THROUGHPUT ANALYSIS OF WIRELESS MESH NETWORKS

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ABSTRACT

Wireless mesh networks are rapidly deployable for many applications. The throughput of such a network depends on the schemes used for medium access control. In this paper, we present some of our latest results on the (intra-network) throughput of large-scale networks. In particular, we show a comparison of the throughput between the opportunistic synchronous array method and the slotted ALOHA under different traffic loading, which reveals that the former yields a higher throughput than the latter except for the region of very low traffic (less than 10% load probability). We further show that a longer distance transmission yields lower throughput than the shortest distance transmission except for the region of extremely low traffic (less than 1% load probability).

***Index Terms*—** Wireless mesh network, Medium access control, adaptive scheduling, Intra-network throughput, Large network

1. INTRODUCTION

Wireless mesh network is a wireless network of relatively stationary nodes. Such a network can serve as a network of virtual base stations for conventional mobile clients in situations where the conventional base stations are not available. In a wireless mesh network, there can be two types of traffic. One is inter-network traffic where the data flows between source nodes and destination nodes involve access to a backbone network. The other is intra-network traffic where the data flows between source nodes and destination nodes stay in the mesh network. This paper is concerned with intra-network traffic.

The capacity scaling law of intra-network traffic of a large network is now well established [1], [2]. Specifically, the maximum achievable throughput of a large 2-D network, in bits-meter/s/Hz/node, is inversely proportional to the square-root of node density. However, the pre-constant of the scaling law, or the throughput in bits-hop/s/Hz/node, highly depends on the medium access control (MAC) schemes to be designed by researchers.

It should be noted that although representing a theoretical challenge to the above stated scaling law, the result shown in [3] requires a very large-scale cooperation (i.e., a virtual MIMO of dimension no less than $2^{14} \times 2^{14}$ according to our calculation) and is highly impractical.

The principles behind many existing MAC schemes are captured by CSMA (carrier sense multiple access) used in IEEE 802.11, MSH-DSCH (mesh distributed scheduler) used in IEEE 802.16, and ALOHA. Designed for cellular networks, CSMA eliminates concurrent co-channel transmissions within an entire radio transmission radius from each receiver, which causes a very large sparseness of concurrent co-channel transmissions and hence yields a very low throughput for large networks [2]. Although proposed for mesh networks, MSH-DSCH is not yet well understood for large networks. ALOHA is a scheme where each transmitting node initiates a transmission to a receiver randomly. Both slotted and un-slotted ALOHA have been well studied for cellular networks. For large mesh networks, slotted ALOHA is recently analyzed in [4] and [5]. Other variations of ALOHA are available in [6], [7]. But analysis of these variations for large networks remains a topic of research. A distributed and cooperative link scheduling algorithm for multi-hop large networks is recently developed in [8].

In addition to the above MAC schemes, there is a scheme called synchronous array method (SAM) which schedules concurrent co-channel transmissions with a pre-determined spacing in a synchronous fashion [2]. It has been shown in [2], [5], [9] that SAM yields a much higher throughput than slotted ALOHA under a full traffic load (i.e., each node has a packet waiting to be transmitted at any time).

In this paper, we extend the previous analysis to the case of any traffic load. In other words, we consider the case where each node has a packet for transmission with a (load) probability p_l. In the next section, We first extend the opportunistic SAM [9] by taking p_l into account and then present a throughput analysis. In section III, we formulate a slotted ALOHA with an adaptive transmission probability. In section IV, we present the numerical evaluations of the network throughput under both ALOHA and SAM. We show that SAM yields a

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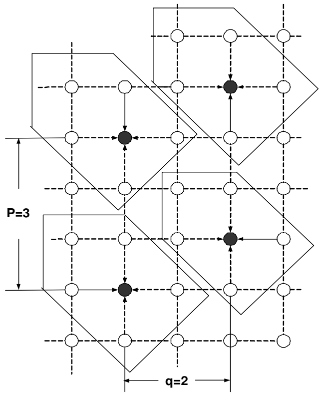

Fig. 1. A large network on square grid is partitioned into subnets. Here, $p = 3$, $q = 2$, $L = 6$ and $n_0 = n_j = 4$.

higher throughput than ALOHA except for the region of very low traffic (i.e., less than 10% load probability). We further demonstrate that by using a longer distance transmission, the throughput in bits-meter/s/Hz/node is even lower except for the region of extremely low traffic (i.e., less than 1% load probability). The network throughput under this load probability is less than half of that under the full traffic load. Such a quantitative insight is a quite surprising result and has a significant implication in practice.

