PRAGMATIC LIFETIME MAXIMIZATION OF COOPERATIVE SENSOR NETWORKS VIA A DECOMPOSITION APPROACH

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ABSTRACT

This paper addresses the problem of lifetime maximization under unequal and time-varying channel conditions, individual battery constraints, and estimation quality requirements at the fusion center. The standard tool for solving this problem (dynamic programming) has exponential complexity with number of sensors and states and needs heavy information exchange with sensors at each iteration. Also errors are introduced via coarse quantization of the parameters, which is forced by complexity concerns. In light of these issues, we propose a pragmatic method via a decomposition: The overall SNR requirement a is "divided" among sensors according to their battery powers and radio link statistics, and then individual sensors transmit powers are carefully controlled to maximize the lifetime. The proposed decomposition drastically reduces the computational requirement, and also allows a semi-distributed control of sensor transmit powers. Simulations verify the viability of this method.

Index Terms— Sensor networks, lifetime maximization, KKT conditions.

1. INTRODUCTION

This paper addresses the problem of the lifetime of sensor networks, under individual battery constraints, individual time-varying channel conditions, and subject to an overall estimation quality constraint (SNR) at the fusion center. The objective is to produce a pragmatic solution that addresses these general conditions while avoiding the computational complexity, global channel state information (CSI) requirements, as well as the quantization issues, that have been pointed out by the past works.

To put this work in perspective, we give a brief outline of the most relevant past work. There has been a significant amount of work in this area, but due to the limited space we can only mention a small subset that we use for comparison or for motivating this paper. We start by mentioning the work of Cui et al. [1] which considers a static network and finds an optimal water-filling solution that minimizes the total energy expenditure of the network. In this solution, the sensors with the worst channel conditions may not transmit, while others transmit with powers corresponding to their channel conditions. This general principle is also known as *minimum total energy (MTE)*. In this approach, it is assumed that there is a common pool of energy from which all sensors are fed, thus it is impossible for an individual sensor to run out of power before others. This approach leads to simplified analysis, but is somewhat unrealistic, since in practice each sensor has an individual battery.

A more general problem formulation includes individual batteries, as well as channel conditions that vary across sensors and over time. The key issue in this situation is that decisions about individual transmit powers must be taken while the nodes have no knowledge of their future transmissions, therefore any optimization will have a stochastic nature. A natural tool for stochastic optimization is dynamic programming, which has been employed in the work of Chen et al [2]. But in order to formulate a pragmatic dynamic programming approach, channel gains and battery powers must be heavily quantized, which introduces errors. Since channel gains and battery powers form part of the state vector, the complexity of dynamic programming is exponentially related to this quantization, as well as the number of sensors, which can be expressed as $\mathcal{O}(M^E Q^M)$, where Q is the number of quantized channel levels, M is the number of sensors, and E is the number of possible values for the residual energy of a sensor. That means for a network consisting of 20 sensors, with 10 possible energy levels and 5 quantized channel levels, the computation complexity is on the order of 10^{27} cycles per transmission interval.

Thus one is motivated to find a more pragmatic solution. We present one such solution that involves two steps. First we determine the portion of the total SNR needed from each sensor, in order to meet the overall SNR constraint. Once the SNR allotment for each sensor is determined, we find the optimal power scheduling to maximize the lifetime. This results in a decomposition of the overall problem into M (simpler) convex optimization problems. A numerical algorithm is proposed to determine the local SNR for each sensor.

A main advantage of this pragmatic approach, in addition to reduced computation, is that it allows each sensor to operate semi-independently: unlike the dynamic programming approach, in our method the control of transmit power of each sensor does not require knowledge of other sensors' *instantaneous* channel conditions (only requires statistics), thus it is more suitable.for field applications.

In the following, we present the system model, formulate the optimization problem, and present numerical simulations. The results are compared with equal-power and minimum total energy solutions. Please note that no dynamic programming simulations are presented, since a dynamic programming solution even with a modest number of sensors, with reasonable quantization of energy levels and channel gains, is beyond the computational capacities of most existing computers.

