LOW COMPLEXITY BLIND ESTIMATION OF THE CARRIER FREQUENCY OFFSET IN MULTICARRIER SYSTEMS

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ABSTRACT

In orthogonal frequency division multiplexing (OFDM), carrier frequency offset (CFO) must be mitigated since it generates interference between received symbols transmitted through different sub-carriers. This paper presents a new algorithm for CFO estimation with reduced computational complexity. The new approach is based on the segmentation of the input-signal autocorrelation matrix into the noise and signal subspaces, the latter being employed to estimate the desired CFO. Simulations validate the effectiveness of the new algorithm in comparison to the traditional parametric estimation technique ESPRIT.

Index Terms— Signal processing, Digital communications, Parameter estimation

1. INTRODUCTION

The increased demand for bandwidth in modern communications systems leads to severe degradation due to intersymbol interference (ISI). One of the most efficient techniques to mitigate ISI is the use of orthogonal carriers for transmission.

Multicarrier communication systems have been used in a variety of scenarios, such as digital television, digital audio broadcasting, or broadband access to the Internet using copper wires [1]. In those applications, the multicarrier modulation is known as OFDM, which uses the inverse discrete Fourier transform (IDFT) to modulate data using orthogonal carriers for the transmission. When used with cyclic prefix (CP) chosen with proper dimensions, multicarrier modulation and demodulation using IDFT and DFT, respectively, may avoid ISI even in a scenario with severe multipath [2].

The orthogonality among the different subcarriers transforms a frequency selective channel into a set of independent memoryless subchannels. The unmatching between frequencies of the local oscillators causes the loss of orthogonality on the receiver side, the so-called CFO. Therefore, there is intercarrier interference (ICI) within a given transmitted block which degrades the overall system performance. The estimation of CFO performed in the receiver is crucial for its compensation and subsequent restoration of orthogonality among the subcarriers. The development of an ESPRIT-based method in [3] represented the first effort in generating a blind parametric algorithm for CFO estimation. The present paper introduces a new algorithm for the same purpose but with reduced computational complexity.

This article is structured as follows: In section 2, we state the CFO problem and define the notation. We describe the blind CFO estimation applying the estimation of parameters via rotational invariance technique (ESPRIT) and the proposed covariance-based (CB) technique in section 3. In section 4, we provide a comparison between the ESPRIT and proposed algorithm with respect to the corresponding computational complexities. Section 5 presents simulations of both techniques illustrating similar performances achieved by both methods, with much less computational effort provided by the CB-CFO algorithm. Section 6 concludes the paper emphasizing its main contributions.

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2. SYSTEM MODELING

2.1. Problem Formulation

In OFDM, data belonging to a single source is first divided in blocks. Then an IDFT is performed on each block and a CP is added with proper length, which must be longer than the length of the channel impulse response. In the receiver, the CP is removed and data is transformed again to the frequency domain using DFT, then a set of 1-tap equalizations is performed. Such multicarrier scenario completely mitigates ISI and ICI distortions as long as the channel model does not change during one OFDM symbol. Considering that only P out of N subcarriers are effectively used, the remaining $(N-P)$ subcarriers are null, denominated *virtual subcarriers* [3].

Consider the vectors $\mathbf{x}(k) \in \mathbb{C}^N$ and $\mathbf{s}(k) \in \mathbb{C}^P$ including only the samples related to the *non-virtual* subcarriers and $k \in \mathbb{Z}$ is the block index. Therefore, those vectors are related by

$$\mathbf{x}(k) = \mathbf{W}_P \mathbf{s}(k),$$

where $\mathbf{W}_P \in \mathbb{C}^{N\times P}$ includes only the first P columns of the $N\times N$ IDFT matrix $\mathbf{W}$, where $[\mathbf{W}]_{m,n} = \frac{1}{\sqrt{N}} e^{j\frac{2\pi mn}{N}}$ and $m,n = 0,1,..,N-1$.

