DISCRETE RATE SPECTRAL EFFICIENCY FOR ADAPTIVE MIMO SYSTEMS

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ABSTRACT

Adaptive modulation scheme has been widely used in multiple-input multiple-output (MIMO) systems to enhance spectral efficiency while maintaining bit-error rate (BER) under a target level. We consider two MIMO schemes, orthogonal space-time block codes (OSTBC) and spatial multiplexing with zero-forcing receiver (SM-ZF), that apply adaptive modulation and derive closed-form expressions of the average spectral efficiency. Based on the closed-form expression of the average spectral efficiency, a low complexity switching method is utilized to gain significant improvement in spectral efficiency over an adaptive modulation system using a single MIMO scheme.

Index Terms— MIMO systems, Communication system operations and management, Communication system performance

1. INTRODUCTION

Channel-adaptive transmission that adjusts the transmitter to enhance the spectral efficiency [1] has been receiving a great deal of attention in the past few years. The idea of channel-adaptive transmission is to feed in the transmitter with Channel State Information (CSI) so that the transmitter can adjust itself accordingly to ensure a robust and spectrum efficient transmission. The application of channel-adaptive scheme in multiple-input multiple-output (MIMO) systems has exhibited substantial benefits on spectral efficiency and link reliability.

Adaptive modulation and transmit power allocation were suggested as means to improve the spectral efficiency in [1]. On the basis of previous work, a new kind of adaptive scheme was suggested in [2, 3] that switches between different MIMO schemes to achieve even higher spectral efficiency. For instance, [3] and [2] switch between a diversity scheme and a spatial multiplexing scheme based on the BER performance, to explore the highest spectral efficiency while maintaining BER under a target level. In contrast to the work of [2], a new approach was proposed in [4, 5] which selects the advantaged MIMO scheme based on the average spectral efficiency. The average spectral efficiencies of the candidate schemes in an uncoded system are evaluated by using closed-form expressions, the so called discrete-rate spectral efficiency (DRSE). The DRSE of Orthogonal Space-Time Block Coding (OSTBC) and Spatial Multiplexing with Zero-Forcing (SM-ZF) were only obtained for 2×2 system in [4, 5]. In this paper, we extend the work described above and generalize the results of DRSE to MIMO systems with arbitrary number of transmit and receive antennas. Specifically, we consider an adaptive MQAM modulation system whose modulation order is updated to the rate of channel variation. To be able to switch between OSTBC and SM-ZF, we obtain the DRSEs of both schemes and select the one with a higher spectral efficiency. The channel environment is assumed to be either independent and identically distributed (i.i.d.) Rayleigh fading or spatially correlated Rayleigh fading with correlation at the transmitter. Although we only consider the switching between OSTBC and SM-ZF, it is a general method that can be applied to more schemes, e.g., beamforming, singular value decomposition, as long as the DRSE of these schemes can be derived.

The remainder of this paper is organized as follows. In section 2, we present the channel model and the candidate MIMO schemes. In section 3, the DRSEs are derived for OSTBC and SM-ZF. In section 5, numerical results from computer simulations are presented for different antenna setups and channel scenarios. Conclusions are drawn in section 6.

2. SYSTEM MODEL

We consider a point to point MIMO system with N_t transmit antennas and N_r receive antennas. There are two types of channels in our consideration: the i.i.d. Rayleigh fading channel, $\mathbf{H}_w$, and the spatially correlated Rayleigh fading channel with correlation at transmitter, $\mathbf{H}_r = \mathbf{H}_w \mathbf{R}_t^{1/2}$. Without loss of generality, the entries of $\mathbf{H}_w$ are assumed to be independent Gaussian random variables, i.e., $[\mathbf{H}_w]_{ij} \sim \mathcal{CN}(0,1)$. The spatial correlation at the transmitter is denoted by $\mathbf{R}_t$, with $[\mathbf{R}_t]_{ij} = \rho^{|i-j|}$, where ρ is the correlation coefficient. $\mathbf{R}_t$ can be diagonalized as: $\mathbf{R}_t = \mathbf{U}_t \Lambda_t \mathbf{U}_t^H$, where $\mathbf{U}_t$ is a unitary matrix and $\mathbf{U}_t^H$ is the Hermitian of it. Λ_t is a diagonal matrix with diagonal entries: $\lambda_i, i = 1, \ldots, N_t$.

