DISTRIBUTION OF ENVELOPE POWER USING SELECTED MAPPING IN OFDM SYSTEMS WITH NONLINEARITY

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ABSTRACT

Selected mapping (SLM) techniques have been used to reduce the high peak-to-average power ratio (PAR) in Orthogonal Frequency Division Multiplexing (OFDM) systems. The capacity of PAR reduction can be increased by improving the PAR statistics at the cost of complexity of the OFDM transmitter. However, it is difficult to clarify the performance degradation in the presence of nonlinearities, such as a power amplifier (PA), with the PAR metric because the statistical distribution of PAR focuses only on the highest peak, even though PAR reduction can help to reduce the required PA backoff. The nonlinear distortion is related to the amount of signal greater than the saturated threshold value; therefore, it is desirable to know the envelope power distribution. In this paper, we derive a closed form expression for the envelope power distribution in SLM/OFDM systems with nonlinearity, and verify the derived expression with simulation results.

***Index Terms*—** Orthogonal frequency division multiplexing (OFDM), peak-to-average ratio (PAR), selected mapping (SLM), envelope power, nonlinearity

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a bandwidth efficient multicarrier modulation technique for high speed wireless data transmission over multipath fading channels. However, due to the large number of subcarriers, high Peak-to-Average power Ratio (PAR) in the time domain is one of the key challenges in OFDM systems. As a result, OFDM is sensitive to nonlinear distortions. To avoid these unwanted nonlinear distortions, high PARs require significantly higher output backoff (OBO) in the power amplifier to ensure that signals are linearly amplified. This high OBO degrades the required signal-to-noise ratio (SNR) and leads to the power inefficiency in the transmitter, which is especially important in a mobile device with limited battery power.

PAR has been extensively researched in the past decade. Many PAR reduction techniques have been proposed in the literature, each with advantages and drawbacks [1]. These techniques can be divided into two classes: peak cancelation and multiple signal mapping.

The peak cancelation class reduces the original waveform with the anti-peak signal. This class includes clipping, tone injection (TI), active constellation extension (ACE), and tone reservation (TR). These techniques, except for TR, allow a slight distortion or modification of the symbol constellation in order to reduce PAR. However, these distortion-bearing schemes result in higher total degradation when the nonlinear effect of the power amplifier is considered [2].

The multiple signal mapping class includes coding, selected mapping (SLM), and partial transmit sequence (PTS). When coding, it is difficult to maintain a reasonable coding rate in multicarrier systems with a large number of subcarriers. SLM and PTS are quite flexible because they modify the data set at the cost of additional side information and complexity. In this class, multiple signals are generated from the same information and of these, the one with the minimum PAR is transmitted.

Previous research has mainly used the distribution of PAR to evaluate the performance of PAR reduction techniques. As a performance criterion of PAR, the complementary cumulative distribution function (CCDF) of PAR is widely used and studied in the literature [3,4]. However, PAR has no direct relationship to the performance degradation of the OFDM system with nonlinearity. Rather, we need the distribution of the envelope power because from it one can compute many system performance metrics. The simulated statistical characteristic of the envelope power in SLM was presented in [5]; however, to the best of our knowledge, an analytic closed form has not yet been derived.

In this paper, we will derive the envelope power distribution function of an OFDM signal in SLM. Deriving the envelope power distribution is important because it will help us directly to compute and analyze the bit-error-rate (BER) of the OFDM system with nonlinearity. Another application of the envelop power distribution is to calculate excess power (EP). Knowledge of EP is important because it is related to the performance degradation of the system with nonlinearity [6].

2. SYSTEM MODEL

Fig. 1 depicts the equivalent-baseband OFDM system model employing SLM for PAR reduction. Let the N-component vector $\mathbf{X}$ denote an OFDM symbol with each component being drawn from an M-ary rectangular QAM. The discrete time-domain complex OFDM symbol $\mathbf{x}$ is given by using the inverse discrete Fourier transform (IDFT) as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{JN} n},\ n = 0, 1, \cdots, N-1 \qquad (1)$$

where n is the discrete time index. Note that the average power is $E[|X_k|^2] = E[|x_n|^2] = \sigma^2$.