Consider the circulant matrix $\mathbf{H} \in \mathbb{C}^{N\times N}$, generated by the use of the CP of length L on the original data, such that

$$\mathbf{\Lambda} = \mathbf{W}^H\mathbf{H}\mathbf{W} \in \mathbb{C}^{N\times N}$$

is diagonal. Selecting the first P rows and P columns of $\mathbf{\Lambda}$, one can form $\mathbf{\Lambda}_P \in \mathbb{C}^{P\times P}$, and the received signal after CP removal can be written as [3]

$$\mathbf{y}(k) = \mathbf{W}_P\mathbf{\Lambda}_P\mathbf{s}(k) \in \mathbb{C}^N. \quad (1)$$

Representing the per-subcarrier 1-tap equalizer by the diagonal matrix $\mathbf{G} \in \mathbb{C}^{P\times P}$, then, after the DFT block at the receiver, one gets

$$\mathbf{z}(k) = \mathbf{G}\mathbf{W}_P^H\mathbf{y}(k) = \mathbf{G}\mathbf{\Lambda}_P\mathbf{s}(k) \in \mathbb{C}^P.$$

If $\mathbf{G}\mathbf{\Lambda}_P = \mathbf{I}_P$, then $\mathbf{s}(k)$ can be recovered from $\mathbf{z}(k)$.

For a normalized (by the subcarrier spacing) deterministic CFO ϕ, symbols belonging to different blocks are cumulatively distorted. Each i^{th} sample of the k^{th} OFDM symbol is modulated by $e^{j((k-1)(N+L_{\text{CP}})+i)\phi}$, for $i = 0,1,\ldots,N-1$.

Taking into account the removed CP, the received signal $\mathbf{y}(k)$, defined in equation (1), becomes

$$\mathbf{y}(k) = \mathbf{E}\mathbf{W}_P\mathbf{\Lambda}_P\mathbf{s}(k)e^{j(k-1)\phi(N+L_{\text{CP}})},$$

where $\mathbf{E} = \text{diag}\left\{e^{jn\phi}\right\}_{n=0}^{N-1} \in \mathbb{C}^{N\times N}$ represents the different multiplying CFO terms inside a given block.

Defining $\tilde{\mathbf{s}}(k) = \mathbf{\Lambda}_P\mathbf{s}(k)e^{j(k-1)\phi(N+L_{\text{CP}})} \in \mathbb{C}^P$, then it follows that

$$\mathbf{z}(k) = \mathbf{G}\mathbf{W}_P^H\mathbf{E}\mathbf{W}_P\tilde{\mathbf{s}}(k) = \mathbf{G}\mathbf{Q}\tilde{\mathbf{s}}(k),$$

where $\mathbf{Q} = \mathbf{W}_P^H\mathbf{E}\mathbf{W}_P \in \mathbb{C}^{P\times P}$ is a circulant matrix. Since $\mathbf{G}$ does not cancel $\mathbf{\Lambda}_p$, due to the presence of $\mathbf{Q}$, ICI arises.

The elimination of the ICI requires estimation of the frequency offset and its proper compensation.

2.2. Signal Subspace Estimation

Estimation of parameters via rotational invariance technique (ESPRIT) [4] is an algorithm which performs parametric estimation when the signal presents some inherent redundancy. In order to use ESPRIT for estimating CFO in OFDM systems, Tureli *et. al* employed in [3] special structures for representing received data, also used in the CB-DoA algorithm.

Consider that $y_i(k)$ is the i^{th} element of the vector $\mathbf{y}(k)$, and define the $(N-M)$ auxiliary forward and backward vectors $\mathbf{y}_{\text{F}}^{(i)}(k), \mathbf{y}_{\text{B}}^{(i)}(k) \in \mathbb{C}^{M+1}$ as in [3]

$$\mathbf{y}_{\text{F}}^{(i)}(k) = \left[y_{i-1}(k), y_i(k), \ldots, y_{i+M-1}(k)\right]^T,$$
$$\mathbf{y}_{\text{B}}^{(i)}(k) = \left[y_{N-i}(k), y_{N-i-1}(k), \ldots, y_{N-i-M}(k)\right]^H,$$

for $i = 1,2,\ldots,N-M$ and $N > M \geq P$.

