CAPACITY ANALYSIS OF MIMO AD HOC NETWORK WITH STREAM NUMBER SELECTION

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***Abstract*—We study the problem of stream number selection in an ad hoc multiple-input multiple-output (MIMO) network, where there are a number of co-existent MIMO transmitter-receiver pairs. We assume that there is no channel state information (CSI) at the transmitters, and the receivers use single-user detection. It is shown that the link capacities at high signal-to-noise ratio (SNR) is mainly limited by the number of degrees of freedom available at the receiver side. In general the number of streams per transmitter should not be more than N_r/L when the interference level is high, where N_r is the number of receive antennas and L is the number of simultaneous links. When the interference level is low, the number of streams should be set equal to the number of transmit antennas.**

***Index Terms*—ad hoc network, MIMO, stream number selection, capacity, degrees of freedom**

I. Introduction

While the capacity of a point-to-point MIMO system is relatively well understood, there are fewer results about the capacity of an ad hoc MIMO network. One reason is that the problem is distributed in nature.

The capacity of MIMO system can be improved if there are channel state information (CSI) available at the transmitter. The capacity of ad hoc network with transmitter side CSI and single-user detection was investigated in [1]. It was shown that the capacity of an individual link is a convex function for fixed covariance matrix of the other users. Feeding back of CSI decreases the system payload. Also, accurate transmitter side CSI may not be available if the channel is fast fading.

Multi-user detection is also useful as the receiver can perform interference cancellation to increase the capacity. However, it is only helpful under weak or strong interference, and it does not outperform orthogonal signal when interference level is comparable to signal level [2].

In this paper, we will discuss the capacity achieved by stream number selection in an ad hoc MIMO network, where there is no CSI at the transmitter. We also assume single-user detection at the receivers. Based on an asymptotic channel capacity analysis and a piecewise linear approximation to the capacity, we will show how to select optimal number of streams to maximize the individual (and overall) capacity.

This paper is organized as follows. In Section II, we present the system model of a MIMO ad hoc network. The asymptotic capacity analysis and a piecewise linear approximation to the capacity is proposed in Section III. We compare the proposed approximation and the numerical calculations in Section IV, and conclude the paper in Section V.

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Notation: boldface uppercase (lowercase) letters denote matrices (vectors). † denotes conjugate transpose.

The identity and all-zero matrices of size $M \times M$ are denoted as $\boldsymbol{I}_M$ and $\boldsymbol{0}_M$, respectively. $\mathcal{C}$ is the complex numbers set. $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes a complex normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ denote the trace and determinant of a matrix, respectively. $\mathrm{diag}(\cdot)$ is diagonal matrix with elements specified in the argument, or a block diagonal matrix if the arguments are matrices. A point-to-point MIMO link with M transmit and N receive antennas is called an $N \times M$ MIMO system.

II. System Model

Consider an ad hoc wireless network, where each node is equipped with multiple antennas. Suppose that there are L effective peer-to-peer communication links in the network, where each link consists of one transmitter, with N_t antennas, and one receiver, with N_r antennas. We assume that all the channels are flat fading, and the received signals are corrupted by additive white Gaussian noise (AWGN).

A. The general model

The received signal of the lth user can be expressed as

$$\boldsymbol{y}_l = \sqrt{\gamma_{l,l}}\boldsymbol{H}_{l,l}\boldsymbol{x}_l + \textstyle\sum_{k=1,k\neq l}^{L} \sqrt{\gamma_{l,k}}\boldsymbol{H}_{l,k}\boldsymbol{x}_k + \boldsymbol{n}_l, \quad (1)$$

where $\boldsymbol{y}_l$ and $\boldsymbol{n}_l$ are vectors in $\mathcal{C}^{N_r \times 1}$ denoting the received signal and additive noise at the lth receiver, respectively; $\boldsymbol{x}_l \in \mathcal{C}^{N_t \times 1}$ is the transmitted signal of the lth transmitter, and $\sqrt{\gamma_{l,k}}\boldsymbol{H}_{l,k} \in \mathcal{C}^{N_r \times N_t}$ is the MIMO channel from kth transmitter to lth receiver.

We assume the following: AS1) $\boldsymbol{n}_k \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{N_r})$. AS2) $\boldsymbol{x}_l \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{Q}_l)$, where the covariance matrix $\boldsymbol{Q}_l$ has an eigen-value decomposition as $\boldsymbol{Q}_l = \boldsymbol{U}_l\boldsymbol{\Lambda}_l\boldsymbol{U}_l^\dagger$. AS3) All entries of all $\boldsymbol{H}_{l,k}$ matrices are i.i.d. $\mathcal{CN}(0,1)$ distributed. AS4) $\boldsymbol{H}_{l,l}$ is known perfectly at the lth receiver.