Since X_k are assumed to be statistically independent and identically distributed (i.i.d.) random variables with zero mean, x_n are asymptotically Gaussian distributed for large N by the central limit theorem. The peak-to-average power ratio (PAR) is defined as

$$\mathrm{PAR}(\mathbf{x}) \triangleq \frac{\max_{0 \le n \le N-1} |x_n|^2}{E[|x_n|^2]}. \qquad (2)$$

In the selected mapping used to reduce the PAR of OFDM signals, it is assumed that M-independent alternative symbols $\{\mathbf{X}^m | m = 1, 2, ..., M\}$ are generated by forming a Hadamard product of $\mathbf{X} = \{X_1, X_2, ..., X_N\}$ and a set of independent and distinct random phase vectors $\{\mathbf{U}^m | m = 1, 2, ..., M\}$ with $U_k^m = \exp(j\phi_k^m),\ \phi_k^m \in [0, 2\pi),\ 0 \le k \le N-1$. The M multiple alternative symbols are expressed as

$$\mathbf{X}^m = \mathbf{X} \circ \mathbf{U}^m,\ 1 \le m \le M \qquad (3)$$

where $\circ$ denotes a Hadamard product (element by element product) of two matrices. We can preserve the signal power by using unit magnitude random phase vectors $\mathbf{U}^m$. The optimal condition for the phase vector in SLM was investigated in [7]. The multiple OFDM sequences $\mathbf{X}^m$ in the frequency domain are transformed into $\mathbf{x}^{(m)}$ by applying the IFFT, and the transmitted sequence $\widetilde{\mathbf{x}}$ is chosen by selecting a signal with the lowest PAR of $\mathbf{x}^{(m)}$:

$$\widetilde{\mathbf{x}} = \{\mathbf{x}^{(m^*)} | m^* = \arg \min_{1 \le m \le M} \mathrm{PAR}(\mathbf{x}^{(m)})\}. \qquad (4)$$

The index m^* should be transmitted to the receiver in order to correctly recover the original message.

Amplitude limiting devices such as digital-to-analog converters (DAC) and PAs are potential sources of nonlinearity. This can cause the performance degradation of OFDM systems. Since the soft limiter is one of the most common models of memoryless nonlinear devices in the transmitter, we will consider the soft limiter as the source of nonlinearity, as shown in Fig. 1. The soft limiter truncates the amplitude of signals that exceed the clipping level as follows,

$$\overline{x}_n = g(\widetilde{x}_n) = \begin{cases} \widetilde{x}_n, & \text{if } |\widetilde{x}| \le A_{\max} \\ A_{\max} e^{(j\angle \widetilde{x}_n)}, & \text{otherwise} \end{cases} \qquad (5)$$

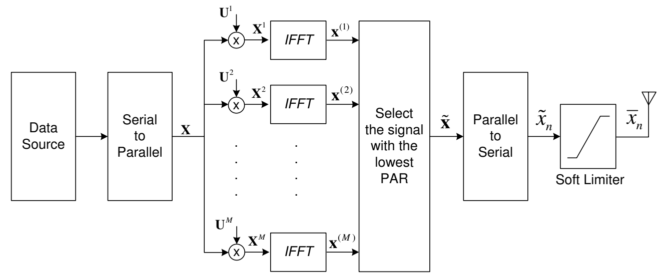

Fig. 1. Block diagram of the SLM OFDM transmitter with the soft limiter

where $A_{\max}$ is the clipping level which can be obtained from the clipping ratio γ defined as

$$\gamma \triangleq \frac{A_{\max}^2}{E[|x_n|^2]}. \qquad (6)$$

3. ENVELOPE POWER DISTRIBUTION FUNCTION OF SLM

As mentioned in the introduction, knowing the distribution of the PAR is not useful in evaluating the performance of OFDM systems using PAR reduction schemes. Knowing the distribution of the output signal $\widetilde{x}_n$ is of much greater value since we can compute many system performance metrics. Thus, in this section, we derive the distribution of the envelope power. In the following section, we will show how one can use the envelope power distribution to compute the excess power.

Assuming the M-candidate signals $\mathbf{x}^{(m)}$ in SLM are i.i.d. complex Gaussian distributed, the probability that the lowest PAR over all M branches is greater than a certain threshold PAR_o (CCDF of the PAR) is given by [4]

$$\Pr\{PAR > PAR_o\} = (1 - (1 - \exp(-PAR_o))^N)^M. \qquad (7)$$

To derive the envelope power distribution function, we consider the transmitted OFDM signal $\widetilde{x}_n$. Note that $\widetilde{x}_n$ is sampled at the Nyquist rate. Let r_i denote the instantaneous normalized sample power of the transmitted signal, $r_i = \frac{|\widetilde{x}_i|^2}{E[|x_n|^2]}, i = 0, 1, ..., N-1$. Then,

$$r_i \le \min_{1 \le m \le M} \frac{\max_{0 \le n \le N-1} |x_n^{(m)}|^2}{E[|x_n|^2]}, \quad \forall i = 0, 1, ...N-1. \qquad (8)$$