Using the auxiliary matrices

$$\mathbf{E}_{M+1} = \text{diag}\left\{e^{jm\phi}\right\}_{m=0}^{M} \in \mathbb{C}^{(M+1)\times(M+1)},$$
$$\mathbf{\Delta} = \text{diag}\left\{e^{j(\phi+p\omega)}\right\}_{p=0}^{P-1} \in \mathbb{C}^{P\times P}, \quad (2)$$

where $\omega = 2\pi/M$, then, the signal $\mathbf{y}_{\text{F}}^{(i)}(k)$ can be expressed as

$$\mathbf{y}_{\text{F}}^{(i)}(k) = \mathbf{E}_{M+1}\mathbf{W}_{M+1}\mathbf{\Delta}^i\tilde{\mathbf{s}}(k),$$

where $\mathbf{W}_{M+1} \in \mathbb{C}^{(M+1)\times P}$ comprises the first $M+1$ rows of W_P.

Consider another auxiliary vector described by

$$\mathbf{r}(k) = e^{-j\phi(N-1)}\text{diag}\left\{e^{jp\omega(N-1)}\right\}_{p=0}^{P-1}\tilde{\mathbf{s}}^*(k).$$

Then, the signal $\mathbf{y}_{\text{B}}^{(i)}(k)$ is given by [3]

$$\mathbf{y}_{\text{B}}^{(i)}(k) = \mathbf{E}_{M+1}\mathbf{W}_{M+1}\mathbf{\Delta}^i\mathbf{r}(k).$$

The matrix $\mathbf{Y}_{\text{E}}^{(i)}(k) \in \mathbb{C}^{(M+1)\times(M+1)}$ used for performing ESPRIT comes from the sum for each i

$$\mathbf{Y}_{\text{E}}^{(i)}(k) = \mathbf{y}_{\text{F}}^{(i)}(k)\left(\mathbf{y}_{\text{F}}^{(i)}(k)\right)^H + \mathbf{y}_{\text{B}}^{(i)}(k)\left(\mathbf{y}_{\text{B}}^{(i)}(k)\right)^H.$$

Defining $\mathbf{A} = \mathbf{E}_{M+1}\mathbf{W}_{M+1} \in \mathbb{C}^{(M+1)\times P}$, then a mean value is described for $\mathbf{Y}_{\text{E}}^{(i)}(k)$,

$$\begin{aligned}\hat{\mathbf{R}}_{yy} &= \frac{1}{K(N-M)}\sum_{k=1}^{K}\sum_{i=1}^{N-M}\mathbf{Y}_{\text{E}}^{(i)}(k) \\ &= \mathbf{A}\sum_{k=1}^{K}\sum_{i=1}^{N-M}\left(\tilde{\mathbf{s}}(k)\left(\tilde{\mathbf{s}}(k)\right)^H + \mathbf{r}(k)\left(\mathbf{r}(k)\right)^H\right)\mathbf{A}^H \\ &= \mathbf{A}\hat{\mathbf{R}}_{\tilde{s}\tilde{s}}\mathbf{A}^H \in \mathbb{C}^{(M+1)\times(M+1)}. \end{aligned} \quad (3)$$

As in eigendecomposition-based methods [5], $\hat{\mathbf{R}}_{yy}$ may be decomposed in signal and noise subspaces. The eigenvectors corresponding to the P largest eigenvalues of $\hat{\mathbf{R}}_{yy}$ are associated to the signal subspace. Consider $\mathbf{U}_{\text{S}} \in \mathbb{C}^{(M+1)\times P}$ the matrix whose columns comprise the P eigenvectors of $\hat{\mathbf{R}}_{yy}$ associated to the signal subspace. Then $\mathbf{U}_{\text{S}}$ and $\mathbf{A}$ span the same subspace. Therefore, they are related by a full-rank linear transformation $\mathbf{T} \in \mathbb{C}^{P\times P}$ such that [4]

$$\mathbf{A} = \mathbf{U}_{\text{S}}\mathbf{T}. \quad (4)$$

3. BLIND CFO ESTIMATION

3.1. ESPRIT-based CFO Estimation

Define the selection matrices

$$\mathbf{J}_1 = [\mathbf{I}_M \quad \mathbf{0}] \in \mathbb{R}^{M\times(M+1)} \text{ and}$$
$$\mathbf{J}_2 = [\mathbf{0} \quad \mathbf{I}_M] \in \mathbb{R}^{M\times(M+1)},$$

where $\mathbf{J}_1$ selects the first M rows of a matrix and $\mathbf{J}_2$ the last M rows.

