A LOW COMPLEXITY SELECTIVE MAPPING TO REDUCE INTERCARRIER INTERFERENCE IN OFDM SYSTEMS

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ABSTRACT

One of major drawbacks of orthogonal frequency division multiplexing is sensitivity to frequency offsets caused by a mismatch between the transmitter and receiver oscillators. This offset destroys the orthogonality of the sub-carriers and introduces intercarrier interference (ICI), reducing the system performance. Previously, selective mapping was considered for reducing the peak interference-to-carrier ratio (PICR). However, this technique has a high computational complexity due to multiple inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) operations. In this paper, we exploit the IFFT/FFT structure to generate phase rotated SLM sequences and obtain a low PICR. This technique significantly reduces the computational complexity while providing a PICR performance close to that of the previous SLM technique.

Index Terms— Orthogonal frequency-division multiplexing (OFDM), peak interference-to-carrier ratio (PICR), selective mapping (SLM).

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is an effective multi-carrier modulation technique that provides efficient bandwidth utilization and robustness against time dispersive channels. However, OFDM is sensitive to frequency offset. This occurs when there is a difference between the transmitter and receiver local oscillators due to oscillator errors and/or doppler spread. Frequency offset introduces intercarrier interference (ICI) and its characteristics are analogous to Gaussian noise [2]. Hence, it degrades the bit error rate (BER) performance.

Selective mapping (SLM) is a technique used to reduce frequency offsets by reducing the peak interference -to-carrier ratio (PICR)[1]. It is adapted from peak-to average power ratio (PAPR) reduction techniques as there is a similarity between the PAPR and PICR. With SLM, multiple OFDM symbols representing the same information are generated and the OFDM symbol with the lowest PICR is chosen for transmission. This results in ICI reduction.

However, SLM for PAPR reduction require multiple IFFTs at the transmitter which results in high computational complexity. When SLM is employed for PICR reduction, this complexity is even worse. This is because ICI occurs at the receiver, hence the PICR computation for each SLM sequence requires an N-point IFFT and an N-point FFT [1]. In this paper, we present a solution to this problem.

To reduce the computational complexity, we use inputs to the middle stages of the IFFT and FFT using a decimation in frequency (DIF) algorithm [3]. Subsets of the inputs to the middle stages of the transforms are phase rotated. To obtain these partial subsets, random sequences are employed. This leads to alternative signal representations. In terms of PICR and complexity reduction, we examine the proposed SLM technique and compare it with the SLM technique in [1]. Numerical results show that there is a slight degradation in PICR reduction compared to the SLM method in [1], but the computational complexity is significantly reduced.

In the next section, we review the PICR and ICI reduction using SLM. Section 3 introduces the proposed SLM technique. Section 4 compares the performance of the proposed technique with that in [1] in terms of PICR and complexity reduction. Some conclusions are given in Section 5.

2. PICR AND SLM

To obtain the OFDM frequency offset, we consider Moose's frequency offset estimation method [4]. Let $\{X(k)\}_{k=0}^{N-1}$ denote the frequency-domain OFDM signal at the transmitter, where N is the number of IFFT points (subcarriers) and k is the frequency index. The time-domain OFDM signal is obtained by taking an N-point inverse discrete Fourier transform (IDFT) of $X(k)$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) T_N^{-nk} \quad 0 \leq n \leq N-1 \qquad (1)$$

where n is the discrete-time index, $T_N = e^{-j2\pi/N}$ (known as the twiddle factor), and $j^2 = -1$. The time-domain samples are converted to an analog signal, up converted to the carrier frequency, and transmitted over the channel. We assume the channel is AWGN. At the receiver, the reverse operations are employed. Since there is a mismatch between carrier frequencies in the transmitter and receiver, the received signal $y(n)$ has a frequency offset. Using (1) and taking the FFT of the received signal in order to recover the original signal, we obtain

$$\begin{aligned} R(k) &= \sum_{b=0}^{N-1} X(b)S(b,k) + n(k) \\ &= X(k)S(k,k) + \sum_{\acute{b}=0}^{N-1} X(\acute{b})S(\acute{b},k) + n(k) \end{aligned} \quad (2)$$

where $\acute{b} \neq k$ and

$$S(\acute{b},k) = \frac{\sin(\pi(\acute{b}-k+\epsilon))}{\pi(\acute{b}-k+\epsilon)} e^{j\pi(\acute{b}-k+\epsilon)}, \quad (3)$$

where ϵ is the normalized frequency offset. The first term in (2) is the original signal shifted by $S(k,k)$. Since $S(k,k)$ is only a function of ϵ, the frequency offset has the same effect on all subcarriers. The second term in (2) represents the ICI on the kth subcarrier and is a function of ϵ and the transmitted data sequence $X = \{X(k)\}_{k=0}^{N-1}$.