The discrete-time baseband equivalent signal model can be written as:

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{x}(n) + \mathbf{z}(n) \tag{1}$$

where $\mathbf{x}(n)$ is an $N_t \times 1$ transmitted vector and $\mathbf{y}(n)$ is an $N_r \times 1$ received vector. $\mathbf{z}(n)$ is the additive white Gaussian noise with covariance $\sigma_n^2\mathbf{I}$. In i.i.d Rayleigh fading channel, $\mathbf{H} = \mathbf{H}_w$; in spatial correlated channel, $\mathbf{H} = \mathbf{H}_r$. The channel is assumed to be quasi-static (channel coefficients remain constant during one time interval, and changes independently in the next interval).

Two MIMO schemes are investigated in this work:

- In case of OSTBC, maximal Ratio Combining (MRC) is applied at the receiver to maximize the effective SNR

$$\gamma = \frac{\gamma_0 \|\mathbf{H}\|_F^2}{N_t R}, \tag{2}$$

where R is the rate of STBC[1], $\|\cdot\|_F$ is the Frobenius norm, $\gamma_0 = P_T/\sigma_n^2$ is the average SNR and P_T is the total transmit power.

- For SM-ZF, the effective SNR of the ith data stream is provided as [6]:

$$\gamma_i = \frac{\gamma_0 \|\mathbf{Q}_i \mathbf{h}_i\|^2}{N_t}, \tag{3}$$

where $\mathbf{Q}_i$ is a $(N_r - N_t + 1) \times N_r$ projection matrix that projects the received signal $\mathbf{y}$ onto a subspace orthogonal to the vectors $\mathbf{h}_1, \ldots, \mathbf{h}_{i-1}, \mathbf{h}_{i+1}, \ldots, \mathbf{h}_{N_t}$.

3. DISCRETE-RATE SPECTRAL EFFICIENCY

The adaptive MQAM modulation [1], which dynamically determines the constellation size based on the effective SNR, is applied to improve the spectral efficiency while keeping the BER under a target level. The relationship between BER and SNR (γ) under a certain modulation order (M_k) for QAM modulation can be approximated as:

$$P_b(\gamma, M_k) \approx c_k e^{-a_k \gamma}, \tag{4}$$

where a_k and c_k can be found numerically by a curve-fitting method. Although (4) is provided to imitate the BER performance in AWGN channel, it can be applied to any type of fading channel when adaptive modulation is used. This is due to that adaptive modulation adjusts the modulation order depending on the instantaneous Channel Quality Information (CQI) so that fading channel is converted to an AWGN channel with fixed channel gain. The SNR thresholds T_k for switching across the modulation orders can be solved from (4):

$$T_k = \frac{1}{a_k} \ln \frac{c_k}{\bar{P}_b}, \tag{5}$$

[1] $R = 1$ for $N_t = 2$ (Alamouti code) and $R < 1$ otherwise

where $\bar{P}_b$ is the target bit error rate for an uncoded system. The DRSE[2] can be expressed as:

$$\begin{aligned} \mathrm{DR} &= \mathbf{E}\left\{\sum_i b_i\right\} = \mathbf{E}\left\{\sum_i \mathcal{Q}\left(\gamma_i | T_k\right)\right\} \\ &= \sum_i \left(\sum_{k=1}^{K} d_k \cdot \mathbf{Pr}(T_k \le \gamma_i < T_{k+1})\right) \\ &= \sum_i \left(\sum_{k=1}^{K} d_k \cdot \int_{T_k}^{T_{k+1}} p(\gamma_i) d\gamma_i\right), \end{aligned} \tag{6}$$

where $\mathcal{Q}(\gamma_i|\Gamma)$ is the slicing function that outputs b_i bits with input γ_i conditioned on the modulation order thresholds T_k. $d_k = \log_2 M_k$ is the number of bits assigned to the sub-channel when the effective SNR falls in the interval: $[T_k, T_{k+1})$. $p(\gamma_i)$ is the probability density function (p.d.f.) of γ_i. Now the problem remained is to determine the p.d.f. of γ_i for different schemes.

