A LOW-COMPLEXITY SYMBOL TIME ESTIMATION FOR OFDM SYSTEMS

Wen-Long Chin and Sau-Gee Chen

Department of Electronics Engineering and Institute of Electronics
National Chiao Tung University
1001 Ta Hsueh Rd, Hsinchu, 30050, Taiwan, ROC

ABSTRACT

Conventional symbol time (ST) synchronization algorithms for orthogonal frequency-division multiplexing (OFDM) systems mostly are based on maximum correlation result of the cyclic prefix. Due to the channel effect, one needs to further identify the channel impulse response (CIR) so as to obtain a better ST estimation. Overall, the required computational complexity is high because it involves correlation operation, as well as the fast Fourier transform (FFT) and inverse FFT (IFFT) operations. In this work, without the FFT/IFFT operations and the knowledge of CIR, a low-complexity time-domain ST estimation is proposed. We first characterize the frequency-domain interference effect as a function of the ST by deriving some analytical equations considering the channel effect. Based on the derivation, the new method locates the symbol boundary at the sampling point with the minimum interference in the frequency-domain. Moreover, for reducing the computational complexity, the proposed frequency-domain minimum-interference metric is converted into a low-complexity time-domain metric by utilizing the Parseval's theorem and the sampling theory. Simulation results exhibit high performances for the proposed algorithm in the multipath fading channels.

***Index Terms*—**OFDM, Symbol Time Estimation

1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is a promising technology for broadband transmission due to its high spectrum efficiency, and its robustness to the effects of multipath fading channels. OFDM has been adopted by many state-of-the-art communication standards such as DVB-T, DAB, xDSL, WLAN systems based on 802.11x standards, and fixed or mobile MAN systems based on 802.16x standards. It has also become a key technology in mobile communication systems beyond 3G. However, it is sensitive to synchronization errors. As a result, one has to achieve as good synchronization as possible in OFDM transmissions.

There are many synchronization issues that should be taken into consideration in OFDM systems. First of all, unknown signal delays introduce the symbol time (ST) offset (STO), and require the coarse symbol time (CST) and fine symbol time (FST) synchronizations. There also exists the carrier frequency offset (CFO) between a transmitter and receiver pair so that the fractional carrier frequency offset (FCFO), integral carrier frequency offset (ICFO) and residual carrier frequency offset (RCFO) have to be eliminated. In addition, the mismatch of sampling clocks between DAC and ADC introduces the sampling clock frequency offset (SCFO).

In [1], the STO and FCFO are jointly estimated by a delayed-correlation algorithm. It is a maximum-likelihood (ML) estimation and only good for the additive white Gaussian noise (AWGN) channels. In [2], a new method making use of training symbols in time-domain was proposed. However, its correlation results exhibit uncertain plateau in multipath fading channels [3]. A remedy for this ambiguity is proposed in [3]. Some techniques [2]-[5] produce good ST performances. However, extra time-domain training symbols are needed. Besides, these algorithms inherently search for the strongest path instead of the first one. Although the technique in [6] can identify ISI-free region in multipath fading channels, for accurate ST estimation, it may involve many symbols. The work in [7] treats ST in multipath fading channels. In [4] and [8], the channel frequency response (CFR) must be estimated first. The inverse fast Fourier transform (IFFT) is then applied to get the channel impulse response (CIR), which is then used to adjust the symbol boundary.

In summary, the mentioned methods are mostly based on the time-domain maximum cyclic-prefix (CP) correlation results combined with channel estimation to achieve satisfactory ST results. In this work, in order to obtain accurate ST estimation with low computational complexity, the proposed scheme is based on a new metric by minimizing interference in the frequency-domain. The theoretical combined interference due to ISI and ICI is derived and applied to accurately locate the best ST with the minimum interference. To reduce the computational complexity, the proposed frequency-domain minimum-interference metric is converted into a low-complexity time-

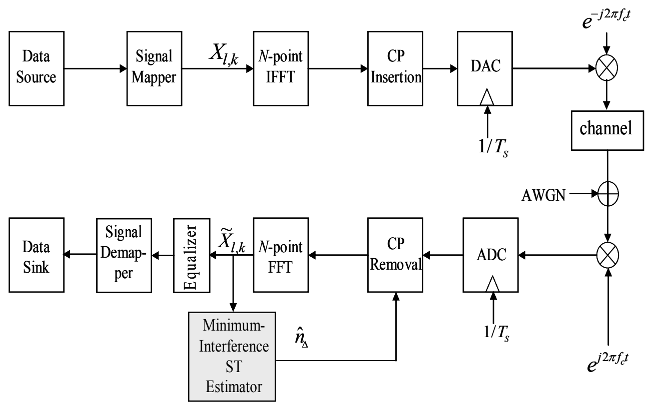

Fig. 1. A simplified OFDM system model.

domain metric by utilizing the Parseval's theorem and the sampling theory. The proposed time-domain approach is low-complexity in the sense that knowledge of the channel profiles and the FFT/IFFT operations are not required.