Let $I(k)$ denote the second term in (2). The PICR of X is defined as [1]

$$\text{PICR}(X) = \frac{\max\limits_{0\leq k\leq N-1} |I(k)|^2}{|S(k,k)X(k)|^2} \quad (4)$$

If (4) is minimized, the ICI is reduced.

Consider a phase rotated version of $X(k)$ given by $P^u(k) = X(k)e^{j\phi^u(k)}$ where the $\phi^u(k)$ are randomly chosen from $\{0,\pi\}$ or $\{0,\pi/2,\pi,3\pi/2\}$. In SLM as a PAPR reduction technique, the time-domain OFDM signal $p^u(n)$ is obtained using the IDFT of $P^u(k)$ according to (1). The lowest PAPR signal $p^{u'}(n)$ is chosen for transmission from among U candidate OFDM signals, including the original $x(n)$. To determine the selected sequence at the receiver, $\log_2 U$ bits of side information must be sent along with the data. SLM as a PAPR reduction technique requires U IDFTs to obtain the sequences $p^u(n)$, which results in high computational complexity for typical values of U.

Considering SLM as a PICR reduction technique, the resulting ICI can be given as

$$I_{SLM}(k) = \sum_{\acute{b}=0,\acute{b}\neq k}^{N-1} e^{j\phi^u(k)} X(\acute{b})S(\acute{b},k) \quad (5)$$

and the lowest PICR is obtained by

$$\text{PICR}_{lowest}(X) = \min_{\phi^1(k),\ldots,\phi^U(k)} \left[\frac{\max\limits_{0\leq k\leq N-1} |I_{SLM}(k)|^2}{|S(k,k)X(k)|^2} \right] \quad (6)$$

Similar to SLM as a PAPR reduction technique, at the transmitter we require U IDFTs to obtain the time-domain sequences $p^U(n)$. However, as seen in (6), we need the PICR in the frequency-domain which requires U DFTs at the transmitter. Hence, this results in extremely high computational complexity and motivates us to propose a new low complexity SLM technique for PICR reduction.

3. PROPOSED SLM TECHNIQUE FOR PICR REDUCTION

We use intermediate signals within an N-point IFFT and FFT using decimation in frequency (DIF) to obtain U partially phase rotated SLM sequences and compute U PICRs, respectively. Hence, we first consider time-domain SLM sequences which are obtained by taking the IFFT of $\{X(k)\}_{k=0}^{N-1}$.

We find U sequences $p^u(n)$ using inputs to intermediate stages within the IFFT. The IFFT computation can be written as [5]

$$x(n) = \frac{1}{N}\left[\sum_{k=0}^{N-1} X^*(k) T_N^{nk}\right]^* \quad (7)$$

Let $y(.)$ represent $X^*(.)$. The expression inside the brackets in (7) is the discrete Fourier transform (DFT) of $X^*(.)$, i.e.

$$Y(k) = \sum_{k=0}^{N-1} y(n) T_N^{nk} \quad (8)$$

Here, symbols and indices for an intermediate signal are represented with different notation: $\widetilde{y}$ and $\widetilde{n}$ for an intermediate signal y and index n, respectively, and $\widetilde{k}$ for a subcarrier index (at the FFT output) corresponding to index $\widetilde{n}$.