2. NETWORK MODEL

We consider a network of M sensors with orthogonal channels between the sensors and the fusion center. . Let $x_i = \theta + n_i$ be the observation at i-th sensor, where θ is the source signal and n_i is the additive complex Gaussian observation noise $n_i \sim \mathcal{CN}(0, \sigma_i^2)$. After the observation is made, our sensor sends this observation to the fusion center. Let h_i denote the i-th channel coefficient between the sensor and the fusion center. We assume that $|h_i|$ has a Rayleigh

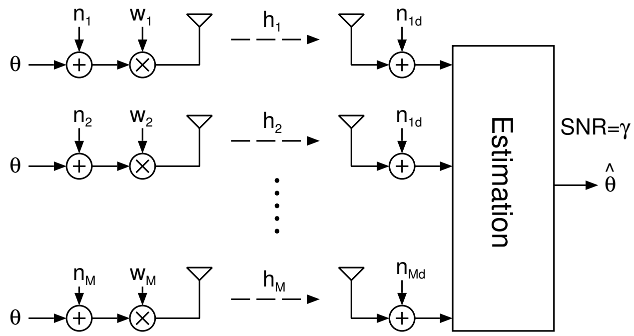

Fig. 1. Network model

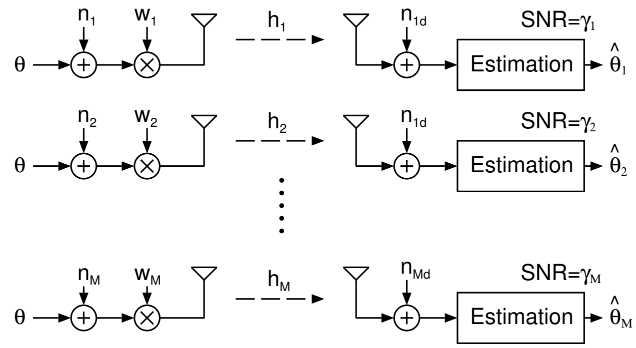

Fig. 2. Network decomposition for the lifetime problem.

distribution.

$$f(|h_i|) = \frac{|h_i| e^{\frac{-|h_i|^2}{2\sigma_{hi}^2}}}{\sigma_{hi}^2} \tag{1}$$

where σ_{hi}^2 is known to us. So the received signal at the destination from the i-th sensor is:

$$y_i = h_i w_i(\theta + n_i) + n_{id}$$

where w_i is the transmission gain for the i-th sensor which is related to transmit power by $p_i = w_i^2(1+\sigma_i^2)$ and $n_{id} \sim \mathcal{CN}(0, \sigma_{id}^2)$ is the destination noise. We also assume that$E[\theta^2] = 1$, where $E[\bullet]$ is the expected value operator. The SNR at destination due to the i-th channel is:

$$\frac{p_i |h_i|^2}{\sigma_i^2 p_i |h_i|^2 + (1+\sigma_i^2)\sigma_{id}^2} \tag{2}$$

The total SNR at the fusion center due to all the sensors is:

$$\sum_{i=1}^{M} \frac{p_i |h_i|^2}{\sigma_i^2 p_i |h_i|^2 + (1+\sigma_i^2)\sigma_{id}^2} \tag{3}$$

Our goal is to maximize the lifetime of our network while keeping the expected value of the SNR greater or equal than a target value γ.

3. PROBLEM FORMULATION

Suppose we measure the channel conditions every T seconds. We assume that N is the number of transmissions before the network runs out of energy. So network lifetime is NT seconds. For simplicity we set $T = 1$. So we can write the problem as :

$$\begin{aligned} &\max \ N \\ &s.t. \ E\left[\sum_{i=1}^{M} \frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2 + (1+\sigma_i^2)\sigma_{id}^2}\right] \geq \gamma \quad j = 1, \ldots, N \\ &p_{ij} \geq 0 \qquad \forall i, j \end{aligned} \tag{4}$$

where $|h_{ij}|$ is the channel coefficient for the i-th channel during the j-th transmission period and p_{ij} is the corresponding transmission power. We now approximate the SNR constraint by twice using the weak law of large numbers [3].