Due to the assumption of cumulative CFO between subcarriers, one can infer from the definition in (2) that

$$\mathbf{J}_1\mathbf{A}\mathbf{\Delta} = \mathbf{J}_2\mathbf{A}, \tag{5}$$

Applying (4) one gets

$$\mathbf{J}_1\mathbf{U}_{\mathrm{S}}\mathbf{T}\mathbf{\Delta} = \mathbf{J}_2\mathbf{U}_{\mathrm{S}}\mathbf{T}. \tag{6}$$

Using the non-singular matrix $\mathbf{\Psi} = \mathbf{T}\mathbf{\Delta}\mathbf{T}^{-1} \in \mathbb{C}^{P\times P}$, (6) becomes

$$\mathbf{J}_1\mathbf{U}_{\mathrm{S}}\mathbf{\Psi} = \mathbf{J}_2\mathbf{U}_{\mathrm{S}}, \tag{7}$$

which is the so-called invariance equation that can be solved by means of *least squares* (LS), *total least squares* (TLS) [4], or *structured least squares* methods (SLS) [6].

Since $\mathbf{\Psi}$ and $\mathbf{\Delta}$ are related via a similarity transformation, both have the same eigenvalues, i.e., the elements in the diagonal of $\mathbf{\Delta}$ are the eigenvalues of $\mathbf{\Psi}$. That means, $\mathbf{\Delta}$ can be obtained via an eigenvalue decomposition (EVD) of $\mathbf{\Psi}$.

After estimating $\mathbf{\Delta}$, the CFO parameter ϕ may be estimated by its first element, or by [3]

$$\phi = \frac{1}{j}\ln\frac{\mathrm{tr}(\mathbf{\Delta})}{\sum_{k=0}^{P-1}\mathrm{e}^{jk\omega}}.$$

3.2. Covariance-Based CFO Estimation

In this subsection, a low-complexity algorithm for CFO parametric estimation is presented. The algorithm originates from another derived in a framework of direction-of-arrival [7], using a data modeling defined by Matrix-Pencil methods [8].

As in section 3.1, consider the matrix $\hat{\mathbf{R}}_{yy}$ defined in (3). Besides that, consider $\mathbf{R}_0 \in \mathbb{C}^{M\times M}$ containing the first M rows and M columns of $\hat{\mathbf{R}}_{yy}$, i.e. $\mathbf{R}_0 = \mathbf{J}_1\hat{\mathbf{R}}_{yy}\mathbf{J}_1^T$.

The EVD of $\mathbf{R}_0$ is given by

$$\mathbf{R}_0 = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H.$$

Defining matrices $\mathbf{\Sigma}_{\mathrm{S}} \in \mathbb{C}^{P\times P}$ and $\mathbf{U}_{\mathrm{S}} \in \mathbb{C}^{M\times P}$ containing the P largest eigenvalues of $\mathbf{R}_0$ and their corresponding eigenvectors, there is a matrix $\mathbf{V} \in \mathbb{C}^{P\times P}$ which gives

$$\mathbf{J}_1\mathbf{A} = \mathbf{U}_{\mathrm{S}}\mathbf{\Sigma}_{\mathrm{S}}\mathbf{V}. \tag{8}$$

Using the cross-correlation matrix $\mathbf{R}_{1,0} \in \mathbb{C}^{M\times M}$, defined as

$$\mathbf{R}_{1,0} = \mathbf{J}_2\hat{\mathbf{R}}_{yy}\mathbf{J}_1^T,$$

in equations (3) and (5), one has that

$$\mathbf{R}_{1,0} = \mathbf{J}_2\mathbf{A}\hat{\mathbf{R}}_{\tilde{s}\tilde{s}}\mathbf{A}^H\mathbf{J}_1^T = \mathbf{J}_1\mathbf{A}\mathbf{\Delta}\hat{\mathbf{R}}_{\tilde{s}\tilde{s}}\mathbf{A}^H\mathbf{J}_1^T. \tag{9}$$

