BLIND CHANNEL SHORTENING IN OFDM SYSTEM USING NULLTONES AND CYCLIC PREFIX

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ABSTRACT

This paper considers the problem of blind channel shortening in OFDM systems. Standard OFDM systems use guard interval (GI) in form of cyclic prefix (CP) and nulltones (NT) redundancy. In this paper, we are interested in exploiting simultaneously the CP and the NT to achieve blindly the channel shortening. We start by proving that the restoration of NT property leads to the desired channel shortening. However, the performance of the shortening based only on the NT is relatively poor. We propose to improve it by using the NT in conjunction with the CP redundancy. Hence, the GI-based shortening criterion is combined with the NT-based criterion via a scalar weighting coefficient. The latter is optimized to improve Symbol Error Rate (SER) performance of the receiver. Simulation results are provided to illustrate the performance of the combined criterion as compared to the GI-Based (MERRY algorithm) or NT-based criteria.

Index Terms— OFDM, Blind channel shortening, Differential encoding.

1. INTRODUCTION

The major asset of Orthogonal Frequency Division Multiplexing is its low equalization complexity as compared to single carrier systems. For this reason, OFDM is very attractive and is used in many standard communication systems including the Digital Video Broadcasting (DVB), Digital Audio Broadcasting (DAB), Asymmetric Digital Subscriber Line (ADSL), etc. The intersymbol interference and the intercarrier interference are removed by using Null Tones redundancy (NT) and the Guard Interval (GI) of length higher than the channel size. Therefore, the effective flow decreases if the channel length increases (i.e, when the GI length increases). To preserve a high effective flow, we must reduce the channel length by using channel shortening techniques. From the literature, different algorithms of blind channel shortening are known: Multicarrier Equalization by Restoration of Redundancy (MERRY) [1], the second order statistics based methods in [2, 3] and the Carrier Nulling Algorithm (CNA) [4] that has been introduced in an adaptive scheme for SIMO (Single-Input Multiple-Output) systems. Recently, in [5]. the authors consider a block differential encoded OFDM system without NT and exploit the knowledge and specific characteristics of the first emitted symbol for channel shortening. Here, we use standard OFDM system (both Null tones and Cyclic prefix (CP) exist) with block differential encoded data. We extend the work of [4] by proving first that the restoration on the NT property allows us to achieve the desired channel shortening in SISO (Single-Input



Fig. 1. System model.

Single-Output) systems ¹. We also propose to improve the blind shortening quality as compared to [1, 4] by properly combining the GI and NT based criteria. The optimal combination is obtained by minimizing the amplitude fluctuation of the received active tones. We use in this paper the following notations: ^T, ^H and * stand for transpose, transconjugate and conjugate respectively. $\mathbf{1}_{a,b}$, $\mathbf{0}_{a,b}$ and \mathbf{I}_a are the $a \times b$ matrix of ones, the $a \times b$ zero matrix and the $a \times a$ identity matrix. $\mathbf{A}_{i,j}$ denotes the (i, j)-th entry of matrix \mathbf{A} . The paper is organized as follows. The next section presents the system model. Section 3 presents blind channel shortening algorithms. Section 4 presents some simulation results and section 5 draws some conclusions.

2. SYSTEM MODEL

In OFDM systems, the transmitted signal s(k) is segmented into blocks of length N (N is the number of frequency bins):

$$\mathbf{s}_{n} = [s(nN), s(nN+1), \cdots, s(nN+N-1)]^{T}$$
(1)

where
$$[s(nN + N - T_n), \cdots, s(nN + N - 1)] = 0$$
 (2)

 T_n and $T_a = N - T_n$ represent respectively the number of nulltones and the number of active tones. Block differential modulation [4, 6, 7] is used as follows: The T_a subcarriers are divided into m groups

¹This result is given for SISO but is obviously true for SIMO systems.

each containing r subcarriers ($T_a = r \times m, r$ and m are integers). The i-th group is given by:

$$\mathbf{s}_{n}^{i} = [s(nN + r(i-1)), \cdots, s(nN + ir - 1)]^{T}.$$
 (3)

