FRAME SYNCHRONIZATION AND JOINT ESTIMATION OF IQ IMBALANCE AND CHANNEL RESPONSE FOR OFDM SYSTEMS

*W.-J. Cho, T.-K. Chang, Y.-H. Chung, S.-M. Phoong**

Dept. of EE & Grad. Inst. of Comm. Engr.
National Taiwan University
Taiwan, ROC

Y.-P. Lin

Dept. Elec. and Contr. Engr.
National Chiao Tung University
Taiwan, ROC

ABSTRACT

In this paper, we first introduce a new method for joint estimation of IQ imbalance and channel response in OFDM systems under the assumption of perfect frame synchronization. The proposed method needs only one OFDM block for training and the solution is given in closed form. Then the method is applied to the scenario when the OFDM frame is not synchronized. A reduced complexity algorithm is derived. With a relatively low complexity, the proposed method can accurately estimate the frame starting position, the IQ imbalance and the channel response simultaneously. Simulation results show that the bit error rate (BER) performance of the proposed method is very close to the ideal case when all these parameters are known perfectly at the receiver.

Index Terms— Frame synchronization, IQ imbalance, channel estimation

1. INTRODUCTION

Recently direct conversion OFDM receivers have attracted a lot attention due to their low cost. However these systems suffer from In-phase/Quadrature-phase (IQ) imbalance at the front-end analog devices [1]. If the IQ imbalance is not properly compensated, the system performance can be seriously degraded. In the literature, there have been many reports on the compensation of IQ imbalance [2]–[7]. In [2], using carefully designed training block, several methods have been proposed for the compensation of IQ imbalance for OFDM systems. In [3], based on the observation that the channel frequency response is usually smooth in most OFDM applications, a frequency-domain approach is proposed for jointly estimating the IQ imbalance and channel frequency response. Using only one OFDM block for training, the authors in [3] demonstrate that good BER performances can be achieved. Based on the circularity property of the baseband signal, a non-data aided blind compensation of IQ imbalance is proposed in [4]. By optimally designing the training block, a method for joint estimation of the IQ imbalance, DC offset and channel response is proposed in [5]. The more general problem of joint compensation of the transmitter and receiver IQ imbalances is studied in [6][7].

In this paper, a new method is proposed for joint estimation of IQ imbalance and channel response in OFDM systems. Using one OFDM block for training, we show how to estimate the IQ imbalance without knowing the channel response. The solution is given in closed form. Based on the estimated IQ imbalance, an estimate of channel response can be obtained by using a simple formula. Unlike many other compensation schemes which assume that the OFDM system has perfect frame synchronization, our method can be applied when the OFDM system is not frame synchronized. A reduced complexity algorithm is derived for joint estimation of the frame starting position, IQ imbalance and channel response. Simulation results show that the BER performance of the proposed method is very close to the ideal case when all these parameters are known perfectly at the receiver.

The paper is organized as follows. Sec. 2 introduces the system model. Under the assumption that the OFDM system has perfect frame synchronization, the proposed method for joint estimation of the IQ imbalance and channel response is derived in Sec. 3. The assumption of perfect frame synchronization is removed in Sec. 4. Monte-Carlo experiments are given in Sec. 5 and conclusions are given in Sec. 6.

Notations. Boldfaced upper case and lower case letters represent vectors and matrices respectively. The matrix $\mathbf{A}^\dagger$ denotes transpose-conjugate of $\mathbf{A}$ and the vector $\mathbf{v}^*$ denotes the complex conjugate of $\mathbf{v}$.

2. SYSTEM MODEL

In an OFDM system, the input block is an $M \times 1$ vector $\mathbf{s}$ consisting of modulation symbols. We first take the M-point normalized IDFT of $\mathbf{s}$ to obtain the time-domain vector $\mathbf{x}$. Then a cyclic prefix (CP) of length L is appended to $\mathbf{x}$. The cyclic prefixed time-domain sequence is transmitted through the channel. In this paper, we assume that the channel does not vary during the transmission of one OFDM block and the channel order does not exceed L. Thus we can write the channel transfer function as