$$\sum_{j=1}^{N} \sum_{i=1}^{M} \frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2 + (1+\sigma_i^2)\sigma_{id}^2} \geq \gamma N$$

$$\sum_{i=1}^{M} NE\left[\frac{p_i|h_i|^2}{\sigma_i^2 p_i |h_i|^2 + (1+\sigma_i^2)\sigma_{id}^2}\right] \geq \gamma N$$

where the expected value is the average SNR due to the i-th channel at the fusion center over the lifetime of the network. Define

$$\gamma_i \triangleq E\left[\frac{p_i|h_i|^2}{\sigma_i^2 p_i |h_i|^2 + (1+\sigma_i^2)\sigma_{id}^2}\right].$$

Then the SNR constraint is $\sum_{i=1}^{M} \gamma_i \geq \gamma$. If over the lifetime of the network i-th sensor provides an average SNR of γ_i at the fusion center such that the lifetime of node is maximized and $\sum_{i=1}^{M} \gamma_i = \gamma$, we have solved the problem. We start by solving the problem of maximizing the lifetime of the i-th node while meeting the average SNR requirement γ_i. Later we propose a numerical algorithm that assigns the proper average SNR to each sensor such that sum SNR requirement is satisfied. Since each sensor has its own energy supply, maximizing the individual lifetime is equivalent to minimizing the power consumption. Thus we have the following convex optimization problem for the i-th sensor.

$$\begin{aligned} &\min \sum_{j=1}^{N} p_{ij} \\ &s.t. \ E\left[\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2 + (1+\sigma_i^2)\sigma_{id}^2}\right] \geq \gamma_i \quad j = 1, \ldots, N \\ &p_{ij} \geq 0 \qquad \forall j \end{aligned}$$

Again, using the weak law of large numbers we can rewrite the problem.

$$\begin{aligned} &\min \sum_{j=1}^{N} p_{ij} \\ &s.t. \ \sum_{j=1}^{N} \left[\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2 + (1+\sigma_i^2)\sigma_{id}^2}\right] \geq \gamma_i N \\ &p_{ij} \geq 0 \quad \forall j \end{aligned} \tag{5}$$

The Lagrangian $\mathcal{L}$ of (5) can be written as:

$$\begin{aligned} \mathcal{L}(\mathbf{p}, \lambda, \nu) = &\sum_{j=1}^{N} p_{ij} - \sum_{j=1}^{N} \lambda_j p_{ij} \\ &+ \nu_i \left(\gamma_i N - \sum_{j=1}^{N} \left(\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2 + (1+\sigma_i^2)\sigma_{id}^2}\right)\right) \end{aligned}$$

The KKT conditions [4] for the problem are given by:

$$\lambda_j \geq 0 \ , \ \lambda_j p_{ij} = 0 \qquad \forall j \ , \ \nu_i \geq 0$$
$$\nu_i \left(\gamma_i N - \sum_{j=1}^{N} \left(\frac{p_{ij}|h_{ij}|^2}{\sigma_i^2 p_{ij}|h_{ij}|^2 + (1+\sigma_i^2)\sigma_{id}^2} \right) \right) = 0$$
$$\frac{\partial \mathcal{L}}{\partial p_{ij}} = 1 - \lambda_j - \nu_i \frac{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2}{(\sigma_i^2 p_{ij}^2 |h_{ij}|^2 + (1+\sigma_i^2)\sigma_{id}^2)^2} = 0 \qquad (6)$$

If $\nu_i = 0$ using the last KKT condition, it would mean that $\lambda_j = 1 \ \forall j$, which from complementary slackness implies that $p_{ij} = 0 \ \forall j$. This result is not acceptable which implies that $\nu_i > 0$. So since $\nu_i > 0$ again because of complementary slackness it implies that the first constraint in (5) is active at the optimal point. The optimal solution is obtained by solving the KKT conditions:

$$\begin{cases} \lambda_j = 1 - \nu_i \frac{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2}{\left(\sigma_i^2 p_{ij}^2|h_{ij}|^2 + \left(1+\sigma_i^2\right)\sigma_{id}^2\right)^2} & \\ \nu_i \leq \frac{\left(\sigma_i^2 p_{ij}^2|h_{ij}|^2 + \left(1+\sigma_i^2\right)\sigma_{id}^2\right)^2}{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2} & \\ p_{ij} > 0 \Rightarrow \lambda_j = 0 & \text{if } \ \nu_i > \frac{((1+\sigma_i^2)\sigma_{id}^2)^2}{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2} \\ \lambda_j > 0 \Rightarrow p_{ij} = 0 & \text{if } \ \nu_i < \frac{((1+\sigma_i^2)\sigma_{id}^2)^2}{|h_{ij}|^2(1+\sigma_i^2)\sigma_{id}^2} \end{cases}$$