Each non-zero element of the diagonal matrix $\hat{\mathbf{R}}_{\tilde{s}\tilde{s}}$ has a different real value, due to the assumption of uncorrelated inputs and the definitions of $\tilde{\mathbf{s}}(k)$ and $\mathbf{r}(k)$. Upon defining $\mathbf{F} = \mathbf{\Sigma}_s^{-1}\mathbf{U}_s^H \in \mathbb{C}^{P\times M}$ and using (8), one gets

$$\mathbf{R}_{1,0} = \mathbf{U}_{\mathrm{S}}\mathbf{\Sigma}_{\mathrm{S}}\mathbf{V}\mathbf{\Delta}\hat{\mathbf{R}}_{\tilde{s}\tilde{s}}\mathbf{V}^H\mathbf{\Sigma}_{\mathrm{S}}^H\mathbf{U}_{\mathrm{S}}^H,$$

or equivalently

$$\mathbf{F}\mathbf{R}_{1,0}\mathbf{F}^H = \mathbf{V}\mathbf{\Delta}\hat{\mathbf{R}}_{\tilde{s}\tilde{s}}\mathbf{V}^H. \tag{10}$$

Since the entries of $\hat{\mathbf{R}}_{\tilde{s}\tilde{s}}$ are real, $\mathbf{\Delta}$ may be obtained by performing an EVD on the left-hand side of (10) and extracting the phase component of each element.

4. COMPUTATIONAL COMPLEXITY

This section presents a comparison between the computational complexity of the proposed covariance-based CFO (CB-CFO) estimation and the most popular implementation of ESPRIT, the TLS-ESPRI[4], in a CFO estimation framework. The main steps required by each algorithm are summarized in Table 1.

Table 1. Comparison between TLS-ESPRIT for CFO estimation [4] and Covariance-Based CFO estimation.

TLS-ESPRIT	Covariance-Based
$[\mathbf{U}_s, \mathbf{\Sigma}_s] = \mathrm{EVD}\left(\hat{\mathbf{R}}_{yy}\right)$	$\mathbf{R}_0 = \mathbf{J}_1\hat{\mathbf{R}}_{yy}\mathbf{J}_1^T$
$\mathbf{E}_0 = \mathbf{J}_1\mathbf{U}_s$	$\mathbf{R}_{1,0} = \mathbf{J}_2\hat{\mathbf{R}}_{yy}\mathbf{J}_1^T$
$\mathbf{E}_1 = \mathbf{J}_2\mathbf{U}_s$	$[\mathbf{U}_s, \mathbf{\Sigma}_s] = \mathrm{EVD}(\mathbf{R}_0)$
$\mathbf{E}_a = \begin{bmatrix}\mathbf{E}_0^H \\ \mathbf{E}_1^H\end{bmatrix}\begin{bmatrix}\mathbf{E}_0 & \mathbf{E}_1\end{bmatrix}$	$\mathbf{F} = \mathbf{\Sigma}_s^{-1}\mathbf{U}_s^H$
$[\mathbf{E}, \mathbf{\Lambda}] = \mathrm{EVD}(\mathbf{E}_a)$	$\mathbf{R}_1 = \mathbf{F}\mathbf{R}_{1,0}\mathbf{F}^H$
$\mathbf{E} = \begin{bmatrix}\mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22}\end{bmatrix}$	
$\mathbf{\Psi} = -\mathbf{E}_{12}\mathbf{E}_{22}^{-1}$	
$[\mathbf{T}, \mathbf{\Delta}] = \mathrm{EVD}(\mathbf{\Psi})$	$[\mathbf{V}, \mathbf{\Delta}] = \mathrm{EVD}(\mathbf{R}_1)$

From Table 1, we verify that TLS-ESPRIT requires: 3 EVDs (1 for a $2P \times 2P$ Hermitian matrix, 1 for an $(M+1)\times(M+1)$ Hermitian matrix and 1 for a $P \times P$ non-Hermitian matrix); 1 full inversion of a $P \times P$ matrix; and 5 matrix multiplications (4 between $P \times M$ matrices and $M \times P$ matrices and 1 between two $P \times P$ matrices).