2. OFDM SYSTEM MODEL

A simplified OFDM system model is shown in Fig. 1. In the figure, $X_{l,k}/\tilde{X}_{l,k}$ is the transmitted/received frequency-domain data on the *k*-th subcarrier, $1/T_S$ is the sampling frequency, f_c is the carrier frequency, and $\hat{n}_\Delta$ is the estimated STO. On the transmitter side, N complex data symbols are modulated onto N subcarriers by using the IFFT. The last N_G IFFT samples are copied to form the CP that is inserted at the beginning of each OFDM symbol. By inserting CP, a guard interval is created so that ISI can be avoided and the orthogonality among subcarriers can be sustained. The receiver uses the fast Fourier transform (FFT) to demodulate received data. Note that the pulse shaping filter and the anti-aliasing filters are not shown in Fig. 1.

As shown in Fig. 2, an estimated ST (associated with a STO n_Δ) generally falls into one of the three depicted regions: the Bad ST1 region, the Good ST region, and the Bad ST2 region in which n_Δ is confined within the ranges of $-N_G \le n_\Delta \le -N_G + \tau_d$, $-N_G + \tau_d + 1 \le n_\Delta \le 0$, and $1 \le n_\Delta \le N-1$, respectively. N_G is the length of CP. That is, the Bad ST1 and Good ST (a.k.a. inter-symbol-interference free) regions are in the guard interval. When the ST is located in the Good ST region, no ISI results; however when the ST is located in the Bad ST1 and Bad ST2 regions, the *l*-th symbol has ISI from the (*l*-1)-th symbol and the (*l*+1)-th symbol, respectively. Furthermore, the CFO and SCFO introduce additional ICI. There is freedom to select the ST in the ISI-free region of the guard interval. This region is obviously defined by the channel length. In the figure, τ_d is the maximum delay spread of the channel. The STO is with reference to the ideal ST of the *l*-th symbol, which is marked by the time index at zero. Detailed analysis

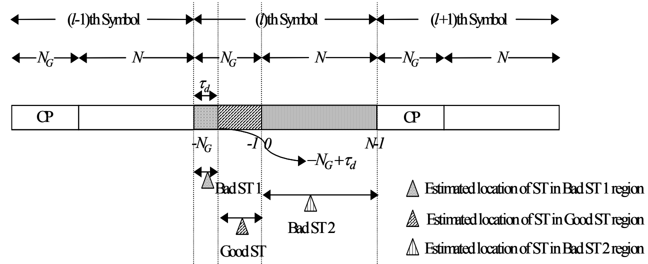

Fig. 2. Three different ST regions.

of the received frequency-domain data on the *k*-th subcarrier in these three regions can be found in [9] and we rewrite the received data in the Bad ST2 region below

$$\tilde{X}_{l,k} = \tilde{X}_{l,k}^{d} + \tilde{N}_k \tag{1}$$

where

$$\tilde{X}_{l,k}^{d} \triangleq \frac{N-n_\Delta}{N} H_k X_{l,k} W_N^{-kn_\Delta} \tag{2}$$

is the desired data,

$$\tilde{N}_k \triangleq \tilde{X}_{l,k}^{ici} + \tilde{X}_{l,k}^{isi} + v \tag{3}$$

is the combined interference plus the AWGN v , $W_N = e^{-j2\pi/N}$, and $N_S = N + N_G$ is the OFDM symbol length including CP. In the latter,

$$\tilde{X}_{l,k}^{ici} = \frac{1}{N}\sum_{m\neq k}\sum_{n'=0}^{N-n_\Delta-1} W_N^{-n'(m-k)} H_m X_{l,m} W_N^{-kn_\Delta} \tag{4}$$

is the ICI, and

$$\tilde{X}_{l,k}^{isi} = \frac{1}{N}\sum_{m}\sum_{n'=N-n_\Delta}^{N-1} W_N^{-n'(m-k)} H_m X_{l+1,m} W_N^{-k(n_\Delta+N_G)} \tag{5}$$

is the ISI. In the equations above, H_m is the CFR on the *m*-th subcarrier. To derive the received data in the Bad ST1 region, the STO n_Δ is replaced with $-N_G + \tau_d - n_\Delta$. Moreover, the desired data, in the Good ST region, has only phase rotation, and there is no ICI and ISI.