3.1. DRSE of OSTBC in i.i.d Rayleigh fading channel

In an i.i.d. Rayleigh fading channel, the p.d.f. of effective SNR can be derived as [6]:

$$p_\gamma(\gamma) = \left(\frac{RN_t}{\gamma_0}\right)^{N_t N_r} \frac{\gamma^{N_t N_r - 1}}{\Gamma(N_t N_r)} e^{-\frac{RN_t}{\gamma_0}\gamma}, \tag{7}$$

where $\Gamma(n) = (n-1)!$. The DRSE of OSTBC can be derived by inserting (7) into (6):

$$\mathrm{DR}_{ostbc}(\gamma_0) = \frac{1/R}{\Gamma(N_t N_r)} \sum_{k=1}^{K} \Delta d_k \Gamma\left(N_t N_r, \frac{RN_t T_k}{\gamma_0}\right) \tag{8}$$

where $\Delta d_k = d_k - d_{k-1}$ and $d_0 = 0$. $\Gamma(a, x)$ is the incomplete Gamma function $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$.

3.2. DRSE of OSTBC in spatially correlated Rayleigh fading channel

In a spatially correlated Rayleigh fading channel, the effective SNR can be written as [5]:

$$\begin{aligned} \gamma &= \frac{\gamma_0}{RN_t} \mathrm{Tr}\left(\mathbf{H}_w \mathbf{R}_t^{1/2} \left(\mathbf{H}_w \mathbf{R}_t^{1/2}\right)^H\right) \\ &= \frac{\gamma_0}{RN_t} \mathrm{Tr}\left(\underbrace{\mathbf{H}_w \mathbf{U}_t}_{\mathbf{H}_{\tilde{w}}} \mathbf{\Lambda}_t \mathbf{U}_t^H \mathbf{H}_w^H\right) \\ &= \frac{\gamma_0}{RN_t} \mathrm{Tr}\left(\mathbf{H}_{\tilde{w}}^H \mathbf{H}_{\tilde{w}} \mathbf{\Lambda}_t\right) = \frac{\gamma_0}{RN_t} \sum_{i=1}^{N_t} \lambda_i \|\mathbf{h}_{\tilde{w}i}\|_2^2 \end{aligned} \tag{9}$$

[2] DRSE means the rate of every transmission is limited to a certain set of integers as the modulation order is. This is in contrast to the continuous rate suggested in [1].

Tr($\mathbf{A}$) denotes the trace of $\mathbf{A}$. The third equation follows from Tr($\mathbf{AB}$) = Tr($\mathbf{BA}$). $\mathbf{h}_{\tilde{w}i}$ is the ith column vector of $\mathbf{H}_{\tilde{w}}$ and $\|\mathbf{h}_{\tilde{w}i}\|_2$ is the Euclidean norm of it. λ_i is the ith eigenvalue of $\mathbf{R}_t$. Because $\mathbf{U}_t$ is a unitary matrix, the distribution of $\mathbf{H}_{\tilde{w}} = \mathbf{H}_w\mathbf{U}_t$ is the same as $\mathbf{H}_w$. Hence, $\|\mathbf{h}_{\tilde{w}i}\|_2^2$ is Chi-square distributed with the d.o.f equal to $2N_r$, whose p.d.f. is given by [6]:

$$p(x_i) = \frac{1}{\Gamma(N_r)} x_i^{N_r-1} e^{-x_i}, \quad (10)$$

where $x_i = \|\mathbf{h}_{\tilde{w}i}\|_2^2$. Then γ is the sum of N_t weighted Chi-square distributed variables, $\gamma = \sum_{i=1}^{N_t} \gamma_i$, where $\gamma_i = \frac{\gamma_0\lambda_i}{RN_t}x_i$. The p.d.f. of γ_i is then:

$$p(\gamma_i) = \left(\frac{RN_t}{\gamma_0\lambda_i}\right)^{N_r} \frac{\gamma_i^{N_r-1}}{\Gamma(N_r)} e^{-\frac{RN_t}{\gamma_0\lambda_i}\gamma_i} \quad (11)$$

The associated Moment-Generating Function (MGF) of $p(\gamma_i)$ is:

$$M_{\gamma_i}(s) = \frac{1}{\left(1 - \frac{\gamma_0\lambda_i}{RN_t}s\right)^{N_r}} \quad (12)$$