2. OPPORTUNISTIC SAM WITH LOAD ADAPTATION

The basic idea of SAM proposed in [2] is that the concurrent co-channel transmissions are scheduled in a synchronous fashion and the spacing between concurrent co-channel transmissions is pre-determined to maximize the throughput. In other words, for each time slot, each of the concurrent co-channel receivers is placed in a subnet of a given dimension. The exact shape of each subnet depends on the network topology. For a network on square grid, the subnet partition is illustrated in Fig. 1. In [2], the transmitters in each time slot are also pre-determined.

In [9], it is proposed that the transmitter in each subnet is chosen opportunistically and the selected transmitter should be the one that has the largest channel gain with respect to the receiver. Furthermore, the largest channel gain must be larger than a pre-determined threshold. In order to achieve this, local channel state information with respect to each receiver must be obtained at the beginning of each time slot. Also, the channel acquisition time must be relatively small compared to the interval of each time slot. This is possible for mesh networks where all nodes are stationary and the channel coherence time is relatively long. However, for long term fairness (or traffic balance), we also require that from slot to slot, all channel coefficients vary randomly. This may not happen naturally for a static network. But we can induce such fading by repositioning the antenna on each node randomly from slot to slot. Such displacement is only in the order of half a wavelength, which does not affect the network topology at microwave frequencies. Alternatively, if multiple antennas are available on each node, we can change the phases of all transmitting antennas on each node randomly from slot to slot and keep the receiving beam vector of each node fixed.

We now extend the idea in [9] to the case where each node has a packet to transmit with the probability p_l. That is, if a node in a subnet (in a given time slot) does not have a packet to transmit to the receiver, then this node is automatically excluded from consideration for transmission to the receiver.

Using subnet 0 as example, the above described scheme can be mathematically expressed as

$$k_0 = \begin{cases} i_{max}, & \text{if } v_{0,0}(i_{max}) \geq \theta \\ \{\phi\}, & \text{otherwise} \end{cases} \tag{1}$$

where k_0 is the node index of the transmitter in subnet 0 selected by the receiver in subnet 0, $i_{max} = \arg\max_i \{v_{0,0}(i), i \in I_0\}$, and $v_{0,0}(i) = |h_{0,0}(i)|^2$. Here, $|h_{0,0}(i)|^2$ denotes the channel gain between the receiver in subnet 0 and the ith potential transmitter in subnet 0, and I_0 is the set of potential transmitters in subset 0 that have a packet to transmit to the receiver. The set I_0 also depends on the network topologies. For square-grid network, I_0 should consist of the four nearest neighboring nodes of the receiver (excluding all nodes that have no packet to transmit to the receiver). If diagonal (apart from horizontal and vertical) transmissions are allowed, the packets to be transmitted diagonally will experience longer delays than packets to be transmitted horizontally and vertically. This is because of the extra distance factor $\sqrt{2}$. But if the network topology is random, the restriction of horizontal or vertical transmission is not necessary because a packet travelling from source to destination will experience a series of hops of varying distances. It is shown in [5] that the network throughput in bits-meter/s/Hz/node does not vary much among regular network topologies as long as the node density remains the same. In this paper, only square-grid is considered.

To compute the throughput of this scheme, we first write the received signal y_0 at the receiving node in subnet 0 as: $y_0 = h_{0,0}x_0 + \sum_{j \neq 0} h_{0,j}x_j + w_0$ where, x_j is the transmitted signal from the transmitter in subnet j, w_0 denotes the white Gaussian noise with zero mean and variance σ^2. The factor $h_{0,j}$ is the channel coefficient between the transmitter in subnet j and the receiver in subnet 0, which is assumed to be a complex Gaussian random variable (i.e., random from slot to slot) with zero mean and variance $E|h_{0,j}|^2 = d_{0,j}^{-\alpha}$. α is the path loss exponent and $d_{0,j}$ is the distance between the transmitter and the receiver. For convenience of analysis, we assume that all nodes transmit with the same power P, i.e. $E|x_j|^2 = P$. Hence, the instantaneous signal to interference and noise ratio (SINR) at the receiver in subnet 0 is $SINR = \frac{v_{0,0}P}{\sum_{j \neq 0} v_{0,j}P + \sigma^2}$ where $v_{0,0} = |h_{0,0}|^2$ and

