TRANSMISSION RATE ALLOCATION IN MULTI-SENSOR TARGET TRACKING

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ABSTRACT

In a multi-sensor target tracking application running on a shared network, at what bit-rates should the sensors send their measurements to the tracking fusion center? Clearly, the sensors cannot use arbitrary rates in a shared network, and a standard network rate control algorithm may not provide rates amenable to effective target tracking. For Kalman Filter-based multi-sensor target tracking, we derive a utility function that captures the tracking Quality of Service as a function of bit-rate. We incorporate this utility function into a network rate resource allocation framework, deriving a distributed rate control algorithm for a shared network that does not require network re-design. In simulation studies, the new rate-control algorithm engenders much better tracking performance than a standard rate-control method.

Index Terms— Target Tracking, Kalman filter

1. INTRODUCTION

Consider a multi-sensor target tracking scenario where a target is tracked by several sensors which send measurements to a fusion center through a shared network. With these measurements, the fusion center estimates the track using Kalman filter based target tracking. In a shared network, what is the optimal set of sensor transmission rates that yields the best target tracking performance? We address this problem in this paper. Standard rate control methods (e.g., TCP protocols) do not consider the QoS (Quality-of-Service) requirements of the multi-sensor target tracking application.

We take the following approach: Starting with the well known Kalman filter-based target tracking equations, in Section 3 we derive *utility functions* for both state vector fusion (SVF) and measurement fusion (MF) methods so that the target position estimation error is minimized. In Section 4, this utility function is used to augment the utility employed in the standard network utility maximization (NUM) approach [1, 2]. We present a distributed rate control protocol to solve this augmented NUM problem using the gradient projection algorithm. The strength of our method is that it can be deployed in existing networks without the need for any redesign. Section 5, presents simulation results that demonstrate the improved track estimation accuracy of the proposed protocol. Section 6 concludes the paper. First, we review the basic notions of Kalman filter based multi-sensor target tracking.

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2. KALMAN FILTER BASED TARGET TRACKING

Suppose the position, velocity, and acceleration of a target along the (x, y) coordinates and at time instant k are $(x(k), y(k))$, $(v_x(k), v_y(k))$, and $(a_x(k), a_y(k))$, respectively. Kalman filter based target tracking utilizes the following plant and measurement equations [3]:

$$X(k+1) = \Phi\, X(k) + \Gamma\, W(k);$$

$$Z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} X(k) + V(k), \qquad (1)$$

where $X(k) = [x(k),\, v_x(k),\, y(k),\, v_y(k)]^T$ is the state vector; plant and measurement noise covariance matrices are $Q(k) = \text{diag}\{q, q\}$ and $R(k) = \text{diag}\,\{r, r\}$, respectively;

$$\Phi = \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix};\ \Gamma = \begin{bmatrix} \frac{\Delta^2}{2} & 0 \\ \Delta & 0 \\ 0 & \frac{\Delta^2}{2} \\ 0 & \Delta \end{bmatrix};\ W(k) = \begin{bmatrix} a_x(k) \\ a_y(k) \end{bmatrix};$$

and Δ is the period of update.

Let N denote the number of sensors, and let the subscript i identify quantities corresponding to sensor i. For multi-sensor target tracking with SVF and basic convex combination [4, 5], the final estimate and error covariance matrix are

$$\hat{X}(k|k) = P(k|k) \sum_{i=1}^{i=N} P_i(k|k)^{-1} \hat{X}_i(k|k);$$

$$P(k|k) = \left[\sum_{i=1}^{i=N} P_i(k|k)^{-1} \right]^{-1}. \qquad (2)$$

MF [4] uses the fused measurement vector and its error covariance matrix as

$$Z(k) = \left[\sum_{i=1}^{i=N} R_i(k)^{-1} \right]^{-1} \sum_{i=1}^{i=N} R_i(k)^{-1} Z_k(k);$$

$$R(k) = \left[\sum_{i=1}^{i=N} R_i(k)^{-1} \right]^{-1}. \qquad (3)$$

3. UTILITY OF TARGET TRACKING

In Kalman filter based estimation, one typically minimizes the trace of the error covariance matrix in order to maximize estimation accuracy. Due to its simplicity of analysis and to place more emphasize on location estimates, the components in the trace that correspond to the location estimates are selected to represent the *utility* $S(k)$, i.e.,

$$S(k) = -P(k+1|k+1)_{1,1} - P(k+1|k+1)_{3,3}. \quad (4)$$

We may maximize the QoS of the multi-sensor target tracking application by maximizing this utility $S(k)$.