The generation of s_n^i is given by the following recursion:

$$\mathbf{s}_{n}^{i} = \begin{cases} \mathbf{K}^{i}(n)\mathbf{s}_{n-1}^{i}; & \text{if } n \ge 1\\ \mathbf{1}_{r,1}; & \text{if } n = 0 \end{cases}$$
(4)

 $K^i(n) \in \mathcal{K}$ transports the information on the main diagonal where \mathcal{K} is a finite group of $r \times r$ unitary and diagonal matrices [4, 6]. If the transmission rate is R bit per symbol, the cardinality of \mathcal{K} is 2^{rR} . After block differential encoding, the block \mathbf{s}_n is transformed into vector \mathbf{x}_n by Inverse Fast Fourier Transform (see Fig. 1):

$$\mathbf{x}_n = \mathbf{F}^H \mathbf{s}_n \tag{5}$$

where **F** represents a $N \times N$ normalized Fourier matrix. Then, the GI redundancy (here, we consider cyclic prefix (CP)) is added to \mathbf{x}_n to form a vector of length $P = N + \nu$, ν being the size of the GI. Due to channel and noise effects, the received signal is given by :

$$y(i) = \sum_{l=0}^{L} h(l)x(i-l) + b(i)$$
(6)

where $\mathbf{h} = [h(0), h(1), \dots, h(L)]^T$ represents a finite impulse response channel, x(i) is the transmitted symbols and b(i) is the observation noise. In this work, the channel memory is larger than the GI (i.e, $L > \nu$) and hence a shortening is needed to reduce the size of the channel and eliminate the ISI. We assume that \mathbf{h} is unknown, the data s(k) and noise b(k) are uncorrelated, zero mean and wide sense stationary processes with variance σ_s^2 and σ_b^2 respectively. Thereafter, the received data is filtered by time domain equalizer (TEQ) $\mathbf{w} = [w(0), w(1), \dots, w(q-1)]^T$ of degree q - 1 (q should not exceed N - L) to obtain the following equalized data:

$$z(i) = \sum_{l=0}^{q-1} w(l)y(i-l).$$
(7)

The combined equalizer-channel impulse response (CIR) is denoted by c(n) = h(n) * w(n) (* being the convolution operator). The goal of the channel shortening is to obtain:

$$\mathbf{c} = [\mathbf{0}_{1,d}, \mathbf{v}^T, \mathbf{0}_{1,(L+q-\nu-d-1)}]^T$$

where

$$\mathbf{c} = [c(0), c(1), \cdots, c(\underbrace{L+q-1}_{L_c})]^T$$
$$\mathbf{v} = [v(0), v(1), \cdots, v(\nu)]^T$$

is the target impulse response (TIR) and d is the equalizer delay. After demodulation (synchronization + removal of the CP and FFT), the received n-th block OFDM can be written as:

$$\tilde{\mathbf{z}}_n = \tilde{\mathbf{v}} \odot \mathbf{s}_n + \tilde{\mathbf{b}}_n$$
 (8)

where \odot denotes the element by element multiplication. $\tilde{\mathbf{v}}, \tilde{\mathbf{b}}_n$ are respectively the TIR and noise in frequency domain. We define the i-th received group as:

$$\tilde{\mathbf{z}}_{n}^{i} = \tilde{\mathbf{v}}^{i} \odot \mathbf{s}_{n}^{i} + \tilde{\mathbf{b}}_{n}^{i}, \quad \forall i \in [0, m-1]$$
(9)

where:

$$\tilde{\mathbf{v}}^{i} = [\tilde{v}(r(i-1)), \cdots, \tilde{v}(ir-1)]^{T}$$
$$\tilde{\mathbf{b}}_{n}^{i} = \left[\tilde{b}_{n}(r(i-1)), \cdots, \tilde{b}_{n}(ir-1)\right]^{T}$$