$v_{0,j} = |h_{0,j}|^2$ which are referred to as channel gains. Assuming that the instantaneous SINR at each receiver is not known by the corresponding transmitter (which is generally true due to random transmissions from other subnets), the network throughput in bits-hops/s/Hz/node (number of bits received over one hop distance per second per Hertz per node) can be defined as : $c_{SAM} = \frac{1}{L} R_\xi P_d$ where L is the node population in each subnet, $R_\xi = \log_2(1+\xi)$ is the packet spectral efficiency, and $P_d = \Pr\{SINR \geq \xi\}$ the probability of packet detection. Strictly speaking, the above expression of the throughput depends on the location of the receiver in the network. But we will consider a receiver in the center of the network, and hence the throughput obtained this way represents a lower bound of the network throughput. The conversion from bits-hop/s/Hz/node to bits-meter/s/Hz/node for each of square, triangle and hexagonal topologies is available in [5]. It is clear that the network throughput must be affected by all three parameters ξ, θ and p_l. A more explicit form of P_d can be obtained as follows:

$$\begin{aligned} P_d &= Pr\{SINR \geq \xi, v_{0,0} \geq \theta\} \\ &= Pr\{v_{0,0} \geq \xi(\sigma^2/P + \sum_{j\neq 0} v_{0,j}), v_{0,0} \geq \theta\} \\ &= Pr\{\sum_{j\neq 0} v_{0,j} \leq \frac{v_{0,0}}{\xi} - \frac{\sigma^2}{P}, v_{0,0} \geq \theta\} \\ &= \int_{\max(\frac{\xi\sigma^2}{P},\theta)}^{\infty} \left(\int_0^{\frac{y}{\xi}-\frac{\sigma^2}{P}} f_{v_I}(x)dx \right) f_{v_{0,0}}(y)dy \quad (2) \end{aligned}$$

where $f_{v_{0,0}}(y)$ is the pdf (probability density function) of $v_{0,0}$, and $f_{v_I}(x)$ is the pdf of $v_I = \sum_{j\neq 0} v_{0,j}$. From the fact that $|h_{0,0}(m)|^2$ is exponentially distributed with the mean $\Gamma_{0,0}(m) = d_{0,0}^{-\alpha}(m)$, where $d_{0,0}(m)$ is the distance between the transmitter and receiver in subnet 0 and α is the path loss exponent, it follows that

$$\begin{aligned} & Pr\{v_{0,0} \leq y\} \\ =& \prod_{m=1}^{n_0} \{(1-\exp\{-y/\Gamma_{0,0}(m)\})p_l + (1-p_l)\} \\ =& \prod_{m=1}^{n_0} (1 - p_l \exp\{-y/\Gamma_{0,0}(m)\}) \\ =& \sum_{k=0}^{n_0} \sum_{w_k} (-p_l)^k \exp(-y \sum_{m\in w_k} \frac{1}{\Gamma_{0,0}(m)}) \qquad (3) \end{aligned}$$

where w_k denotes a subset of $\Omega_0 = \{1, 2, \ldots, n_0\}$ with the cardinality k, and $n_0 \leq L-1$ is the maximum number of potential transmitters to the receiver in subnet 0. It then follows that

$$\begin{aligned} f_{v_{0,0}}(y) = & \sum_{k=1}^{n_0} (-1)^{k+1} p_l^k \sum_{\omega_k} [(\sum_{m\in\omega_k} \frac{1}{\Gamma_{0,0}(m)}) \\ & \exp\{\sum_{m\in\omega_k} \frac{-y}{\Gamma_{0,0}(m)}\}] + (1-p_l)^{n_0}\delta(y) \qquad (4) \end{aligned}$$