To derive expressions for $S(k)$, we need

Definition 1 *A* 4×4 *matrix* P *that takes the form* $P = \begin{bmatrix} \Phi & \emptyset \\ \emptyset & \Phi \end{bmatrix}$, *where* Φ *is a* 2×2 *symmetric matrix, is said to have a* Φ-block diagonal form.

3.1. State Vector Fusion (SVF) Method

Suppose the initial error covariance matrix $P_i(0|0)$ for sensor i is $\Phi_i(0)$-block diagonal with arbitrary $\Phi_i(0)$.

Claim 1 *With SVF, the following are true:*

(i) The error covariance matrix $P_i(k|k)$ *is* $\Phi_i(k)$*-block diagonal with* $\Phi_i(k) = \begin{bmatrix} p1_i(k) & p2_i(k) \\ p2_i(k) & p3_i(k) \end{bmatrix}$.

(ii) The final error covariance matrix $P(k|k)$ *is* $\Phi(k)$*-block diagonal with* $\Phi(k) = \begin{bmatrix} p1(k) & p2(k) \\ p2(k) & p3(k) \end{bmatrix}$.

(iii) The Utility *can be expressed as*

$$S_{SVF}(k) = \frac{-2\,\rho 1(k)}{\rho 1(k)\, r' + \rho 1(k)\,\rho 3(k) - \rho 2(k)^2}.$$

Furthermore, $\rho 1(k)\,\rho 3(k) - \rho 2(k)^2 > 0$, $\forall k > 0$, *if* $P_i(0|0) = I$, $\forall i$.

Proof. By induction. Suppose the claim is true for k.

(i) Use Kalman update equations to show that $P_i(k+1|k+1)$ is $\Phi_i(k+1)$-block diagonal with

$$p1_i(k+1) = \frac{a_i(k)\, r_i}{a_i(k) + r_i}; \quad p2_i(k+1) = \frac{b_i(k)\, r_i}{a_i(k) + r_i};$$
$$p3_i(k+1) = \frac{-b_i(k)^2 + a_i(k)\, c_i(k) + c_i(k)\, r_i}{a_i(k) + r_i}.$$

Here, $a_i(k) = p1_i(k) + 2\,p2_i(k)\,\Delta + p3_i(k)\,\Delta^2 + (q/4)\,\Delta^4$; $b_i(k) = p2_i(k) + p3_i(k)\,\Delta + (q/2)\,\Delta^3$; and $c_i(k) = p3_i(k) + q\,\Delta^2$.

(ii) Use (i) in (2) to show that $P(k+1|k+1)$ is $\Phi(k+1)$-block diagonal with

$$p1(k+1) = \frac{\rho 1(k)}{\rho 1(k)\,\rho 3(k) + \rho 1(k)\, r' - \rho 2(k)^2};$$
$$p2(k+1) = \frac{\rho 2(k)}{\rho 1(k)\,\rho 3(k) + \rho 1(k)\, r' - \rho 2(k)^2};$$
$$p3(k+1) = \frac{\rho 3(k) + r'}{\rho 1(k)\,\rho 3(k) + \rho 1(k)\, r' - \rho 2(k)^2}.$$

Here, $r' = \sum_{i=1}^{i=N} \frac{1}{r_i}$; $\rho 1(k) = \sum_{i=1}^{i=N} \frac{a_i(k)}{\delta_i(k)}$; $\rho 2(k) = \sum_{i=1}^{i=N} \frac{b_i(k)}{\delta_i(k)}$; and $\rho 3(k) = \sum_{i=1}^{i=N} \frac{c_i(k)}{\delta_i(k)}$, with $\delta_i(k) \equiv a_i(k)\, c_i(k) - b_i(k)^2$.

(iii) This follows from (4). QED

3.2. Measurement Fusion (MF) Method

Suppose the initial error covariance matrix $P(0|0)$ is $\Phi(0)$-block diagonal with arbitrary $\Phi(0)$.