Using (4) and (9), we obtain the recursive relation between $\tilde{\mathbf{z}}_n^i$ and $\tilde{\mathbf{z}}_{n-1}^i$:

$$\tilde{\mathbf{z}}_{n}^{i} = \mathbf{K}^{i}(n)\tilde{\mathbf{z}}_{n-1}^{i} + \tilde{\mathbf{b}}_{n}^{i}$$
(10)

where $\mathbf{\tilde{b}}_{n}^{i} = \mathbf{\tilde{b}}_{n}^{i} - \mathbf{K}^{i}(n)\mathbf{\tilde{b}}_{n-1}^{i}$ represents the global noise term in i-th group. To decode the information symbols, we use the MMSE estimator as follows:

$$\hat{\mathbf{K}}^{i}(n) = \underset{\mathbf{K}\in\mathcal{K}}{\operatorname{arg\,min}} \quad \left\| \tilde{\mathbf{z}}_{n}^{i} - \mathbf{K}\tilde{\mathbf{z}}_{n-1}^{i} \right\|^{2}$$
(11)

where $\|.\|$ denotes the Frobenius norm. We note that the information can be decoded without the knowledge of the TIR thanks to block differential encoding. Equation(11) can be solved either by exhaustive search or by using fast lattice decoding algorithm [8].

Next, we recall the basic principle of GI-Based and NT-based shortening techniques.

3. BLIND SHORTENING ALGORITHMS

3.1. GI-Based shortening (MERRY):

The MERRY algorithm consists in minimizing the square of the difference between two z values separated by the data block of length N. This allows to restoring the CP-redundancy. In [1], the authors propose the following cost function:

$$J_{merry} = E \left[|z(nP + d + \nu) - z(nP + d + \nu + N)|^2 \right]$$
(12)

 J_{merry} can be written as:

$$J_{merry} = \mathbf{w}^{H} E\left[\check{\mathbf{y}}_{n} \check{\mathbf{y}}_{n}^{H}\right] \mathbf{w}$$
(13)

where

$$\check{\mathbf{y}}_{n} = \begin{bmatrix}
y^{*}(nP+d+\nu) - y^{*}((n+1)P+d) \\
y^{*}(nP+d+\nu-1) - y^{*}((n+1)P+d-1) \\
\vdots \\
y^{*}(nP+d+\nu-q+1) - y^{*}((n+1)P+d-q+1)
\end{bmatrix}.$$
(14)

Equation (13) is minimized subject to the unit norm constraint, i.e. $\|\mathbf{w}\| = 1$, to avoid the trivial solution $\mathbf{w} = \mathbf{0}$. Hence, \mathbf{w} is obtained as the least eigenvector of the quadratic form:

$$\mathbf{Q}_{merry} = \sum_{n=1}^{N_b} \check{\mathbf{y}}_n \check{\mathbf{y}}_n^H \tag{15}$$

 N_b being the number of the OFDM symbols in batch (block) processing. In an adaptive scheme, w can be computed using a stochastic gradient algorithm as suggested [1].

3.2. NT-Based shortening (CNA):

We consider here the NT-based criterion used in [4] for channel shortening:

$$J_{cna} = \sum_{i=T_a+1}^{N} E\left[|\tilde{z}_n(i)|^2 \right]$$
(16)

which consists in the restoration of the "zero energy" of the null tones. We start first by proving theoretically that minimizing criterion (16) leads to the desired channel shortening.

Lemma: Assume that $T_a > L + q - \nu - 1$. Then, in noiseless case, criterion J_{cna} satisfies:

$$J_{cna} = 0 \Leftrightarrow \mathbf{c} = [\mathbf{v}^T, \mathbf{0}_{1,(L_c-\nu)}]^T$$

where
$$\mathbf{v} = [c(0), c(1), \cdots, c(\nu)]^T$$
.