$$H(z) = \sum_{n=0}^{L} h(n) z^{-n}. \tag{1}$$

At the receiver, the first L samples of each block is removed to eliminate interblock interference and the result is an $M \times 1$ received vector $\mathbf{r}$. It is well-known that when there is no IQ imbalance and the OFDM frame is perfectly synchronized, the received vector $\mathbf{r}$ is given by

$$\mathbf{r} = \mathbf{H}_{cir}\mathbf{x} + \mathbf{q}, \tag{2}$$

where $\mathbf{q}$ is the noise vector and $\mathbf{H}_{cir}$ is an $M \times M$ circulant matrix with the first column

$$\mathbf{h} = [h(0)\ h(1)\ \dots\ h(L)\ 0\ \dots\ 0]^T. \tag{3}$$

As circulant matrices are diagonalized by DFT matrices, we can recover the vector $\mathbf{s}$ by taking normalized DFT of $\mathbf{r}$ and using a set of one-tap frequency domain equalizers.

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In the presence of receiver IQ imbalance, the received vector is no longer equal to the vector $\mathbf{r}$ in (2). It is known [2] that the IQ corrupted received vector $\widetilde{\mathbf{r}}$ is related to the desired vector $\mathbf{r}$ as

$$\widetilde{\mathbf{r}} = \mu\mathbf{r} + \nu\mathbf{r}^*, \tag{4}$$

where $\mathbf{r}^*$ is the complex conjugate of $\mathbf{r}$. The IQ parameters μ and ν are respectively related to the oscillator's amplitude mismatch ϵ and phase mismatch ϕ as

$$\mu = \frac{1 + (1+\epsilon)e^{-j\phi}}{2} \text{ and } \nu = \frac{1 - (1+\epsilon)e^{j\phi}}{2}. \tag{5}$$

For convenience, we define the IQ parameter

$$\alpha \stackrel{\triangle}{=} \nu/\mu^*. \tag{6}$$

When there is no IQ imbalance, we have $\epsilon = 0$ and $\phi = 0$. Or equivalently, $\mu = 1$, $\nu = 0$ and $\alpha = 0$. It can be shown that if α is known at the receiver, we can recover a scaled version of the desired vector $\mu\mathbf{r}$ from the IQ corrupted vector $\widetilde{\mathbf{r}}$ as

$$\mathbf{r}_0 \stackrel{\triangle}{=} \mu\mathbf{r} = \frac{\widetilde{\mathbf{r}} - \alpha\widetilde{\mathbf{r}}^*}{1 - |\alpha|^2}. \tag{7}$$

As we will see in the next section, the estimated channel response will also be scaled by μ and hence the scaled factor μ in $\mathbf{r}_0$ will be canceled by the equalizer. In Sec. 3, we will propose a new method for joint estimation of the IQ parameter α and the channel response. The topic of frame synchronization will be studied in Sec. 4.

3. JOINT ESTIMATION OF IQ PARAMETER AND CHANNEL RESPONSE

In this section, it is assumed that the OFDM frame is perfectly synchronized. Suppose we send a known OFDM input block $\mathbf{s}$ for training. In other words, the receiver knows $\mathbf{x}$ which is the IDFT of $\mathbf{s}$. For the purpose of channel estimation, we write the vector $\mathbf{r}$ in (2) as

$$\mathbf{r} = \mathbf{X}_{cir}\mathbf{h} + \mathbf{q}, \tag{8}$$

where $\mathbf{h}$ is given in (3) and $\mathbf{X}_{cir}$ is $M \times M$ circulant with the first column $\mathbf{x}$. Therefore when there is no IQ imbalance, one can obtain an estimate of the channel as $\widehat{\mathbf{h}} = \mathbf{X}_{cir}^{-1}\mathbf{r}$ (assuming that $\mathbf{X}_{cir}$ is invertible, which is true if and only if the entries of $\mathbf{s}$ are nonzero).