Solving for $p_{ij} > 0$ and $\lambda_j = 0$ from Eq. (6), we get the following water-filling solution.

$$p_{ij} = \begin{cases} \frac{|h_{ij}|\sqrt{\nu_i}\sqrt{(1+\sigma_i^2)\sigma_{id}^2} - (1+\sigma_i^2)\sigma_{id}^2}{\sigma_i^2|h_{ij}|^2} & |h_{ij}|^2 > \frac{(1+\sigma_i^2)\sigma_{id}^2}{\nu_i} \\ 0 & |h_{ij}|^2 < \frac{(1+\sigma_o^2)\sigma_{id}^2}{\nu_i} \end{cases} \qquad (7)$$

The main challenge of the problem now lies in finding the value of ν_i since we do not know the values for $|h_{ij}|$s. We only know they are i.i.d distributed according to (2). In order to find the value of ν_i, first we need to express the SNR (S) in terms of our optimal power transmission solution.

$$S_{ij} = \begin{cases} \frac{1}{\sigma_i^2} - \frac{\alpha_i}{\sigma_i^2}\frac{1}{|h_{ij}|} & |h_{ij}| > \alpha_i \\ 0 & |h_{ij}| < \alpha_i \end{cases} \qquad (8)$$

where S_{ij} is the received SNR at the fusion center from the i-th sensor at the j-th transmission period and $\alpha_i \equiv \sqrt{\frac{(1+\sigma_i^2)\sigma_{id}^2}{\nu_i}}$. We need to find the expected value of the the SNR due to the i-th sensor at the fusion center over the lifetime of the network, which we denote by $E[S_i]$. A assuming that $|h_i|$ has pdf defined in (1), we can find the $E[S_i]$. In general if we have random variables X and Y such that:

$$Y = \begin{cases} C + g(X) & X > b \\ 0 & X < b \end{cases}$$

then

$$E[Y] = C \int_b^\infty f(x)dx + \int_b^\infty f(x)g(x)dx \qquad (9)$$

where, $f(x)$ is the pdf of X. Using Eq. (9) and assuming that $|h_i|$ has pdf defined in (1), then

$$E[S_i] = \frac{1}{\sigma_i^2}\exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2}\right) - \frac{\alpha_i}{\sigma_i^2}\int_{\alpha_i}^\infty \frac{1}{\sigma_{hi}^2}\exp\left(\frac{-x^2}{2\sigma_{hi}^2}\right)dx$$
$$E[S_i] = \frac{1}{\sigma_i^2}\exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2}\right) - \frac{\alpha_i}{\sigma_i^2}\left(\frac{\sqrt{2\pi}}{2\sigma_{hi}}\left(1 - \text{erf}\left(\frac{\alpha_i}{\sqrt{2}\sigma_{hi}}\right)\right)\right) \qquad (10)$$

where erf$(\cdot)$ is the one-sided error function. From Eq. (4), we know that $E[S_i] = \gamma_i$, so to find the value of ν_i, we need to solve the following problem:

$$\gamma_i = \frac{1}{\sigma_i^2}\exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2}\right) - \frac{\alpha_i}{\sigma_i^2}\left(\frac{\sqrt{2\pi}}{2\sigma_{hi}}\left(1 - \text{erf}\left(\frac{\alpha_i}{\sqrt{2}\sigma_{hi}}\right)\right)\right) \qquad (11)$$