Proof: The received signal after filtering by TEQ w and CP removal can be written as:

$$\tilde{\mathbf{z}}_n = \mathbf{F}\mathbf{C}_1\mathbf{x}_n + \mathbf{F}\mathbf{C}_2\mathbf{x}_{n-1} \tag{17}$$

a(1)

where C_1 and C_2 are $N \times N$ matrices defined as:

$$\mathbf{C}_{1} = \begin{bmatrix} c(0) & 0 & \rightarrow & \rightarrow & \rightarrow & 0 & c(\nu) & \cdots & c(1) \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 & c(L_{c}) & \cdots & c(L_{c} - \nu + 1) \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & c(L_{c}) \\ c(L_{c}) & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & c(L_{c}) \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 1 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \frac{1}{0} & \rightarrow & \rightarrow & 0 & c(L_{c}) & \cdots & \cdots & c(0) \\ \end{bmatrix}$$
and
$$\mathbf{C}_{2} = \begin{bmatrix} 0 & \cdots & c(L_{c}) & \cdots & c(\nu + 1) \\ \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \rightarrow & \rightarrow & c(L_{c}) \\ \mathbf{0} & \rightarrow & \rightarrow & \rightarrow & \mathbf{0} \end{bmatrix}$$

In the case, where $\mathbf{C}_2 = \mathbf{0}$, i.e. $\mathbf{c} = [\mathbf{v}^T, \mathbf{0}_{1,(L_c - \nu)}]^T$ vector $\tilde{\mathbf{z}}_n$ can be expressed (in the noiseless case) as:

$$\tilde{\mathbf{z}}_{n} = \begin{bmatrix} \tilde{\mathbf{z}}_{n}(1) \\ \vdots \\ \tilde{\mathbf{z}}_{n}(T_{a}) \\ \tilde{\mathbf{z}}_{n}(T_{a}+1) \\ \vdots \\ \tilde{\mathbf{z}}_{n}(N) \end{bmatrix} = \begin{bmatrix} \tilde{c}(1) \\ \vdots \\ \vdots \\ \tilde{c}(N) \end{bmatrix} \odot \begin{bmatrix} s_{n}(1) \\ \vdots \\ s_{n}(T_{a}) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$
(18)

where $[\tilde{c}(1), \cdots, \tilde{c}(N)] = \mathbf{F}[c(0), \cdots, c(\nu), 0, \cdots, 0]^T$. This proves the first part of the equivalence:

$$= [\mathbf{v}^T, \mathbf{0}_{1,(L_c-\nu)}]^T \Rightarrow J_{cna} = 0.$$

Since \mathbf{x}_n and \mathbf{x}_{n-1} are decorrelated, we have:

$$J_{cna} \geq \sum_{i=T_a+1}^{N} E\left[|\mathbf{F}\mathbf{C}_{2}\mathbf{x}_{n-1}(i)|^{2}\right]$$

= $Trace\left\{\mathbf{F}_{2}\mathbf{C}_{2}\mathbf{F}^{H}E\left[\mathbf{s}_{n-1}\mathbf{s}_{n-1}^{H}\right]\mathbf{F}\mathbf{C}_{2}^{H}\mathbf{F}_{2}^{H}\right\}$
= $\sigma_{s}^{2}Trace\left\{\mathbf{F}_{2}\mathbf{C}_{2}\mathbf{F}^{H}\mathbf{P}\mathbf{F}\mathbf{C}_{2}^{H}\mathbf{F}_{2}^{H}\right\}$
= $\sigma_{s}^{2}Trace\left\{\mathbf{F}_{2}\mathbf{C}_{2}\mathbf{F}_{1}^{H}\mathbf{F}_{1}\mathbf{C}_{2}^{H}\mathbf{F}_{2}^{H}\right\} = \sigma_{s}^{2}\left\|\mathbf{F}_{2}\mathbf{C}_{2}\mathbf{F}_{1}^{H}\right\|^{2}$

where $\mathbf{P} = \begin{bmatrix} \mathbf{I}_{T_a} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_{T_n, T_n} \end{bmatrix}$, \mathbf{F}_1 and \mathbf{F}_2 are submatrices of \mathbf{F} defined as: $\mathbf{F} = \begin{array}{c} T_a \left\{ \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix} \right.$ (20)