Since γ_i are independent to each other, the MGF of $p(\gamma)$ is the product of the MGFs of $p(\gamma_i)$:

$$M_\gamma(s) = \prod_{i=1}^{N_t} \frac{1}{\left(1 - \frac{\gamma_0\lambda_i}{RN_t}s\right)^{N_r}} \quad (13)$$

By applying the partial fraction expansion, the MGF of $p(\gamma)$ can be split as:

$$M_\gamma(s) = \sum_{i=1}^{N_t}\sum_{j=1}^{N_r} \frac{\varphi_{i,j}}{\left(1 - \frac{\gamma_0\lambda_i}{RN_t}s\right)^{j}} = \sum_{i=1}^{N_t}\sum_{j=1}^{N_r} \frac{\varphi_{i,j}}{(1-\alpha_i s)^j} \quad (14)$$

where the coefficients $\varphi_{i,j}$ can be solved by Heaviside Cover-Up method:

$$\varphi_{i,j} = \frac{(-1/\alpha_i)^{N_r-j}}{(N_r-j)!} \frac{d^{N_r-j}}{ds^{N_r-j}} \Big[(1-\alpha_i s)^{N_r} M_\gamma(s)\Big]\Big|_{s = 1/\alpha_i} \quad (15)$$

The p.d.f. of γ can now be calculated by inverse Laplace transform:

$$\begin{aligned} p_\gamma(\gamma) &= \mathcal{L}^{-1}\Big(M_\gamma(-s)\Big) \\ &= \sum_{i=1}^{N_t}\sum_{j=1}^{N_r} \frac{\varphi_{i,j}}{\Gamma(j)\alpha_i^j} \gamma^{j-1} e^{-\frac{\gamma}{\alpha_i}} \end{aligned} \quad (16)$$

where $\mathcal{L}^{-1}$ denotes the inverse Laplace transform and $\alpha_i = \dfrac{\gamma_0\lambda_i}{RN_t}$. By inserting the p.d.f. derived above into (6), the DRSE of STBC is obtained as:

$$\mathrm{DR}_{ostbc}(\gamma_0, \rho) = R\sum_{k=1}^{K}\sum_{i=1}^{N_t}\sum_{j=1}^{N_r} \frac{\varphi_{i,j}}{\Gamma(j)} \Delta d_k \Gamma\big(j, \frac{T_k}{\alpha_i}\big), \quad (17)$$

where $\Delta d_k = d_k - d_{k-1}$ and $\Gamma\big(a, x\big)$ is the incomplete Gamma function.

3.3. DRSE of SM-ZF

The effective SNR on the ith stream of the ZF receiver is distributed as [5]:

$$p(\gamma_i) = \frac{\sigma_i^2 e^{-\gamma_i\sigma_i^2/\frac{\gamma_0}{N_t}}}{\frac{\gamma_0}{N_t}\Gamma(N_r - N_t + 1)} \left(\frac{\gamma_i\sigma_i^2}{\gamma_0/N_t}\right)^{N_r-N_t} \quad (18)$$

where σ_i^2 is the ith diagonal entry of $\mathbf{R}_t^{-1}$. The DRSE can be obtained by inserting (18) into (6):

$$\mathrm{DR}_{zf}(\gamma_0, \sigma_i^2) = \sum_{i=1}^{N_t}\sum_{k=1}^{K} \Delta d_k \frac{\Gamma(N_r - N_t + 1, \frac{\sigma_i^2 T_k}{\gamma_0/N_t})}{\Gamma(N_r - N_t + 1)} \quad (19)$$

If the channel is spatially uncorrelated, then $\rho = 0$ and $\sigma_i^2 = 1$. The resulting DRSE of the uncorrelated channel is:

$$\mathrm{DR}_{zf}(\gamma_0) = \sum_{i=1}^{N_t}\sum_{k=1}^{K} \Delta d_k \frac{\Gamma(N_r - N_t + 1, \frac{T_k}{\gamma_0/N_t})}{\Gamma(N_r - N_t + 1)} \quad (20)$$

4. A LOW COMPLEXITY ADAPTATION SCHEME

In a conventional adaptive system [1], the modulation order at the transmitter is updated periodically depending on the instantaneous CQI. In this paper, we extend the scope of adaptation by adding a scheme selection method to further improve spectral efficiency.