There is no closed form solution of the above equation and it must be solved numerically. Having found ν_i for $i = 1, \ldots, M$ then

$$p_{ij} = \frac{\sqrt{(1+\sigma_i^2)\,\sigma_{id}^2}}{\sigma_i^2|h_{ij}^2}\left(\sqrt{\nu_i}|h_{ij}| - \sqrt{(1+\sigma_i^2)\,\sigma_{id}^2}\right)^+$$

where we define $(x)^+ = \max\{x, 0\}$. We can also find the $E[p_i]$:

$$E[p_i] = \frac{\sqrt{\pi\nu_i(1+\sigma_i^2)\sigma_{id}^2}}{\sqrt{2}\sigma_{hi}\sigma_i^2}\left[1 - \text{erf}\left(\frac{\alpha_i}{\sqrt{2}\sigma_{hi}}\right)\right] - \frac{(1+\sigma_i^2)\sigma_{id}^2}{2\sigma_i^2\sigma_{hi}^2}E_1\left(\frac{\alpha_i^2}{2\sigma_{hi}^2}\right)$$

where $E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n}dt, (x > 0, n = 0, 1, ...)$is the exponential integral function.

Once we have found the $E[p_i]$, the expected lifetime of the node can also be determined. Assume that the i-th node has as initial energy of $\mathcal{E}_i$. Also for simplicity take each transmission period to be $1s$. Thus the lifetime of the node is simply the number of transmissions. $\mathcal{E}_i = \sum_{j=1}^N p_{ij}$. Assuming $N \gg 1$ it follows from the Law of Large Numbers [3] that $\sum_{j=1}^N p_{ij} = NE[p_i]$, so $N = \frac{\mathcal{E}_i}{E[p_i]}$. Thus

$$N = \mathcal{E}_i \times \left\{ \frac{\sqrt{\pi\nu_i(1+\sigma_i^2)\sigma_{id}^2}}{\sqrt{2}\sigma_{hi}\sigma_i^2}\left[1 - \text{erf}\left(\frac{\alpha_i}{\sqrt{2}\sigma_{hi}}\right)\right] - \frac{(1+\sigma_i^2)\sigma_{id}^2}{2\sigma_i^2\sigma_{hi}^2}E_1\left(\frac{\alpha_i^2}{2\sigma_{hi}^2}\right) \right\}^{-1} \qquad (12)$$

We can also find the pdf of S_i of the sensor at the fusion center. Finding the pdf S_i allows us to do outage analysis for the sensor. The pdf of S_i, which we denote by $f_{S_i}(s)$, is defined by the following expression:

$$f_{S_i}(s) = \left[1 - \exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2}\right)\right]\delta(s) + \frac{\sigma_i^2\alpha_i^2}{\sigma_{hi}^2(1-s\sigma_i^2)^3}\exp\left(\frac{-\alpha_i^2}{2\sigma_{hi}^2(1-s\sigma_i^2)^2}\right) \qquad (13)$$

where $\delta(s)$ is the Dirac delta function. It can be seen from the pdf that S_i is a mixed random variable and its support region is $[0, \frac{1}{\sigma_i^2})$.

4. NETWORK POWER SCHEDULING

In the previous section we found the optimal solution that will maximize the lifetime of a sensor such that the expected value of the received SNR at the fusion center will be a given value. Now we consider the question of how to assign an expected value for each sensor (γ_i) such that $\sum_{i=1}^M \gamma_i = \gamma$ and all the nodes have the same expected lifetime (N) . We assume that our network consists of M sensors. We would like to maximize the lifetime of our network such that the expected value of the total SNR at the fusion center be equal to γ. We use the algorithm in Fig. 5 to find N and $\gamma_1, \gamma_2, ..., \gamma_M$.