Claim 2 *With MF, the following are true:*

(i) The error covariance matrix $P(k|k)$ *is* $\Phi(k)$*-block diagonal.*

(ii) The Utility *can be expressed as*

$$S_{MF}(k) = \frac{-2a(k)}{a(k)\sum_{i=1}^{i=N} \frac{1}{r_i} + 1}.$$

Proof. By induction. Suppose the claim is true for k.

(i) Use Kalman update equations to show that $P(k+1|k+1)$ is $\Phi(k+1)$-block diagonal with

$$p1(k+1) = \frac{a(k)}{a(k)\sum_{i=1}^{i=N} \frac{1}{r_i} + 1};$$
$$p2(k+1) = \frac{b(k)}{a(k)\sum_{i=1}^{i=N} \frac{1}{r_i} + 1};$$
$$p3(k+1) = \frac{c(k) + [a(k)\, c(k) - b(k)^2]\sum_{i=1}^{i=N} \frac{1}{r_i}}{a\sum_{i=1}^{i=N} \frac{1}{r_i} + 1}.$$

Here, $a = p1(k) + 2\,p2(k)\,\Delta + p3(k)\,\Delta^2 + \frac{q}{4}\Delta^4$; $b = p2(k) + p3(k)\,\Delta + \frac{q}{2}\Delta^3$; and $c = p3(k) + q\,\Delta^2$.

(ii) This follows from (4). QED

3.3. Properties of $S(k)$

Claim 3 *The* Utility $S(k)$ *(for both SVF and MF) are concave with respect to* $\mathbf{x}$ *(the vector of all the source rates) if the measurement error covariance of sensor* i *is* $r_i = r + d/x_i$*. Here,* x_i *is the transmission rate of sensor* i*,* r *is the actual error covariance of the sensors, and* d *is a constant.*

Proof. Note that $S(k) = -2r_j/[1 + A_j(k) r_j]$, $\forall j$, where

$$A_j(k) = \begin{cases} \frac{\rho 1(k)\,\rho 3(k) - \rho 2(k)^2}{\rho 1(k)} + \sum_{i=1}^{i=N} \frac{1}{r_i} - \frac{1}{r_j}, & \text{for SVF;} \\ \frac{1}{a(k)} + \sum_{i=1}^{i=N} \frac{1}{r_i} - \frac{1}{r_j}, & \text{for MF.} \end{cases}$$

The claim follows because $\rho 1(k)\,\rho 3(k) - \rho 2(k)^2 > 0$. QED

The inclusion of the term d/x_i in r_i is motivated by several issues: as $x_i \to \infty$, we need $r_i \to r$; as $x_i \to 0$, we need $r_i \to \infty$; and r_i should be a decreasing function of x_i. So, this term captures the reality of the non-availability of infinite bandwidth. Alternate terms that satisfy all these features (e.g., the rate distortion function of a Gaussian random variable) are of course available; however, one may not be able to guarantee concavity of $S(k)$.

With N sensors and $\sum_{i=1}^{N} x_i \leq C$, where C is a constant, let us examine where $S(k)$ achieves its maximum: equal transmission rates if each sensor has the same r value; if not, then sensors with lower r values get higher transmission rates; if there are sensors with $r = 0$, then only those sensors get non-zero transmission rates; sensors with infinite r get zero transmission rates.

4. RATE ALLOCATION IN TARGET TRACKING

The challenge one encounters in reality is how to find the optimal set of transmission rates that maximizes the *Utility* $S(k)$. The rate allocation problem, expressed within the NUM framework, is

$$\max_{\mathbf{x}\geq 0} \sum_{j\in\mathfrak{T}} U_j(\mathbf{x}) \text{ subject to } \sum_{j\in\mathfrak{T}} R_{j\ell} x_j \leq C_\ell,\ \forall \ell \in \mathfrak{L}, \quad (5)$$

where each source j measures its utility using a concave function $U_j(\mathbf{x})$. The set $\mathfrak{L}$ indexes all the links in the network, the term $R_{j\ell} = 1$ if the flow of source j utilizes link ℓ, and $R_{j\ell} = 0$ otherwise. This framework leads to a convex optimization problem that possesses a unique global maximum. To include QoS requirements of the multi-sensor target tracking application into the NUM framework, we augment the ordinary utility function with an additive term.