Using (19), we can rewrite as:

$$J_{cna} \geq \sigma_s^2 \left\| \mathbf{F}_{21} \mathbf{T} \mathbf{F}_{12}^H \right\|^2 \tag{21}$$

where F_{12} is a submatrix of F_1 given by its last T_a columns vectors (resp. \mathbf{F}_{21} is a submatrix of \mathbf{F}_2 given by its first T_a columns vectors). **T** is a $T_a \times T_a$ submatrix of **C**₂ defined as:

$$\mathbf{T} = \begin{bmatrix} 0 & \cdots & c(L_c) & \cdots & c(\nu+1) \\ \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \rightarrow & \rightarrow & c(L_c) \\ \mathbf{0} & \rightarrow & \rightarrow & \rightarrow & \mathbf{0} \end{bmatrix}.$$

Consequently, $J_{cna} = 0$ only if: $\mathbf{F}_{21}\mathbf{TF}_{12}^H = \mathbf{0}$.

Since \mathbf{F}_{12} is a $T_a \times T_a$ full rank (Vandermonde) matrix, the latter equality leads to: $\mathbf{F}_{21}\mathbf{T} = \mathbf{0}$.

Given the triangular structure of T and the fact that the first column of \mathbf{F}_{21} is equal to $\mathbf{1}_{T_n,1}$, we get finally:

$$c(\nu + 1) = c(\nu + 2) \dots = c(L_c) = 0$$

We conclude that restoration of the null tones is equivalent to channel shortening at length $\nu + 1$. \diamond

Minimizing (16) under unit norm constant, $\|\mathbf{w}\| = 1$, leads to quadratic optimization problem as shown below:

$$\mathbf{w}_{opt} = \arg\min_{\mathbf{w}} E\left[\sum_{i=T_a+1}^{N} |\tilde{z}_n(i)|^2\right] \text{ subject to (s.t.)} \quad \|\mathbf{w}\| = 1.$$
(22)

The equalized received block \mathbf{z}_n can be written as:

$$\mathbf{z}_n = \mathbf{Y}_n \mathbf{w} \tag{23}$$

where \mathbf{Y}_n is the received matrix defined by:

$$\mathbf{Y}_n = \begin{bmatrix} y(nP+\nu) & \cdots & y(nP+\nu-q+1) \\ \vdots & \ddots & \vdots \\ y(nP+\nu+N-1) & \cdots & y(nP+\nu+N-q) \end{bmatrix}.$$

The received tones $\tilde{\mathbf{z}}_n$ are given by: $\tilde{\mathbf{z}}_n = \mathbf{F}(\mathbf{Y}_n.\mathbf{w}) = (\mathbf{F}\mathbf{Y}_n)\mathbf{w}$ and hence equation (22) becomes:

$$\mathbf{w}_{opt} = \operatorname*{arg\,min}_{\mathbf{w}} \mathbf{w}^{H} E \left[\mathbf{Y}_{n}^{H} \mathbf{F}_{2}^{H} \mathbf{F}_{2} \mathbf{Y}_{n} \right] \mathbf{w} \text{ s.t. } \|\mathbf{w}\| = 1.$$
(24)

The desired TEQ \mathbf{w}_{cna} is the unit eigenvector corresponding to the least eigenvalue of the quadratic form:

$$\mathbf{Q}_{cna} = \sum_{n=1}^{N_b} \mathbf{Y}_n^H \mathbf{F}_2^H \mathbf{F}_2 \mathbf{Y}_n \tag{25}$$

3.3. Combined technique

We suggest here to combine both the restoration of the NT and the restoration of the CP properties. The GI-based shortening criterion in combined with the NT-based criterion via a scalar weighting coefficient $0 \le \rho \le 1$, leading to:

$$J_{com} = \rho J_{merry} + (1 - \rho) J_{cna} \tag{26}$$

Using (13) and (25), J_{com} is expressed as:

$$J_{com} = \mathbf{w}^{H} \left(\rho \mathbf{Q}_{merry} + (1 - \rho) \mathbf{Q}_{cna} \right) \mathbf{w} = \mathbf{w}^{H} \mathbf{Q}_{com} \mathbf{w}.$$
 (27)