Let $\Psi(\gamma)$ denote the envelope power distribution function, which is the probability that the instantaneous normalized power is greater than the clipping ratio γ. The maximum value of r_i is always equal to the PAR given in SLM. In addition, based on the assumption that the components of the candidates $\mathbf{x}^{(m)}$ are i.i.d with a complex Gaussian distribution, r_i is inherently i.i.d; however, its distribution function may not be the chi-square distribution with two degrees of freedom due to the selected mapping for the PAR reduction. However, the CCDF of the maximum value of **r** is identical to (7)

$$\begin{aligned}\Pr(\max_{0\le i\le N-1} r_i > \gamma) &= 1-(1-\Pr(r_0>\gamma))(1-\Pr(r_1>\gamma)) \\ &\quad\cdots(1-\Pr(r_{N-1}>\gamma)) \\ &= 1-(1-\Psi(\gamma))^N \\ &= (1-(1-\exp(-\gamma))^N)^M.\end{aligned} \tag{9}$$

Consequently, the envelope distribution function can be obtained by

$$\begin{aligned}\Psi(\gamma) &= \Pr(r_i>\gamma) \\ &= 1-(1-(1-(1-\exp(-\gamma))^N)^M)^{\frac{1}{N}}.\end{aligned} \tag{10}$$

For $(1-\exp(-\gamma))^N << 1$,

$$\Psi(\gamma) \approx 1 - M^{1/N}(1-\exp(-\gamma)). \tag{11}$$

For large N, $M^{1/N} \longrightarrow 1$; (11) can be simplified as

$$\Psi(\gamma) \approx \exp(-\gamma). \tag{12}$$

When the clipping ratio γ is severely low, the envelope power distribution behaves as (12). On the other hand, as γ increases, the distribution function is highly affected by parameters M and N, as in (10).

4. DETERMINATION OF THE EXCESS POWER

We now demonstrate an application of the envelope power distribution derived in the previous section by computing the excess power.

As more samples are saturated or clipped, more nonlinear distortion is generated [8]. The nonlinear distortions include two primary undesirable effects: (i) spectral regrowth (out-of-band radiation), which causes unacceptable interference to users in neighboring RF channels, and (ii) distortion of the desired signal, which results in a BER performance degradation. PAR does not characterize these nonlinear effects because PAR focuses on the highest value among N samples in an OFDM symbol.

As an alternative to PAR, the excess power (EP) of the OFDM symbol can be used to predict the effect of the nonlinear distortions including both the spectral regrowth and in-band distortion, and is defined in [6] as

$$\mathrm{P}_{\text{excess}} = E[(|\widetilde{x}_n| - |\overline{x}_n|)^2 | (|\widetilde{x}_n| > |\overline{x}_n|)]. \tag{13}$$

This excess power is related to the performance degradation of a system with nonlinearity in [6]. The EP reduction can improve the error-probability performance of OFDM systems in the presence of nonlinearities. We can compute the excess power using equation (10).

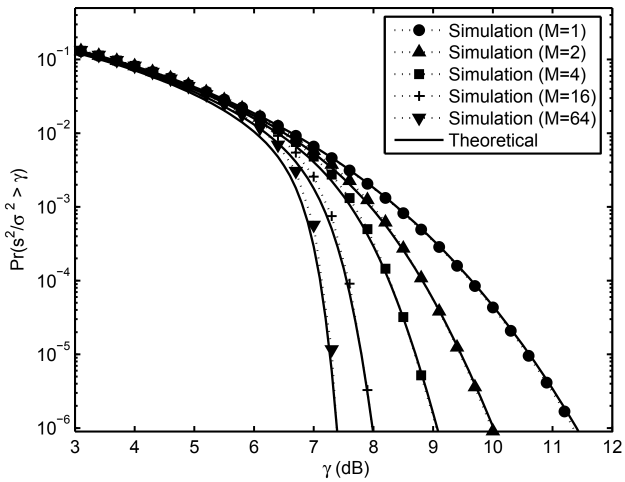

Fig. 2. The envelope power distribution function with the SLM technique for different clipping ratios and M = 1, 2, 4, 16, and 64, when the number of subcarrier N is 512

Let $s = |\widetilde{x}_n| = \sigma\sqrt{r_n}$ denote the envelope of the transmitted signal $\widetilde{x}_n$. The probability density function (pdf) for s is proportional to the derivative of the $\Psi(\gamma)$ in (10) with respect to γ, where $\gamma = \frac{s^2}{\sigma^2}$ as follows:

$$\begin{aligned}p_s(s) =& 2\frac{s}{\sigma^2}\{-\frac{\partial\Psi(\gamma=\frac{s^2}{\sigma^2})}{\partial\gamma}\} \\ =& 2\frac{s}{\sigma^2}M\exp(-\frac{s^2}{\sigma^2})\times \\ & (1-(1-(1-\exp(-\frac{s^2}{\sigma^2}))^N)^M)^{1/N-1}\times \\ & (1-(1-\exp(-\frac{s^2}{\sigma^2}))^N)^{M-1}(1-\exp(-\frac{s^2}{\sigma^2}))^{N-1}.\end{aligned} \tag{14}$$