On the other hand, the CB-CFO estimation method requires 2 EVDs (1 for an $M \times M$ Hermitian matrix and 1 for a $P \times P$ Hermitian matrix); 1 inversion of a $P \times P$ diagonal matrix; and 3 matrix multiplications (1 between a $P \times P$ matrix and a $P \times M$ matrix, 1 between a $P \times M$ matrix and an $M \times M$ matrix and 1 between a $P \times M$ matrix and an $M \times P$ matrix).

A more direct comparison between the number of math operations required by both TLS-ESPRIT and CB-CFO is presented in Table 2, where asymptotic complexity for each operation is stated according to [9]. It is possible to verify that the most computationally complex operations are the EVD and matrix multiplications. The proposed CB-CFO requires less computationally demanding operations than ESPRIT. Besides that, while ESPRIT requires inversion of a full-matrix, CB-CFO requires the inversion of a diagonal matrix.

Table 2. Summarized comparison of TLS-ESPRIT and CB-CFO in terms of number of matrix operations.

Operation	Compl. [9]	ESPRIT	CB-CFO
Non-Herm. Eigend.	$\mathcal{O}(25n^3)$	1	–
Herm. Eigend.	$\mathcal{O}(n^2)$	2	2
Full Inversion	$\mathcal{O}(2n^3/3)$	1	–
Diag. Inversion	$\mathcal{O}(n)$	–	1
Multiplication	$\mathcal{O}(n^3)$	5	3

5. SIMULATIONS

Some computer simulations were performed in order to assess the error performance of TLS-ESPRIT and CB-CFO algorithms. The metrics used for assessment is the normalized mean-square error, as defined in [3]

$$\text{NMSE} = \frac{1}{Q}\sum_{i=1}^{Q}\left(\frac{\hat{\phi}-\phi}{\omega}\right)^2, \tag{11}$$

where Q denotes the number of Monte-Carlo runs. In our framework, $Q = 50$. The other parameters used for simulation were based on the Long-Term Evolution standard for 3GSM networks, $N = 512$ and $P = 310$. The CFO estimation was performed for 200 OFDM blocks. The channel used for simulation is the 12-tap urban channel (U) and the rural channel (R) standardized for GSM systems in [10]. Moreover we have empirically set $M = 511$ and cyclic prefix length $L = 55$ for the urban channel and $L = 11$ for the rural channel. The results are presented in Fig.1. As can be observed, for a wide range of values of SNR the CB-CFO meets the performance of ESPRIT with much reduced computation demand.

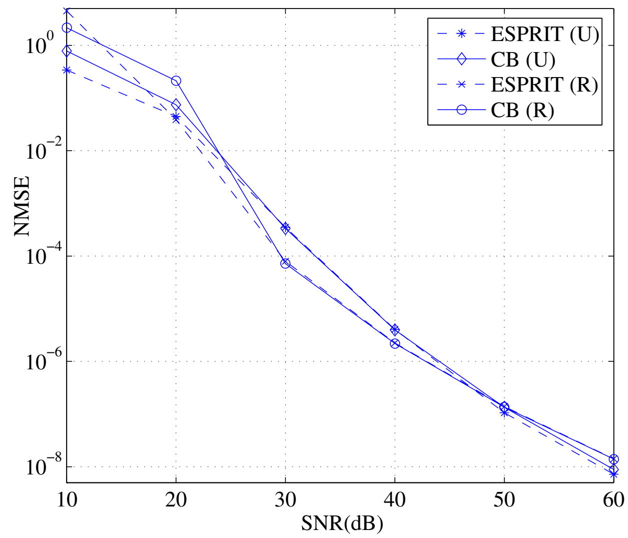

Fig. 1. NMSE for ESPRIT and CB-CFO.

6. CONCLUSIONS

In this article, we presented a blind algorithm for estimating the carrier frequency offset (CFO) in OFDM systems. The so-called CB-CFO scheme is characterized by a substantially lower computational complexity when compared to the standard parametric method TLS-ESPRIT. Despite such a simplified implementation, CB-CFO algorithm presents equivalent NMSE performance in comparison to the TLS-ESPRIT.

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