Again, we use the constraint $\|\mathbf{w}\| = 1$ to avoid the null solution. So that, the desired TEQ \mathbf{w}_{com} is equal to the least eigenvector of \mathbf{Q}_{com} . Now, our objective is to properly choose the weighting parameter ρ in such a way we improve the system performance. If the channel on the target impulse response is known, one can optimize ρ by minimizing the symbol error rate expression given in [9]. In our context, such information is not available and hence we precede "blindly" to choose the weighting parameter ρ . For that, we exploit the fact that the differential encoder we have considered corresponds to constant modulus emitted symbols on each active subcarrier. Therefore, we choose to optimize ρ by minimizing the averaged amplitude fluctuation over N_b OFDM symbols. Indeed, the amplitude fluctuation of the active tones is mainly due the residual interference noise after channel shortening. Hence, minimizing this fluctuation would reduce the residual shortening noise. We define the amplitude fluctuation at i-th subcarrier by:

$$\mathcal{F}_{i} = \frac{\max_{n} |\tilde{\mathbf{z}}_{n}(i)|}{\max_{n} |\tilde{\mathbf{z}}_{n}(i)|} - 1$$
(28)

and its average value by: $\mathcal{F} = \frac{1}{T_a} \sum_{i=1}^{T_a} \mathcal{F}_i$ (29)

We have used Matlab Optmization Toolbox to minimize \mathcal{F} .

4. SIMULATION RESULTS

In this section, a series of simulations is conducted to study and compare the performance of the considered algorithms. We consider a SISO OFDM system. N = 64 represents the number of subcarriers, $T_N = 7$ and $\nu = 8$ are respectively the length of the NT and CP. A channel of length L = 20, is generated randomly such that its taps are zero-mean complex Gaussian variables with variances $\sigma_l^2 = \lambda . exp(-0.5l), \ l = 0 \cdots L - 1$ where λ ensure the unit energy of **h**. The SNR is defined as: $SNR = \frac{\sigma_s^2 \|\mathbf{h}\|^2}{\sigma_s^2}$. We compute the optimal value ρ that minimizes (29). $N_b = 50^{\circ b}$ OFDM symbols are generated at each run using either a scalar differential encoder with $K \in e^{\frac{j\pi}{4}}, e^{\frac{-j\pi}{4}}, e^{\frac{3j\pi}{4}}, e^{\frac{5j\pi}{4}}$ a 3- dimensional dif-ferential block encoder corresponding to the finite group [7]: $\mathcal{K} = \left\{ K^l | K = \text{diag}[e^{\frac{j\pi}{4}}; e^{\frac{j\pi}{4}}; e^{\frac{3j\pi}{4}}], l = 0 \cdots 7 \right\}$ A TEQ with q = 24taps is used and the delay parameter is chosen equal to d = 0. For each SNR, the SER is measured by observing a minimum of 100 symbol errors. Fig.2. illustrates the original channel, the CIR obtained in using the CNA, MERRY and combined methods at SNR = 25dB. Fig.3. displays the overall SER performance corresponding to an SNR range of [5, 25] dB for scalar differential encoding. Starting from SNR=10db, the proposed method has a lower SER as compared to MERRY and CNA methods. Fig.4 represents a similar results but with a block differential encoder using the group \mathcal{K} defined above. Again, the combined method has a lower SER compared to MERRY and CNA methods especially at high SNR values.

5. CONCLUSION

This paper provides a theoretical proof that the restoration of the null tones allows us to achieve the desired channel shortening. Then, we introduce a new shortening method that combines CNA and MERRY criteria in such a way to minimize the averaged amplitude fluctuation of the active tones signals. This combined method outperforms the two previous ones.



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