As a result, the excess power in (13) can be obtained as follows:

$$\begin{aligned}\mathrm{P}_{\text{excess}} =& E[(s-A_{\max})^2|(s>A_{\max})] \\ =& \frac{\int_{A_{\max}}^{\infty}(s-A_{\max})^2 p_s(s)ds}{\int_{A_{\max}}^{\infty}p_s(s)ds}\end{aligned} \tag{15}$$

It is clear that the excess power depends on the envelope power distribution function, which in turn is determined by the SLM. In addition, we can apply the derived expression for the envelope power distribution function to other nonlinear models as well as to the soft limiter in (5). In the next section, we plot the excess power as a function of clipping ratio γ.

5. SIMULATION RESULTS AND DISCUSSION

In this section, we consider OFDM systems with 512 subcarriers and 16 QAM using SLM for PAR reduction. To get better performance from SLM, i.i.d. random phase vectors $\mathbf{U}^m$ with zero mean are used [7].

Figure 2 shows the envelope power distribution functions of the transmitted signal $\widetilde{x}_n$ using SLM with M = 1, 2, 4, 16, and 64. Our theoretically derived expression in (10) shows good agreement with the simulation results. Note that the derived expression (10) captures the effect of the variation of M. This envelope power distribution function (10) is the CCDF of the instantaneous normalized power, which is the probability that the instantaneous normalized power is greater than the clipping ratio γ. From the figure, we observe that at fixed $\Pr\left(s^2/\sigma^2 > \gamma\right)$, the required threshold value γ decreases as M increases, which means that the envelope power fluctuation is reduced by increasing the number of candidate signals $\mathbf{x}^{(m)}$, M. For example, the threshold instantaneous normalized power at the probability 10^{-6} is 11.4, 10, 9.1, 8, and 7.4dB for M = 1, 2, 4, 16, and 64, respectively. Meanwhile, the envelope power distribution functions converge independent of M at low clipping ratios.

To show the validity of the derived distribution (14) for different values of M and γ, EP (15) is plotted in Fig 3, along with the corresponding simulation results. As we mentioned before, excess power is the average power of the clipping signal when signal is greater than some clipping level $A_{\max}$. This EP quantity is difficult to obtain from the existing PAR metric. In Fig. 3, we observe that the required clipping ratio γ dramatically decreases as M increases when the amount of tolerable excess power is fixed. Because EP is quantified by the envelope power distribution function (10), EP decreases as M increases, which is consistent with the trend of the envelope power distribution function shown in Fig. 2. For example, the required γ is dramatically reduced by increasing M at 0.05 EP; the required γ is 9.34, 6.6, 5.9, 5.3 and 4.9dB for M = 1, 2, 4, 16, and 64, respectively. These simulation results are in good agreement with the theoretical expression based on the derived distribution (10).

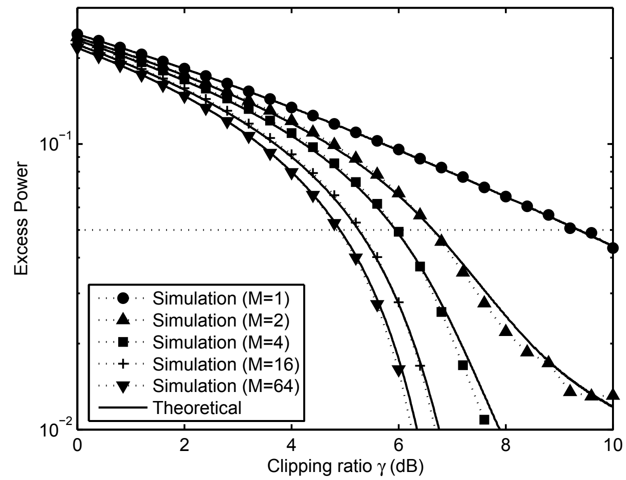

Fig. 3. Excess power of SLM for M = 1, 2, 4, 16, and 64 with respect to clipping ratio γ, together with the corresponding simulation results

6. CONCLUSION

In this paper we have derived the distribution of the envelope power of the OFDM signals, assuming that M candidate signals generated by using SLM are i.i.d. Gaussian random variables. By comparing the simulation results, our derived expression well approximates the statistical behavior of the envelope power in SLM. We also verified the accuracy of the our derived distribution by quantifying the excess power associated with the nonlinear soft limiter. The result of this paper will help us directly to analyze the performance degradation of OFDM systems with nonlinearity.

7. REFERENCES

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