where $\delta(y)$ is a Dirac's delta function. Since v_I is a summation of independent exponential random variables, the pdf of v_I can be obtained by using the characteristic function: $\mathcal{F}_{v_I}(\mathcal{U}) = E_{v_I}[e^{-\mathcal{U}v_I}] = \prod_{j=1}^{S} \mathcal{F}_{v_{0,j}}(\mathcal{U})$. The cdf (cumulative distribution function) of $v_{0,j}$ in subnet j is $Pr\{v_{0,j} \leq x\} = \{\overline{P_j} + \sum_{l=1}^{n_j} P_j^l Pr\{|h_{0,j}(l)|^2 \leq x\}\}U(x)$ where $U(x)$ is a step function, $\overline{P_j}$ is the probability that there is no transmission in subnet j, and P_j^l is the probability that the lth node in subnet j transmits. By the assumption that $|h_{0,j}(l)|^2$ is also exponentially distributed with mean $\Gamma_{0,j}(l) = d_{0,j}^{-\alpha}(l)$, we can obtain: $P_j^l = \int_\theta^\infty \frac{p_l}{\Gamma_{j,j}(l)} e^{-\frac{x}{\Gamma_{j,j}(l)}} \prod_{k\neq l}^{n_j}(1 - p_l e^{\frac{-x}{\Gamma_{j,j}(k)}})dx$, $\overline{P_j} = 1 - \sum_l P_j^l$, and $Pr\{|h_{0,j}(l)|^2 \leq x\} = 1 - e^{-x/\Gamma_{0,j}(l)}$. It then follows that $f_{v_{0,j}}(x) = \overline{P_j}\delta(x) + \sum_{l=1}^{n_j} P_j^l \frac{1}{\Gamma_{0,j}(l)} e^{\frac{-x}{\Gamma_{0,j}(l)}} U(x)$ and $\mathcal{F}_{v_I}(\mathcal{U}) = \mathcal{B} + \sum_{j=1}^{S} \sum_{l=1}^{n_j} \frac{\mathcal{A}_j^l}{\mathcal{U} + 1/\Gamma_{0,j}(l)}$ where $\mathcal{A}_j^l = \frac{P_j^l}{\Gamma_{0,j}(l)} \prod_{k\neq 0,j}(\overline{P_k} + \sum_{m=1}^{n_k} P_k^m \frac{\frac{1}{\Gamma_{0,k}(m)}}{\frac{1}{\Gamma_{0,k}(m)} - \frac{1}{\Gamma_{0,j}(l)}})$ and $\mathcal{B} = \prod_{j=1}^{S} \overline{P_j}$. In the above, we have applied the fractional decomposition. Here we assume that the roots of the common denominator are distinct, i.e. $\Gamma_{0,j}(l) \neq \Gamma_{0,k}(m)$, $\forall j \neq k, l \neq m$. By taking the inverse Laplace transform, the pdf of v_I:

$$f_{v_I}(x) = \mathcal{L}^{-1}\{\mathcal{F}_{v_I}(\mathcal{U})\} = \mathcal{B}\delta(x) + \sum_{j=1}^{S} \sum_{l=1}^{n_j} \mathcal{A}_j^l e^{\frac{-x}{\Gamma_{0,j}(l)}} \qquad (5)$$

Plugging $f_{v_{0,0}}(x)$ and $f_{v_I}(y)$ into (2), we can obtain:

$$\begin{aligned} P_d = & 1 - \prod_{m=1}^{n_0} \left(1 - e^{-\frac{\max(\frac{\sigma^2\xi}{P},\theta)}{\Gamma_{0,0}(m)}}\right) + \sum_{j=1}^{S} \sum_{l=1}^{n_j} e^{\frac{-\max(\frac{\theta}{\xi} - \frac{\sigma^2}{P}, 0)}{\Gamma_{0,j}(l)}} \\ & C_j^l \sum_{k=0}^{n_0} (-p_l)^k \sum_{\omega_k} \frac{\sum_m \frac{1}{\Gamma_{0,0}(m)}}{\sum_m \frac{1}{\Gamma_{0,0}(m)} + \frac{1}{\xi\Gamma_{0,j}(l)}} e^{\sum_{m\in\omega_k} \frac{-\max(\frac{\xi\sigma^2}{P},\theta)}{\Gamma_{0,0}(m)}} \end{aligned}$$

where $C_j^l = \mathcal{A}_j^l \Gamma_{0,j}(l)$.

3. SLOTTED ALOHA WITH LOAD ADAPTATION

This scheme is as follows. During each time slot, each node in the network transmits a packet with the probability p_t provided that the node has a data packet to transmit. Since p_l is the probability that each node has a packet to transmit, the effective probability that a packet is transmitted from each node is $p_l p_t$. Then, as shown in [4], [5], the network throughput in bits-hop/s/Hz/node is $c_{ALOHA} = (1 - p_l p_t) p_l p_t R_\xi P_d$ where $R_\xi = \log_2(1+\xi)$ and $P_d = e^{-\xi \frac{\sigma^2}{\mu_0 P}} \prod_{i\geq 1} [p_l p_t \frac{1}{\xi \frac{\mu_i}{\mu_0} + 1} + (1 - p_l p_t)]$. Here, $\mu_i = d_i^{-\alpha}$ and d_i is the distance between the ith interferer and the receiver. It is clear that the network throughput of this scheme depends ξ and the product $p_t p_l$. For each ξ, there is an optimal value p_{opt} for the product $p_t p_l$. Assuming that each node knows p_l, p_t should be chosen as follows: $p_t = 1$ if $p_l \leq p_{opt}$, and $p_t = \frac{p_{opt}}{p_l}$ if $p_l > p_{opt}$.