In the presence of IQ imbalance, we know that the desired received vector $\mathbf{r}_0 = \mu\mathbf{r}$ is related to the IQ corrupted received vector $\widetilde{\mathbf{r}}$ as in (7). Define the scaled version of $\mathbf{h}$ as

$$\mathbf{h}_0 \stackrel{\triangle}{=} \mu\mathbf{h}. \tag{9}$$

If we have an estimate of the IQ parameter $\widehat{\alpha}$ at the receiver, one can get an estimate of $\mathbf{h}_0$ using (7) and it is given by

$$\widehat{\mathbf{h}}_0 = \mathbf{X}_{cir}^{-1}\frac{\widetilde{\mathbf{r}} - \widehat{\alpha}\widetilde{\mathbf{r}}^*}{1 - |\widehat{\alpha}|^2}. \tag{10}$$

The above channel estimate is a function of $\widehat{\alpha}$, different $\widehat{\alpha}$'s give different $\widehat{\mathbf{h}}_0$'s. Below we will show how to estimate α without knowing the channel.

Recall from the definition of $\mathbf{h}_0$ that its last $(M-L-1)$ entries are zero. If $\widehat{\alpha}$ is an accurate estimate of α, one would expect the last $(M-L-1)$ entries of $\widehat{\mathbf{h}}_0$ in (10) to be small. Define the sum of squares of these entries as

$$\mathcal{J}(\widehat{\alpha}) = \sum_{n=L+1}^{M-1} |\widehat{h}_0(n)|^2. \tag{11}$$

Figure 1 shows a contour plot of $10\log\mathcal{J}(\widehat{\alpha})$ where the x and y axes represent the real and imaginary parts of $\widehat{\alpha}$ respectively. In this example, $M = 64$, $L = 3$ and the channel responses are $h(0) = 0.3903 + j0.1049$, $h(1) = 0.6050 + j0.1422$, $h(2) = 0.6050 + j0.1422$ and $h(3) = 0.0714 + j0.5002$. The SNR is 30 dB and the IQ parameter is $\alpha = -0.0480 - j0.0873$ (indicated by $\triangle$ in the plot). As we can see from the figure that the cost function $\mathcal{J}(\widehat{\alpha})$ has a very nice bowl shape and it has a minimum at $\widehat{\alpha} = \alpha$. This example demonstrates that $\widehat{\alpha}$ should be chosen so that $\mathcal{J}(\widehat{\alpha})$ is minimized. Below we shall see that the optimal $\widehat{\alpha}$ is given in closed form.

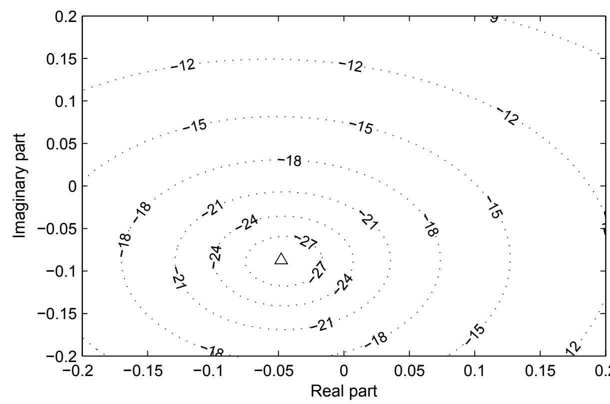

Fig. 1. A contour plot of the cost function $\mathcal{J}(\widehat{\alpha})$ in dB. The desired IQ parameter α is indicated by $\triangle$.

Define the $(M-L-1) \times M$ matrix

$$\mathbf{E} = [\,\mathbf{0}\ \ \mathbf{I}_{M-L-1}\,]. \tag{12}$$

In practice, the OFDM block size M is usually much larger than the CP length L. It is reasonable to make the assumption $M > L+1$ so that $\mathbf{E}$ is not a zero matrix. The cost function can be expressed as $\mathcal{J}(\widehat{\alpha}) = \|\mathbf{E}\widehat{\mathbf{h}}_0\|^2$. Using (10), we get

$$\mathcal{J}(\widehat{\alpha}) = \|\mathbf{E}\mathbf{X}_{cir}^{-1}\frac{\widetilde{\mathbf{r}} - \widehat{\alpha}\widetilde{\mathbf{r}}^*}{1 - |\widehat{\alpha}|^2}\|^2. \tag{13}$$