There are basically two types of users on the network: (1) *target tracking users* who are running the target tracking application; and (2) *ordinary users* who are running ordinary data transfer applications. Now consider the following new utility function that all network users use:

$$U_j(\mathbf{x}) = V_j(x_j) + KS(k),\ \forall j \in \mathfrak{T}, \quad (6)$$

The first term $V_j(x_j)$ is a concave function of the source data rate x_j and it addresses rate maximization, and is from the standard (original) utility function. The second term addresses the target tracking QoS requirements since $S(k)$ is the quantity we should maximize to maximize the target tracking QoS (see Section 3). Claim 3 establishes concavity of $S(k)$. The real parameter $K > 0$ determines the emphasis placed upon target tracking QoS over the standard utility maximization. Choosing $K = 0$ for ordinary users, their utility function reduces to the original rate maximization utility function.

To solve (5), we take the primal-dual approach [2], and consider the Lagrangian

$$L(\mathbf{x},\mathbf{p}) = \sum_{j\in\mathfrak{T}} \left(U_j(\mathbf{x}) - x_j \sum_{\ell\in\mathfrak{L}} R_{j\ell} p_\ell \right) + \sum_{\ell\in\mathfrak{L}} C_\ell p_\ell. \quad (7)$$

Here, $\mathbf{p}$ is a column vector of Lagrange multipliers. The sum $\sum_{\ell\in\mathfrak{L}} R_{j\ell} p_\ell$ is the sum of all the prices that source j incurs by using its particular links in the network. Now the original constrained optimization problem has been converted into a Lagrangian dual problem. To proceed, we need to specify $V(x_j)$ within $U_j(\mathbf{x})$. We let $V(x_j) = \alpha \log(x_j)$, which is the utility function of TCP Vegas [2]. Note that we can use the utility functions of any other rate control TCP variants, as long as the chosen function is concave. One may show that

$$\frac{\partial L(\mathbf{x},\mathbf{p})}{\partial x_j} = \frac{\alpha}{x_j} - q_j + I_j NK \frac{\partial S(k)}{\partial x_j} = \frac{\alpha}{x_j} - q_j + F_j(\mathbf{x}), \quad (8)$$

where $I_m = 1$ if source m is a target tracking source and 0 otherwise;

$$\sum_{\ell\in\mathfrak{L}} R_{j\ell} p_\ell = q_j;\ r_j = r + d/x_j;\ \partial r_j/\partial x_j = -d/x_j^2 R;$$

$$F_j(\mathbf{x}) = -2 I_j NK \frac{\partial r_j/\partial x_j}{(1 + A_j(k) r_j)^2}. \quad (9)$$

For ordinary sources, this reduces to $\alpha/x_j - q_j$. Here, $F_j(\mathbf{x})$ is the scalar feedback information sent by the sink node to the source j. Note that, $F_j(\mathbf{x}) = 0$ whenever $K_j = 0$.

Using the gradient projection algorithm, which enables one to achieve the optimum in an iterative manner by updating the transmission rates of individual sources in a distributed manner, and (8), we can get the rate update function. We proceed by noting that $x_j(t) = w_j(t)/RTT_j(t)$, where $w_j(t)$ and $RTT_j(t)$ are the current window size and Round-Trip Time (RTT) corresponding to source j, respectively. However, $RTT_j(t+1)$ is not available at the time of window update. Hence, for implementation purposes, we use the approximation $RTT_j(t+1) \approx RTT_j(t)$. Then we can derive the window update function

$$w_j(t+1) = [w_j(t) + s\, RTT_j(t)\,(\alpha\, RTT_j(t)/w_j(t) - q_j(t) + F_j(\mathbf{x}(t)))]^+, \quad (10)$$

where s is a constant. The queuing delay $q_j(t)$ is estimated at the source as the difference between the current RTT and the minimum observed RTT [2, 6]. An ordinary source updates its rate using its current rate, and the current queueing delay measurement, i.e., TCP Vegas. A target tracking source updates its rate using its current rate, the queueing delay feedback, and the feedback $F_j(\mathbf{x}(\mathbf{t}))$ from the sink node.

