performance curves (BER vs SNR) for an uncoded 64QAM 2x2 OFDM system. Every channel realization is independent of the previous one and the BER results depicted are obtained by averaging the BER curves over $10^4$ independent channels. In the presence of RF impairments with no compensation scheme in place, the OFDM system is completely unusable. Even for the case when there is only frequency independent IQ imbalance, the BER is very high. When the compensation scheme is in place, the performance is very close to the ideal case. The compensation performance depends on how accurately the adaptive equalizer coefficients can converge to the ideal values. It should be noted that the PTEQ scheme proposed here can also be generalized for the case of insufficient CP length [6]. In this case the PTEQ length may have to be increased further to also compensate for the inter-block-interferences (IBI) between the OFDM symbols.

## 5. CONCLUSION

In this paper the joint effect of transmitter and receiver frequency selective IQ imbalance, CFO and multipath channel distortions has been studied for MIMO OFDM systems. A compensation scheme has been developed for such distortions in the digital domain. We also consider multi-user scenarios where each user may have a different frequency offset. The compensation scheme provides a very efficient, post-FFT equalization with performance very close to the ideal case.

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