It is well known that radix FFT algorithms are efficient implementations of the DFT. The FFT output corresponding to the intermediate signal $\widetilde{y}$ using DIF radix-r at a particular stage v is given by [3]

$$\begin{aligned} Y(rk+k_0) &= \sum_{\alpha=1}^{r^{v-1}} Y^\alpha(r\widetilde{k}+k_0) = \sum_{\alpha=1}^{r^{v-1}} \sum_{\widetilde{n}=0}^{\frac{N}{r^v}-1} \\ &\quad ((\sum_{i=0}^{r-1} \widetilde{y}^{\,\alpha}(\widetilde{n}+\frac{N}{r^v}i) T_r^{ik_0}) T_{N/r^{v-1}}^{\widetilde{n}\,\widetilde{k}_0}) T_{N/r^v}^{\widetilde{k}\,\widetilde{n}} \end{aligned} \quad (9)$$

where $k_0, 0 \leq k_0 \leq r-1$, is the index of the twiddle factors, $\widetilde{k} = 0,\ldots,N/r^v - 1$, $\widetilde{n} = 0,\ldots,N/r^v - 1$ and $\alpha, \alpha = 1,\ldots,r^{v-1}$, denotes a particular N/r^{v-1}-point DFT at stage v, $v = 1,\ldots,m$. A phase rotated version of $\widetilde{y}^\alpha(\widetilde{n}+\frac{N}{r^v}i)$ is given by

$$P_\alpha^u(\widetilde{n}) = \widetilde{y}^\alpha(\widetilde{n}+\frac{N}{r^v}i)e^{j\phi^u(\widetilde{n})} \quad (10)$$

and the phase rotated SLM sequences within the IFFT are then

$$P^u(\widetilde{n}) = [P_1^u(\widetilde{n}), P_2^u(\widetilde{n}),\ldots,P_\alpha^u(\widetilde{n}),\ldots,P_{r^{v-1}}^u(\widetilde{n})]. \quad (11)$$

We propose a low complexity SLM technique where a subset of the $P_\alpha^u(\tilde{n})$ are phase rotated so the time-domain $p^u(n)$ are obtained using partial $r^{v-1} \times N/r^{v-1}$-point DFTs.

Let λ denote the number of subsets of N/r^{v-1}-point DFTs where $\lambda \leq U$, $2 \leq \lambda < r^{v-1}$ and λ is a power of 2. When $\lambda = 2^0$, there are no partial subsets. We first construct a pseudo-random sequence $S_0 = [s_0\, s_1\, \ldots s_{r^{v-1}}]$ over Z_λ, the integers modulo λ. Then, we obtain the matrix $C = \left[S_0\, S_1\, \ldots S_u \ldots S_{\frac{U}{\lambda}-1}\right]^T$ of dimension $\frac{U}{\lambda} \times r^{v-1}$, where

$$S_{\beta+1} = S_\beta + \lambda I \bmod U, \quad 0 \leq \beta \leq \frac{U}{\lambda} - 2 \tag{12}$$

and I is the identity vector of dimension r^{v-1}. The elements in C represent the N/r^{v-1}-point DFTs corresponding to the uth SLM sequence. As an example, consider $N = 32$, $v = 4$, $r = 2$, $U = 4$, and $\lambda = 2$. We generate $S = [01001110]$ over Z_2 and obtain

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 2 & 3 & 2 & 2 & 3 & 3 & 3 & 2 \end{bmatrix} \tag{13}$$

Assume the initial sequence at stage $v = 4$ is $P^I(\tilde{n}) = \left[P_1^I(\tilde{n})P_2^I(\tilde{n})P_3^I(\tilde{n})P_4^I(\tilde{n})P_5^I(\tilde{n})P_6^I(\tilde{n})P_7^I(\tilde{n})P_8^I(\tilde{n})\right]$. From (13), the 4 phase rotated sequences are

$$\left[P_1^0(\tilde{n})P_2^I(\tilde{n})P_3^0(\tilde{n})P_4^0(\tilde{n})P_5^I(\tilde{n})P_6^I(\tilde{n})P_7^I(\tilde{n})P_8^0(\tilde{n})\right],$$
$$\left[P_1^0(\tilde{n})P_2^1(\tilde{n})P_3^0(\tilde{n})P_4^0(\tilde{n})P_5^1(\tilde{n})P_6^1(\tilde{n})P_7^1(\tilde{n})P_8^0(\tilde{n})\right],$$
$$\left[P_1^2(\tilde{n})P_2^1(\tilde{n})P_3^2(\tilde{n})P_4^2(\tilde{n})P_5^1(\tilde{n})P_6^1(\tilde{n})P_7^1(\tilde{n})P_8^2(\tilde{n})\right],$$
$$\left[P_1^2(\tilde{n})P_2^3(\tilde{n})P_3^2(\tilde{n})P_4^2(\tilde{n})P_5^3(\tilde{n})P_6^3(\tilde{n})P_7^3(\tilde{n})P_8^2(\tilde{n})\right].$$

The proposed technique has v stages with one N-point DFT, and $(m - v)$ stages with $(\frac{U}{\lambda} + 1)r^{v-1} \times \frac{N}{r^{v-1}}$-point DFTs. Note that the first sequence is computed over all r^{v-1}-point DFTs. This is illustrated in Fig. 1 and shows a significant complexity reduction compared to the SLM in [1], which requires m stages of $U \times N$-point DFTs.