In practice, the IQ parameter $\widehat{\alpha}$ is small so we can make the approximation $1 - |\widehat{\alpha}|^2 \approx 1$. Moreover this term simply scales the cost function by a factor. Thus the approximation has little effect on the optimal solution. Using this approximation, the optimal estimate $\widehat{\alpha}$ is given by

$$\widehat{\alpha}_{opt} = \arg\min_{\widehat{\alpha}} \mathcal{J}(\widehat{\alpha}) = \arg\min_{\widehat{\alpha}} \|\mathbf{E}\mathbf{X}_{cir}^{-1}(\widetilde{\mathbf{r}} - \widehat{\alpha}\widetilde{\mathbf{r}}^*)\|^2. \tag{14}$$

The solution to the above problem is well-known and it is given in closed form

$$\widehat{\alpha}_{opt} = \frac{(\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}^*)^\dagger(\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}})}{\|\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}^*\|^2}. \tag{15}$$

The minimized cost function is

$$\mathcal{J}_{min} = \|(\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}})\|^2 - \frac{|(\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}^*)^\dagger(\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}})|^2}{\|\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}^*\|^2}. \tag{16}$$

Substituting $\widehat{\alpha}_{opt}$ into (10), we obtain the channel estimate $\widehat{\mathbf{h}}_0$. Notice that there is no constraint on the training block $\mathbf{s}$ and it can consist of any nonzero entries. Moreover the computational complexity is low. As the circulant matrix $\mathbf{X}_{cir}^{-1}$ is fixed and known, it can be precomputed and its first column can be stored at receiver. The main computation of $\widehat{\alpha}_{opt}$ is the calculation of two vectors $\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}$ and $\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}^*$, which can be computed efficiently using FFTs with a complexity of $M \log_2 M$.

4. FRAME SYNCHRONIZATION

In previous discussion, it is assumed that the starting position of the training block is known at the receiver. Let i_0 denote the starting position. Below we will show how to estimate i_0, α and $h_0(n)$ simultaneously. Let the IQ corrupted received vector with starting position i be given by

$$\widetilde{\mathbf{r}}_i = [\widetilde{r}(i)\ \widetilde{r}(i-1)\ \ldots \widetilde{r}(i-M+1)]^T. \quad (17)$$

For each vector $\widetilde{\mathbf{r}}_i$, we evaluate the cost function

$$\mathcal{J}_{min,i} = \|(\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i)\|^2 - \frac{|(\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i^*)^\dagger(\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i)|^2}{\|\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i^*\|^2}. \quad (18)$$

Then the estimate of i_0 is obtained as

$$\widehat{\imath}_0 = \arg \min_{0 \le i < M} \mathcal{J}_{min,i}. \quad (19)$$

The IQ parameter $\widehat{\alpha}_{opt}$ can be obtained by substituting $\widetilde{\mathbf{r}}_{\widehat{\imath}_0}$ into (15). Note that in the above optimization, one has to carry out the matrix-vector multiplications of $\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i$ and $\mathbf{E}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i^*$ for $0 \le i < M$. The complexity is in the order of M^3 (for direct multiplication) or $M^2 \log_2 M$ (for FFT implementation). By using the circulant property of $\mathbf{X}_{cir}^{-1}$, we will show that the complexity can be reduced to the order of M^2. To do this, let us define the $M \times M$ matrix

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & \ldots & 0 & 1 \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & & & \\ \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & \ldots & 1 & 0 \end{pmatrix}. \quad (20)$$

Then the circulant matrix $\mathbf{X}_{cir}^{-1}$ can be expressed as

$$\mathbf{X}_{cir}^{-1} = \sum_{k=0}^{M-1} a_k \mathbf{C}^k, \quad (21)$$

where a_k is the k-th entry of the first column of $\mathbf{X}_{cir}^{-1}$. Note that the matrix $\mathbf{C}$ satisfies $\mathbf{C}^M = \mathbf{I}_M$ and multiplying a vector $\mathbf{v}$ by the matrix $\mathbf{C}$ simply circularly rotates the entries of $\mathbf{v}$ downward by one. Using these facts, we can write

$$\widetilde{\mathbf{r}}_{i+1} = \mathbf{C}\widetilde{\mathbf{r}}_i + d_i\mathbf{1}, \quad (22)$$