5. SIMULATION

In a $4000 \times 4000\text{ft}^2$ test area, 16 sensors track a target that moves in a pre-determined path. All sensors measure the (x, y) target location. Each sensor measurement error is modeled via a zero mean white noise source with covariance $\gamma^2 \propto D^2$; here D is the distance between the sensor and the target. All 16 sensors send their readings to the common sink node through a shared network. The frequency at which the sensors send readings are limited by the allocated bandwidth of the sensor. The sink node uses MF as discussed earlier for track estimation. The topology of the simulated network is given in Fig. 1. Twelve Flows go via 5 intermediate routers while other four go via 4 intermediate routers. Moreover, all

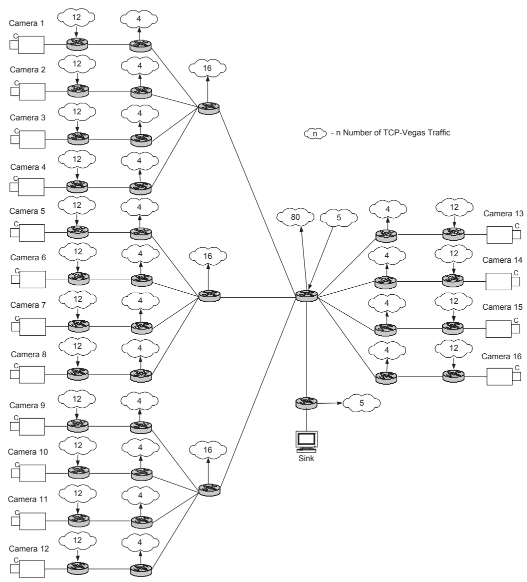

Fig. 1. Network topology for target tracking application.

16 flows go via a common link. Each link is congested with regular TCP Vegas traffic. The 'cloud' next to each router identifies the exact number of incoming and outgoing regular TCP Vegas traffic at that router. Regular Vegas flows start and stop transmission randomly. All simulations use ns-2.

Two scenarios were considered: (1) *TCP Vegas*: All flows run the ordinary data transfer protocol TCP Vegas; and (2) *Proposed protocol*: The coordinated flows (others use TCP Vegas) run the new protocol with $K = 300$ and $s = 25$.

A representative section of the target's track, together with the reconstructed tracks, for the two scenarios appear in Fig. 2. Notice the better performance provided by the

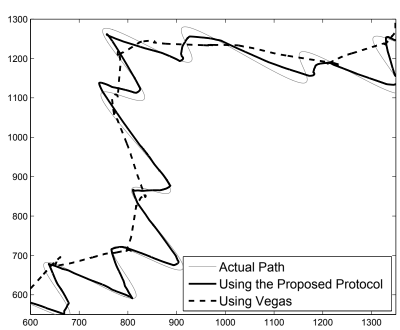

Fig. 2. Actual and reconstructed tracks of the target.

proposed protocol.

The next experiment is a target catching experiment. Here, the sink periodically sends the estimates of the target's position and velocity to a mobile agent which is attempting to catch the target. When the mobile agent gets a new estimate, it calculates its new direction. The agent starts its pursuit at time 300s from the $(2000, 1600)$ location. The tracks of the agent appear in Fig. 3. The target is considered to be 'caught'

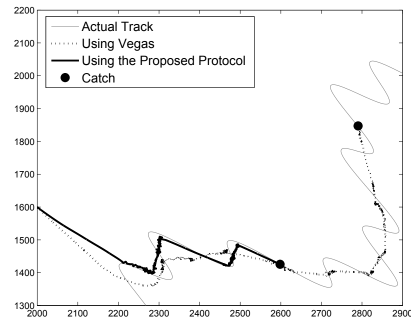

Fig. 3. Tracks of the mobile agent.

when the mobile reaches it within 5ft. It is clear from the results that the agent catches the target with the new protocol with shorter time (49.3s) than TCP Vegas (118.7s).

6. CONCLUDING REMARKS

In this paper, we have examined the problem of rate control of multi-sensor target tracking algorithms operating over a network shared by other applications. This work has attempted to bridge the gap between target tracking QoS objectives and the network rate allocation. The standard utility function is augmented with the target tracking *Utility*. This target tracking *Utility* is derived in order to maximize the estimation accuracy. A distributed rate control algorithm is derived by solving a convex optimization problem. As we have seen, this new rate control algorithm provided better target tracking performance than a more conventional rate control algorithm.

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