In order to calculate the frequency-domain SLM sequences, as indicated in [1], we require a value for ϵ at the transmitter to obtain $I(k)$. However, the offset frequency ϵ is not known at the transmitter. Thus, to obtain the PICR, a worst case ϵ is considered, ϵ_{wc}, where $|\epsilon| < \epsilon_{wc}$ [1]. We assume the inputs and outputs of the IFFT are in reverse and normal order, respectively. Hence, the FFT computation is symmetric to the IFFT computation. As a consequence, to obtain the PICR of U SLM sequences, we require v stages of $(\frac{U}{\lambda}+1)r^{v-1} \times \frac{N}{r^{v-1}}$-point DFTs and $(m - v)$ stages of $U \times N$-point DFTs. A block diagram illustrating this computation for new SLM is depicted in Fig. 1. For the new SLM PICR reduction technique, we can compute the ICI based on (5), and the lowest PICR is obtained using (6).

It is obvious that the same FFT computation as above is used for recovery of the transmitted signal at the receiver. The locations of the phasing and de-phasing sequences $P^u(\tilde{n})$ are designed once (offline) for both the FFT and IFFT. As a consequence, the new SLM does not introduce any complexity to the system and requires the same side information as the technique in [1].

The effect of $m - v$ and λ on the PICR reduction is considered in the next section.

4. PERFORMANCE RESULTS

To obtain the computational complexity, let M_v and A_v denote the number of twiddle factors $T_{N/r^{v-1}}^{\tilde{n}k0}$ and $T_r^{ik_0}$ [1] (representing complex multiplications) and addition at stage v for the DIF algorithm, respectively [3]. The overall multiplicative and additive complexity of the IFFT computation for new SLM (see Fig. 1) are

$$M_{total}^{IFFT} = \sum_{\beta=1}^{v-1} M_\beta + (\frac{U}{\lambda} + 1)\sum_{\beta=v}^{m} M_\beta \tag{14}$$

and

$$A_{total}^{IFFT} = A_v[v + (\frac{U}{\lambda} + 1)(m - v)] \tag{15}$$

respectively. Similarity, we can obtain the corresponding complexities for the FFT.

To compare the multiplicative and additive complexity between two PICR techniques, we define the reduction ratios $R^{mul} = 1-(M_{total}^1/M_{total}^2)$ and $R^{add} = 1-(A_{total}^1/A_{total}^2)$, respectively. Tables I and II summarize the multiplicative and additive complexity reductions for different λ and $m - v$ values with $N = 1024$ and $r = 2$. For numerical results, we consider BPSK modulated OFDM signals with $N = 1024$, $U = 8$, and $\epsilon = 0.1$ [1]. The complementary cumulative distribution function (CCDF) of the PICR (4) is used to evaluate the PICR performance. We randomly choose the phase sequences $\phi^u(\tilde{n})$ from $\{0, \pi\}$.

Fig. 2 shows the CCDF of new SLM with $\lambda = 2^0$ (no partial SLM) and $r = 2$. For $m - v = 5$ and $m - v = 6$, there is a slight performance degradation with new SLM compared to SLM in [1]. We found through simulation that $m - v = 5$ gives similar results for $256 \leq N \leq 2048$. The CCDF of PICR for different λ and $m - v = 5$ is shown in Fig. 3. For $\lambda = 2$, new SLM performs close to SLM in [1]. In this case, the proposed SLM achieves multiplicative and additive complexity reductions of $R^{mul} = 71\%$ and $R^{add} = 66\%$, respectively, for the IFFT compared with the SLM technique in [1]. The corresponding complexity reductions for the FFT are $R^{mul} = 27\%$ and $R^{add} = 22\%$.