where $d_i = \widetilde{r}(i+1) - \widetilde{r}(i-M+1)$ and the $M \times 1$ vector $\mathbf{1} = [1\ 0\ \ldots\ 0]^T$. Using the above expression and the fact that the matrix $\mathbf{C}$ commutes with circulant matrices, we can write

$$\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_{i+1} = \mathbf{C}\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i + d_i\mathbf{a}, \quad (23)$$

where $\mathbf{a} = [a_0\ a_1\ \ldots\ a_{M-1}]^T$. Using the above formula, we need only M multiplications to compute $\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_{i+1}$ from $\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i$. Thus the complexity of computing $\mathbf{X}_{cir}^{-1}\widetilde{\mathbf{r}}_i$ for all $0 \le i < M$ is in the order of M^2.

5. SIMULATION RESULTS

We carry out Monte-Carlo experiments to verify the performance of the proposed method. The OFDM block size is $M = 64$, the CP length and channel order are the same and they are equal to L. Two cases, $L = 3$ and $L = 15$, are studied. A total of 300 random channels for each case are generated and the channel taps are iid Gaussian random variables. The amplitude and phase mismatches are respectively $\epsilon = 0.1$ and $\phi = 10^0$. The corresponding IQ parameter is $\alpha = -0.048 - 0.0873j$. The MSEs of the IQ parameter and channel response are respectively defined as

$$MSE(\alpha) = E[|\widehat{\alpha}_{opt} - \alpha|^2] \quad (24)$$

$$MSE(h) = \sum_{i=0}^{L} E|\widehat{h}_0(n) - h_0(n)|^2. \quad (25)$$

The modulation symbols are QPSK.

First we assume that the system has perfect frame synchronization. Figure 2 show the MSE results. For comparison, we also plot the MSE results of the IQ-FD method [3]. From the figure, we see that for $L = 3$, both methods have comparable performance at a moderate SNR and the proposed method has a better performance at high SNR. When L increases from 3 to 15, the MSEs of our method only increase slightly whereas the MSEs of IQ-FD method increase significantly. The reason is that the IQ-FD method makes the assumption that the channel frequency response is smooth, which is no longer valid when $L = 15$. Fig. 3 shows the BER performance. For both $L = 3$ and $L = 15$, the BER floors at around $4 * 10^{-3}$ when the IQ imbalance is not compensated. In both cases, the BER performance of the proposed method is very close to the ideal case when α and $h_0(n)$ are known perfectly at the receiver. For the IQ-FD method, when $L = 3$ its BER performance is satisfactory but when $L = 15$, its BER performance is only slightly better than the case of no compensation.

Next we consider joint estimation of frame starting position i_0, IQ parameter α and channel response $h_0(n)$. For the case when the actual starting position of the frame is $i_0 = 32$ and the SNR is 10 dB, Fig. 4 shows probability of estimated starting position $\widehat{\imath}_0$ for $0 \le \widehat{\imath}_0 < 63$. The figure shows that $\widehat{\imath}_0 = i_0$ with a probability of around 0.93. The synchronization error almost never exceeds 1 sample. From our simulations, it is found that the BER performance is also very close to the ideal case when all the parameters, i_0, α and $h_0(n)$, are known perfectly at the receiver.

6. CONCLUSIONS

In this paper, a new method for joint estimation of IQ imbalance and channel response is proposed. The solution is given in closed form. Moreover the method can be extended to the scenario when the OFDM frame is not synchronized. By using only one OFDM block for training, we are able to accurately estimate the frame starting position, the IQ parameters and the channel response simultaneously.