The selection between OSTBC and SM-ZF is based on the DRSE, which can be used to evaluate the average spectral efficiencies of the two schemes. DRSE depends on the statistical channel information–the average SNR and spatial correlation coefficient. Since the statistical information does not change as time elapses, the selection of the optimal scheme only needs to be done once. The following operation to determine the modulation order is exactly the same as the conventional adaptive system once the scheme is determined. Therefore, only one bit (the choice for the scheme) is fed back via link adaptation to determine the scheme, which adds limited complexity to the existing adaptive modulation systems with a single MIMO scheme.

5. SIMULATION AND NUMERICAL RESULTS

We consider adaptive MQAM modulation with the constellation size $M = \{0, 2, 4, 16, 64\}$. The target BER is set to 0.1%. The average spectral efficiencies are obtained based on $10,000$ channel realizations.

5.1. DRSEs of OSTBC and SM-ZF

Spectral efficiencies of OSTBC and SM-ZF in i.i.d. Rayleigh fading channel are plotted in Fig. 1, where both the empirical results from computer simulation and the theoretical results from (8) and (20) are presented. In case of 3×3 MIMO systems, we use the orthogonal codes with $R = 1/2$. This explains that spectral efficiencies level out at 3 *bits/channel use* when $N_t = 3$. As can be seen from the Fig. 1, the theoretical results match very well with the simulation results for all cases, indicating that DRSE is an accurate estimation of average spectral efficiency that can be achieved by applying adaptive modulation.

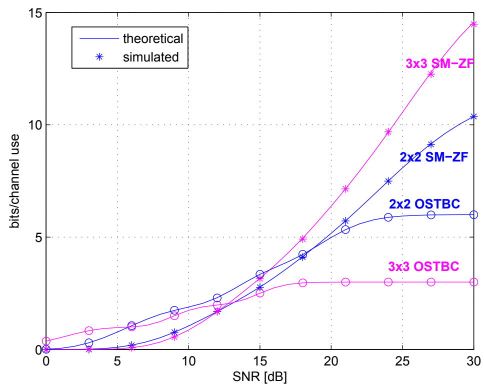

Fig. 1. Spectral efficiencies in an i.i.d. Rayleigh fading channel

Similarly, the performance of OSTBC and SM-ZF in spatially correlated Rayleigh fading channel is shown in Fig. 2, where the DRSE of OSTBC is given by (17) and the DRSE of SM-ZF is provided in (19). As expected, the theoretical results provide accurate estimation for the simulation results.

Since DRSE is an accurate estimation of average spectral efficiency, it can be applied to calculate the switching point of OSTBC and SM-ZF. Although a closed-form expression for the switching point is not available, a numerical result can be computed by using Newton-Ralphson method. In spit of incomplete Gamma function in the DRSE, the result converges fast because there is only one root for equation:

$$\mathrm{DR}_{zf} = \mathrm{DR}_{ostbc} \tag{21}$$

6. CONCLUSIONS

In this paper, we considered an adaptive MQAM modulation system that maximizes achievable spectral efficiency. Perfect CSI is assumed at receiver and CQI is available at transmitter. Closed-form expressions of average spectral efficiency, termed as DRSE, is derived for OSTBC and SM-ZF with N_t transmit antennas and N_r receive antennas in different channel conditions. Furthermore, a low complexity switching scheme that switches between OSTBC and SM-ZF based on the DRSE has the potential to dramatically increase the achievable spectral efficiency.

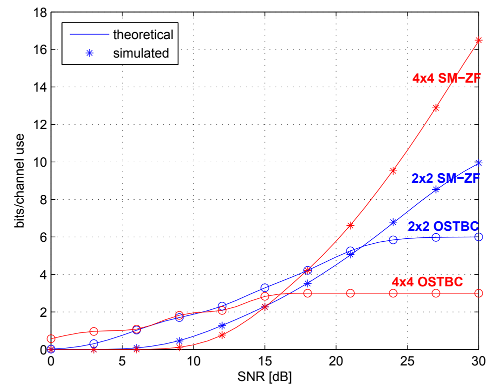

Fig. 2. *Spectral efficiencies achieved by OSTBC and SM-ZF in a spatially correlated Rayleigh fading channel with $\rho = 0.5$*

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7. REFERENCES

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