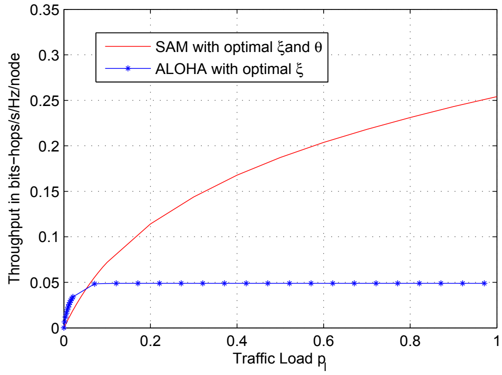

Fig. 2. Throughput of SAM and ALOHA versus load probability. $SNR = P/\sigma^2 = 10dB$.

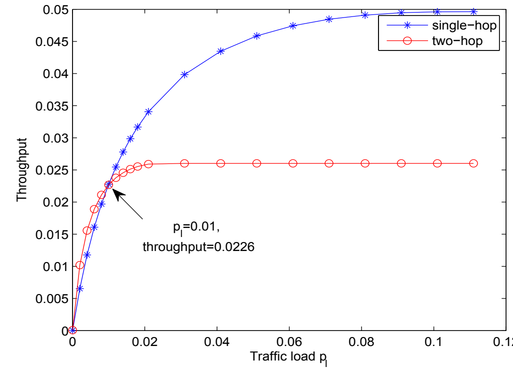

Fig. 3. Distance weighted throughput of ALOHA with different transmission ranges: 1-hop range and 2-hop range. $P/\sigma^2 = 10dB$ for 1-hop range, and $2^{-\alpha}P/\sigma^2 = 10dB$ for 2-hop range. $\xi = \xi_{opt}$ and $p_t = p_{opt}$.

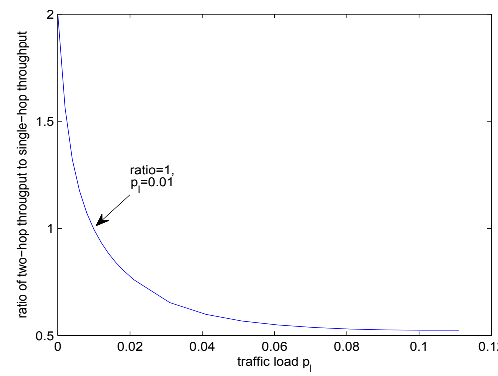

Fig. 4. Ratio of distance weighted throughput of different transmission ranges: 2-hop range over 1-hop range.

4. THROUGHPUT COMPARISON

To illustrate the throughput of the schemes presented earlier, we consider a large network of 294 nodes on a square grid. The distance between adjacent nodes is fixed to be one. The path loss exponent is $\alpha = 4$. The nominal SNR is $SNR = P/\sigma^2 = 10$. For SAM, we choose the size of each subnet as shown in Fig. 1 where $L = 6$. Fig. 2 shows the throughput (in bits-hop/s/Hz/node) of SAM and ALOHA versus p_l. For SAM, the throughput is optimized over ξ and θ. For ALOHA, the throughput is optimized over ξ. We see that as long as $p_l > 10\%$, SAM yields higher throughput. In other words, when the traffic load is very low, ALOHA has a higher throughput.

The scenario where the traffic load is very low prompts one to think of long distance transmissions in hope to increase the throughput in bits-meter/s/Hz/node. In Fig. 3, we compare the distance-weighted throughput of ALOHA for 1-hop distance transmission and 2-hop distance transmission. Here, the throughput for each of the two transmission distances is optimized over ξ for each given p_l. We see that as long as $p_l > 1\%$, the shortest (1-hop) distance transmission has a higher throughput. Only when $p_l < 1\%$, the longer (2-hop) distance transmission starts to yield a higher throughput. In Fig. 4, we show the ratio of the long distance throughput over the short distance throughput. Clearly, this ratio is upper bounded by two, which is approached when p_l is near zero.

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