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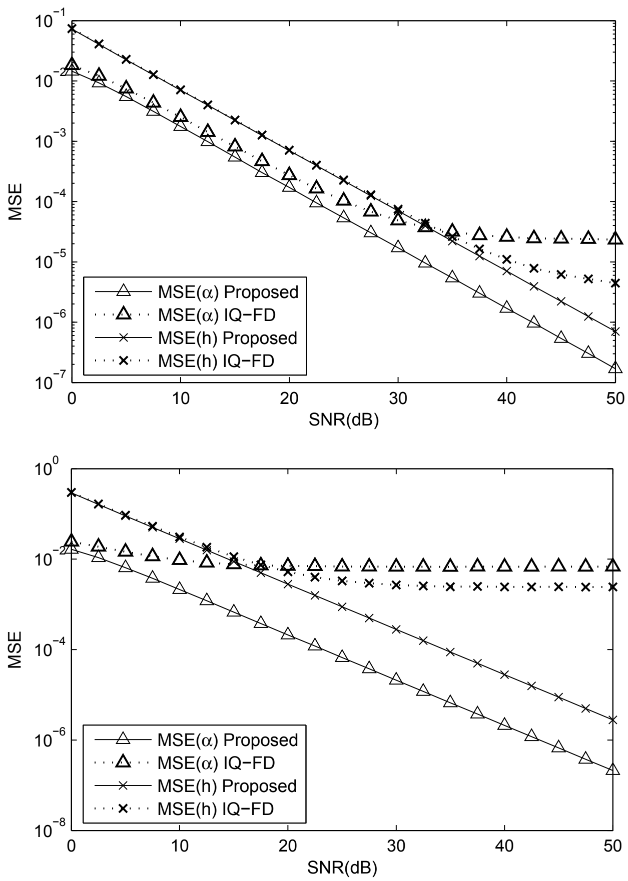

Fig. 2. MSEs: $L = 3$ (top) and $L = 15$ (bottom).

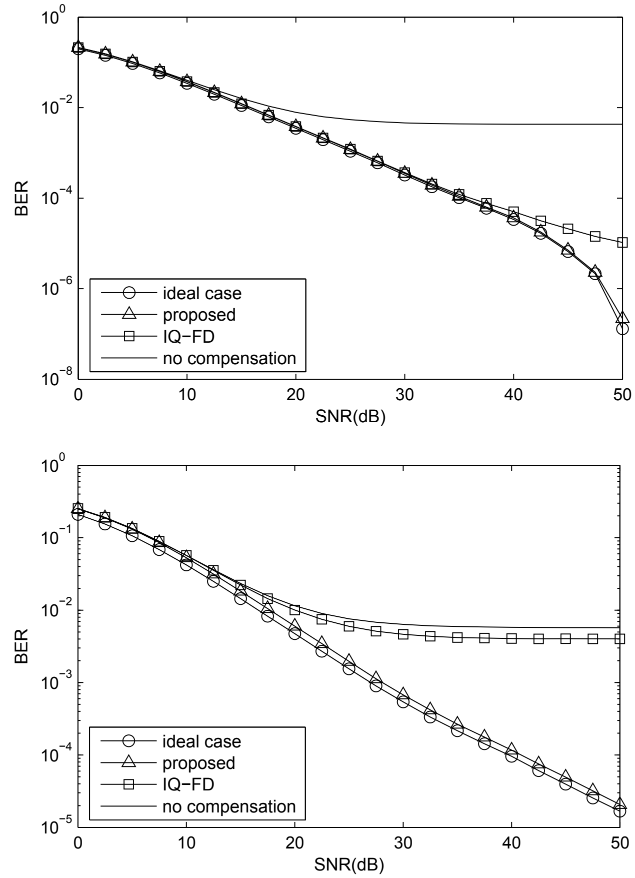

Fig. 3. BER: $L = 3$ (top) and $L = 15$ (bottom).

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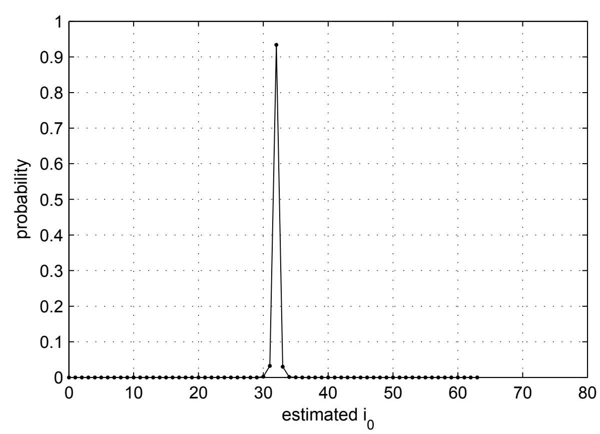

Fig. 4. Probability of the estimated i_0, where the desired $i_0 = 32$.