3. PROPOSED MINIMUM-INTERFERENCE ST ESTIMATION

3.1. Proposed Frequency-Domain Estimation

First, we assume that in data transmission there are M uniformly-spaced pilot subcarriers in each OFDM symbol. Besides, the pilot values in different OFDM symbols are the same for the same pilot subcarrier index. Moreover, by assuming that the channel is quasi-stationary over two consecutive symbols, the received pilot subcarriers will be the same in the ISI-free region because of nonexistence of the ISI and ICI. Otherwise, the received pilot subcarriers will be different in the Bad ST1 and Bad ST2 regions. By

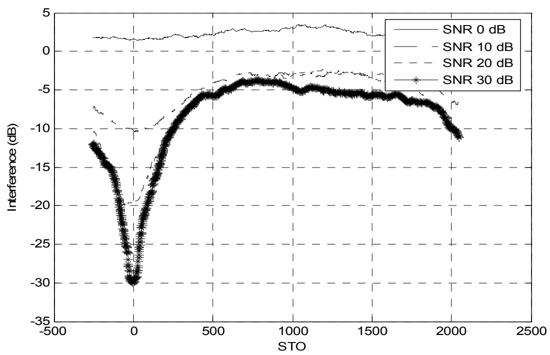

Fig. 3. Interference detected by the minimum-interference estimation of (13).

taking advantage of the property, we propose the following ST estimation metric:

$$\hat{n}_\Delta = \arg\min_{n_\Delta} \sum_{k\in P} \left|\tilde{X}_{l+1,k} - \tilde{X}_{l,k}\right|^2, \;\; 0 \le n_\Delta < N_S. \quad (6)$$

where P is the pilot set. Note that the dependence of $\tilde{X}_{l,k}$ on n_Δ is dropped for clarity. The sampling point that has the minimum value for the square-error sum (SES) metric in (6) is the minimum-interference symbol boundary in the Good ST region, as analyzed below.

3.2. Analysis of the Frequency -Domain Estimation

From Section 2, the expectation value of the SES metric in (6), conditioned on the channel states, in the Good ST region can be easily shown to be

$$E\left[\sum_{k\in P}\left|\tilde{X}_{l+1,k} - \tilde{X}_{l,k}\right|^2\right] = 2M\sigma_v^2 \quad (7)$$

where σ_v^2 is the AWGN power. Likewise, due to the fact that the transmitted data of the l-th and (l+1)-th symbols are uncorrelated, the expectation value of the SES metric in (6) in the Bad ST2 region can be shown to be

$$\begin{aligned} &E\left[\sum_{k\in P}\left|\tilde{X}_{l+1,k} - \tilde{X}_{l,k}\right|^2\right] \\ &= 2\sum_{k\in P}\left(\sigma^2_{ICI,k} + \sigma^2_{ISI,k} + \sigma^2_v\right) \\ &= 2\sum_{k\in P}\left[\frac{m_{|X|^2}}{N^2}\left(\sum_{m\ne k}\sum_{n_1=0}^{N-n_\Delta-1}\sum_{n_2=0}^{N-n_\Delta-1} W_N^{-(n_1-n_2)(m-k)}\left|H_m\right|^2 \right.\right. \\ &\left.\left. + \sum_{m\ne k}\sum_{n_1=N-n_\Delta}^{N-1}\sum_{n_2=N-n_\Delta}^{N-1} W_N^{-(n_1-n_2)(m-k)}\left|H_m\right|^2\right)\right] + 2M\sigma_v^2 \end{aligned} \quad (8)$$

where $\sigma^2_{ICI,k}$ and $\sigma^2_{ISI,k}$ are the ICI and ISI power on the k-th subcarrier, respectively; and $m_{|X|^2} = E\left[\left|X_{l,k}\right|^2\right] = E\left[\left|X_{l+1,k}\right|^2\right]$ is the transmitted signal power. The SES metric in the Bad ST1 region can be obtained by replacing the STO n_Δ with $-N_G + \tau_d - n_\Delta$. From (7)-(8), it is obvious that the SES metric in (6) in the Good ST region has minimum value. Therefore, the proposed estimation locates the symbol boundary at the minimum-interference sampling point in the Good ST region.