[1] The twiddle factors $T_r^{ik_0}$ do not introduce any multiplicative complexity as they are $(\pm 1, \pm j)$ if we use radix-2 and/or radix-4.

Table 1. Multiplicative and Additive Complexity Reduction with $N = 1024$, $U = 8$, $\lambda = 2$, and $2 \leq m - v \leq 5$

IFFT				
$m - v$	New SLM compared to SLM in [1], IFFT		New SLM compared to SLM in [1], FFT	
	$R^{mul}(\%)$	$R^{add}(\%)$	$R^{mul}(\%)$	$R^{add}(\%)$
2	85	79	41	35
3	81	74	37	30
4	76	70	32	26
5	71	66	27	22

5. CONCLUSIONS

The main drawback in using the SLM technique in [1] to reduce the peak interference-to-carrier ratio (PICR) is the computational complexity of the multiple inverse fast Fourier transforms (IFFTs) and fast Fourier transforms (FFTs). To reduce this complexity, we generated time-domain SLM sequences using partial inputs to the middle stages of the IFFT. A comparison between this new technique and the approach in [1] was presented in terms of PICR reduction and computational complexity. Our technique significantly reduces the computational complexity compared to that in [1], while providing approximately the same PICR performance.

6. REFERENCES

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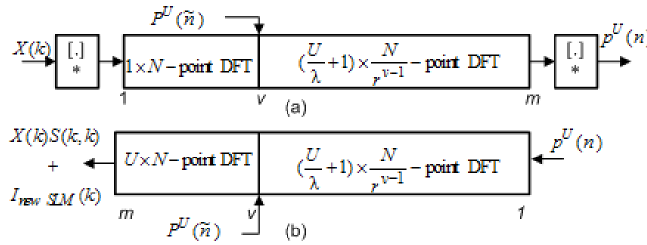

Fig. 1. PICR computation for the new SLM sequences. (a) IFFT (b) FFT.

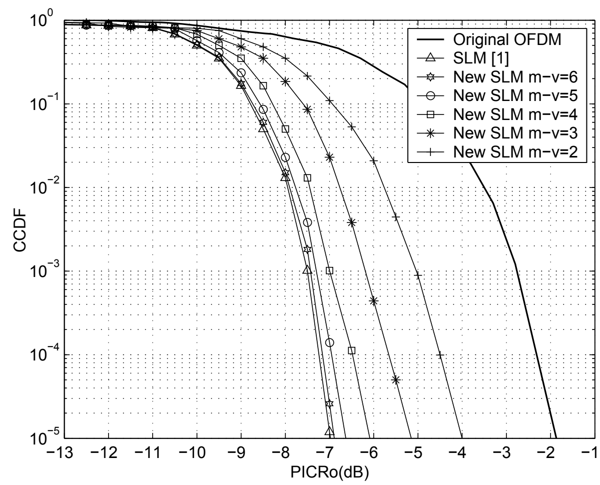

Fig. 2. PICR CCDF for the proposed SLM compared with SLM in [1] for various $m-v$, with $\epsilon = 0.1$, $r = 2$, $N = 1024$, and $\lambda = 2^0$ (no partial SLM).

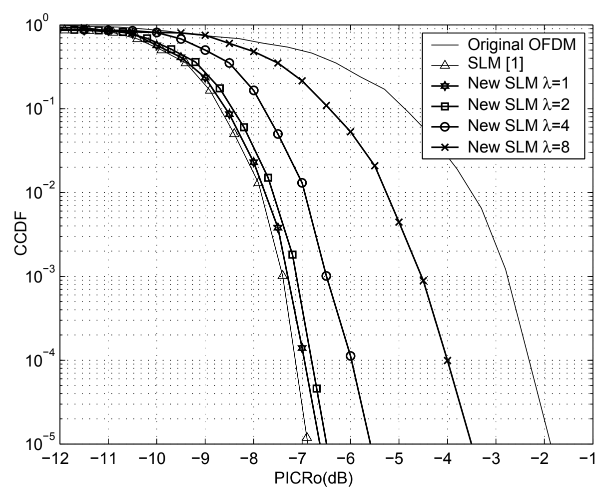

Fig. 3. PICR CCDF for the proposed SLM compared with SLM in [1] for various λ, with $\epsilon = 0.1$, $r = 2$, $N = 1024$, and $m - v = 5$.