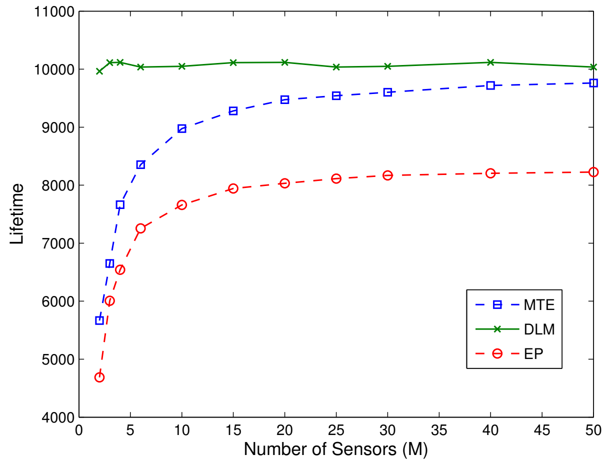

Fig. 3. Lifetime comparisons for identical sensors

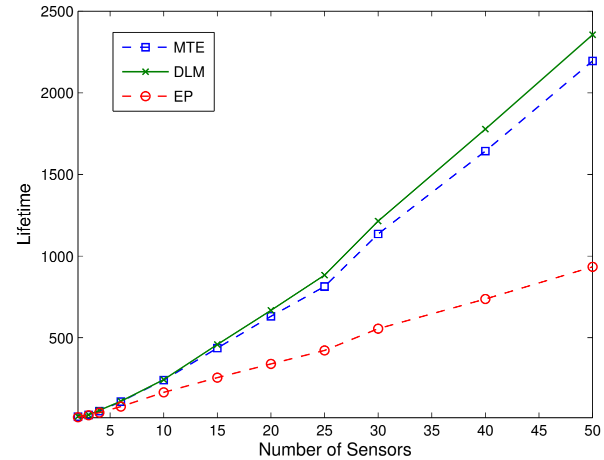

Fig. 4. Lifetime Comparisons for randomly chosen sensors

$N = N_0$
Use (12) to find ν_i for $i = 1, 2, ..., M$
Use (10)to find $E[s_i]$ for $i = 1, 2, ..., M$
while $\left|\sum_{i=1}^{M} E[S_i] - \gamma\right| > \epsilon$
$N \leftarrow N \frac{\gamma}{\sum_{i=1}^{M} E[S_i]}$
Use (12) to find ν_i for $i = 1, 2, ..., M$
Use (10)to find $E[s_i]$ for $i = 1, 2, ..., M$
end
$\gamma_i = E[S_i]$

Fig. 5. Algorithm for allocation of partial SNR's

5. NUMERICAL RESULTS

For our numerical results, we consider two different cases and in each case we compare the performance of our decomposition lifetime maximizing (DLM) method against the minimum-total-energy (MTE) [1] and equal-power (EP) strategies. For the EP strategy, we assign power to each sensor according to the residual energy left in the sensor.

In the first experiment, we show the effect of increasing number of sensors M, under equal statistics, where the required SNR at destination is normalized to the number of sensors, i.e., $M\gamma_0$. For our method this means that each sensor on average will provide an SNR of γ_0 over its lifetime. This normalization removes the effect of additional energy injected into the network via additional sensors. The results show the consistence of the performance of our method across different sizes. (See Fig. 3). From the numerical results one suspects that our method provides an upper bound for the MTE algorithm as the number of sensors approaches infinity, but we have no theoretical results at the present to support this conjecture.

For the second part we generate σ_i^2 , $\mathcal{E}_i$, and σ_{hi} randomly such that, $\sigma_i^2 \in \mathcal{U}[0.05, 0.1]$, $\mathcal{E}_i \in \mathcal{U}[250, 500]$, and $\sigma_{hi} \in \mathcal{U}[0.1, 0.2]$. Where $\mathcal{U}[a, b]$ denotes uniform distribution between a and b. For simplicity we take all the destination noise variances to be 0.08 and we require an average SNR of $10dB$ at the receiver. (See Fig. 4).

Please note that no dynamic programming simulations are presented due to their immense computational complexity that is beyond most existing computational facilities. We note that the main point of this work is not to claim any improvement over dynamic programming, but rather to provide a pragmatic method with fewer requirements.

6. CONCLUSION

This paper develops a solution for maximizing the lifetime of a cooperative sensor network, subject to an average SNR requirement, in an environment where each sensor has an individual battery, as well as time-varying channel conditions. The motivation for this work has been to address three key difficulties that are associated with a full-scale stochastic optimization, which is often performed with dynamic programming: (a) unwieldy computational complexity, and as a result: (b) quantization of the parameters, and (c) strong CSI requirements. We propose a two-step approximation. First, the SNR requirement is divided between the sensors according to their channel statistics and battery power, and secondly, each sensor's transmission is controlled for best behavior across time. The solution allows each sensor to operate independently of the other sensors, which greatly reduces the implementation overhead compared to MTE and dynamic programming algorithms.

7. REFERENCES

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