3.3. Proposed Low-Complexity Time-Domain Estimation: The Frequency-Domain Counterpart

The computational complexity of the ST estimation in (6) is high because the pilot subcarriers must be derived for each searched sampling point within a symbol duration. To reduce the complexity, one can transform (6) into a time-domain form by the Parseval's theorem and the DTFT sampling theory as follows.

It is assumed that the pilots are located at $\{0, \delta, ..., (M-1)\delta\}$ where $\delta = N/M$ is the pilot subcarrier spacing. Considering the discrete-time Fourier transform (DTFT) of the received time-domain signal $\tilde{x}_{l,n}$

$$\tilde{X}_l(e^{j\omega}) = \sum_{n=0}^{N-1} \tilde{x}_{l,n} \cdot e^{-j\omega n} \quad (9)$$

where $0 \le \omega < 2\pi$. For each n_Δ , to obtain the pilot subcarrier responses, one needs to uniformly sample M data points of $\tilde{X}_l(e^{j\omega})$ at the pilot subcarrier frequency of $\omega_k = 2\pi k/N$, where $k \in P$ and $k = 0, \delta, ..., (M-1)\delta$. From the DTFT sampling theory, this results in the following sub-sampling of the DTFT (9):

$$\tilde{X}_{l,k'} = \sum_{n'=0}^{M-1} \hat{x}_{l,n'} \cdot W_M^{k'n'}, \;\; 0 \le k' < M \quad (10)$$

where k' is the sub-sampling index of k and $\hat{x}_{l,n'}$ is the aliased version of $\tilde{x}_{l,n'}$:

$$\hat{x}_{l,n'} = \sum_{i=0}^{\delta} \tilde{x}_{l,n'+iM}, \;\; 0 \le n' < M. \quad (11)$$

To find the optimum ST by using (6), assuming a specific ST n_Δ, we first need to compute the aliased time-domain signals $\hat{x}_{l,n'}$ and $\hat{x}_{l+1,n'}$ of $\tilde{x}_{l,n}$ and $\tilde{x}_{l+1,n}$, respectively, according to (11). Then, we calculate the difference of the aliased samples as $\Delta\hat{x}_{l+1,n'} \triangleq \hat{x}_{l+1,n'} - \hat{x}_{l,n'}, \;\; 0 \le n' < M$. Finally, the metric (6) reduces to

$$\sum_{k\in P}\left|\tilde{X}_{l+1,k} - \tilde{X}_{l,k}\right|^2 = \sum_{k'=0}^{M-1}\left|\sum_{n'=0}^{M-1}\Delta\hat{x}_{l+1,n'} \cdot W_M^{k'n'}\right|^2 \quad (12)$$

where $\sum_{n'=0}^{M-1}\Delta\hat{x}_{l+1,n'} \cdot W_M^{k'n'}, \; 0 \le k' < M$ can be realized by the FFT operation. The process is repeated for each n_Δ in the ragne of $0 \le n_\Delta < N_S$. As a result, it needs N_S FFTs to acquire pilot subcarriers. To further reduce the huge amount of computation, we can utilize the Parseval's theorem, and

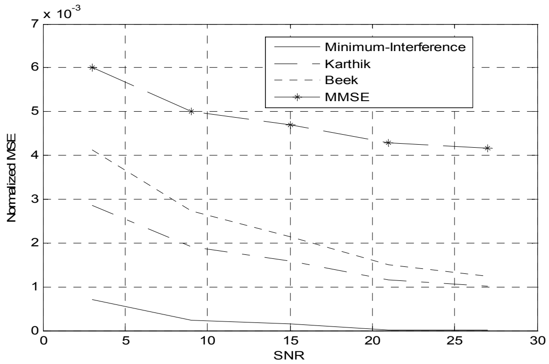

Fig. 4. Normalized MSE against SNR.

map the frequency-domain minimum-interference ST estimation of (6) into to the following time-domain minimum-interference ST estimation:

$$\begin{aligned}\hat{n}_\Delta &= \arg \min_{n_\Delta} \sum_{n'=0}^{M-1} \left|\hat{x}_{l+1,n'} - \hat{x}_{l,n'}\right|^2, \ \ 0 \le n_\Delta < N_S \\ &= \arg \min_{n_\Delta} \sum_{n'=0}^{M-1} \left|\Delta\hat{x}_{l+1,n'}\right|^2, \ \ 0 \le n_\Delta < N_S.\end{aligned} \quad (13)$$

Compared with conventional algorithms, the proposed time-domain algorithm does not need to estimate the CIR and perform the FFT/IFFT operations.

The simulated SES metric of (13) under various signal-to-noise ratios (SNRs) is shown in Fig. 3. We assume an OFDM system of N=2048, N_G=N/8=256, and 64 pilot subcarriers, in a 250-path channel whose CIR is randomly generated by complex Gaussian random variables. The modulation scheme is QPSK. The signal bandwidth is 10 MHz and the radio frequency is 800 MHz. The subcarrier spacing is 4.88 kHz. The OFDM symbol duration is 230.4 μs. As can be seen, the metric has the minimum value in the Good ST region ranging from the -6th to the 0th samples.

4. SIMULATIONS

Monte Carlo simulations were conducted to evaluate the performance of the minimum-interference estimator in the above-mentioned OFDM system. We evaluate the performance of the estimators by means of the estimators' normalized mean-squared error (MSE). In each simulation run, 10,000 symbols are tested. For transmission efficiency consideration, we will focus on comparing the proposed ST technique with those in [1], [6], [7] that do not need extra time-domain training symbols.

The performance of the algorithms is shown in Fig. 4. Fig. 4 is the normalized MSE of the proposed minimum-interference estimator compared with the Beek's estimator [1], the Karthik's estimator [6] and the MMSE estimator [7]. From Fig. 4, the normalized MSE of the proposed minimum-interference estimator are lower than those of the compared estimators because the compared time-domain estimators depend highly on the length of the CP and exploit the redundancy in the CP. In contrast, the proposed technique is developed in frequency-domain and utilizes the special pilot characteristic.

5. CONCLUSION

In this work, a new highly efficient ST estimation technique for OFDM systems is proposed and analyzed. The proposed technique has low-complexity properties: the knowledge of the channel profiles and the FFT/IFFT operations are not required; and the time-domain estimation operates directly on the time-domain samples. It was shown that the proposed technique acquires the ST under low SNR with high accuracy in multipath fading channels.

6. REFERENCES

[1] J. J. van de Beek, M. Sandell, and P. O. Börjesson, "ML estimation of time and frequency offset in OFDM systems," *IEEE Trans. Signal Process*., vol.45, no.7, pp. 1800-1805, Jul. 1997.

[2] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Trans. Commun.*, vol.45, no.12, pp. 1613-1621, Dec. 1997.

[3] B. Park, H. Cheon, C. Kang, and D. Hong, "A novel timing estimation method for OFDM systems," *IEEE Commun. Letters*, vol.7, no.5, pp. 239-241, May 2003.

[4] H. Minn, V. K. Bhargava, and K. B. Letaief, "A robust timing and frequency synchronization for OFDM systems," *IEEE Trans. Wireless Commun.*, vol.2, no.4, pp. 822-839, Jul 2003.

[5] K. Shi and E. Serpedin, "Coarse frame and carrier synchronization of OFDM systems: a new metric and comparison," *IEEE Trans. Wireless Commun.*, vol.3, no.4, pp. 1271-1284, Jul. 2004.

[6] Karthik Ramasubramanian, Kevin Baum, "An OFDM Timing Recovery Scheme with Inherent Delay-Spread Estimation," GLOBECOM'01. IEEE. vol.5, pp. 3111-3115, Nov. 2001.

[7] D. Lee and K. Cheun, "Coarse symbol synchronization algorithms for OFDM systems in multipath channels," *IEEE Commun. Letters*, vol.6, no.10, pp. 446-448, Oct. 2002.

[8] M. Speth, S. Fechtel, G. Fock, and H. Meyer, "Optimum receiver design for OFDM-based broadband transmission-part II: a case study," *IEEE Trans. Commun.*, vol.49, no.4, pp. 571-578, Apr. 2001.

[9] M. Speth, S. Fechtel, G. Fock, and H. Meyer, "Optimum receiver design for wireless broad-band systems using OFDM-part I," *IEEE Trans. Commun.*, vol.47, no.11, pp. 1668-1677, Nov. 1999.