The constants $\{\gamma_{l,k} : k, l \in \{1, \dots, L\}\}$ control the channel magnitudes. We define an $L \times L$ matrix $\boldsymbol{\Gamma}$ whose (l,k)th entry is $\gamma_{l,k}$. The diagonal entries of $\boldsymbol{\Gamma}$ control the signal-to-noise ratios (SNR) and the off-diagonal entries control the interference-to-noise ratios (INR).

Define $\tilde{\boldsymbol{H}}_l = [\boldsymbol{H}_{l,1}, \dots, \boldsymbol{H}_{l,L}]$, $\tilde{\boldsymbol{Q}} = \mathrm{diag}(\boldsymbol{Q}_1, \dots, \boldsymbol{Q}_L)$, $\tilde{\boldsymbol{\Gamma}} = \mathrm{diag}(\gamma_{l,1}\boldsymbol{I}_{Nt}, \dots, \gamma_{l,L}\boldsymbol{I}_{Nt})$ and

$$\tilde{\boldsymbol{H}}_{-l} = [\boldsymbol{H}_{l,1}, \dots, \boldsymbol{H}_{l,l-1}, \boldsymbol{H}_{l,l+1}, \dots \boldsymbol{H}_{l,L}] \quad (2)$$

$$\tilde{\boldsymbol{Q}}_{-l} = \mathrm{diag}(\boldsymbol{Q}_1, \dots, \boldsymbol{Q}_{l-1}, \boldsymbol{Q}_{l+1}, \dots, \boldsymbol{Q}_L) \quad (3)$$

$$\tilde{\boldsymbol{\Gamma}}_{-l} = \mathrm{diag}(\gamma_{l,1}\boldsymbol{I}_{Nt}, \dots, \gamma_{l,l-1}\boldsymbol{I}_{Nt}, \gamma_{l,l+1}\boldsymbol{I}_{Nt}, \dots \gamma_{l,L}\boldsymbol{I}_{Nt}). \quad (4)$$

We assume that single user detection is used at each receiver, which means that the interferences are treated as noise. The interference plus noise covariance matrix at receiver l is given by $\boldsymbol{R}_l = \boldsymbol{I}_{N_r} + \tilde{\boldsymbol{H}}_{-l}\tilde{\boldsymbol{\Gamma}}_{-l}\tilde{\boldsymbol{Q}}_{-l}\tilde{\boldsymbol{H}}_{-l}^{\dagger}$. The capacity of the lth link with single user detection is given by [3], [1]

$$C^{(l)} = \mathrm{E}\left\{\log_2\left[\det\left(\boldsymbol{I}_{N_r} + \tilde{\boldsymbol{H}}_l\tilde{\boldsymbol{\Gamma}}\tilde{\boldsymbol{Q}}\tilde{\boldsymbol{H}}_l^{\dagger}\boldsymbol{R}_l^{-1}\right)\right]\right\} \quad (5)$$

The quantity $C^{(l)}$ is only a mutual information and an achievable rate for the lth link. The Shannon capacity may be larger, with optimized $\boldsymbol{Q}_l$ and multi-user detection. However, for convenience, we still call $C^{(l)}$ capacity in the following.

The rank of $\boldsymbol{Q}_l$ determines the *number of streams* sent out simultaneously by the lth transmitter, and is denoted as B_l. Due to the rotational invariance in the statistical distribution of the $\{\boldsymbol{H}_{l,k}\}$ matrices, $\{\boldsymbol{U}_l\}$ do not affect the capacities. The value of $C^{(l)}$ in (5) does not change if we replace $\tilde{\boldsymbol{Q}}$ and $\tilde{\boldsymbol{Q}}_{-l}$ with $\tilde{\boldsymbol{\Lambda}}$ and $\tilde{\boldsymbol{\Lambda}}_{-l}$, respectively, where $\tilde{\boldsymbol{\Lambda}}$ and $\tilde{\boldsymbol{\Lambda}}_{-l}$ are similarly defined as $\tilde{\boldsymbol{Q}}$ and $\tilde{\boldsymbol{Q}}_{-l}$. Selecting the number of streams will affect the capacity, though.

B. Simplified model

Having presented the general system model, we specialize the model to a simplified case that we will consider in the paper. In addition to the assumptions we made in the general model, we further assume that AS5) the number of streams per link is the same: $B_l = B, \forall l$. AS6) The SNRs and INRs are the same for the links:

$$\gamma_{l,k} = \begin{cases} \gamma, & \text{if } l = k, \\ \eta & \text{if } l \neq k, \end{cases} \qquad l,k = 1,2,\ldots,L. \quad (6)$$

AS7) all the transmitted powers are one: $\mathrm{tr}(\boldsymbol{Q}_l) = 1, \forall l$. AS8) The power is equally allocated among all active streams: $\boldsymbol{Q}_l = \mathrm{diag}[(1/B)\boldsymbol{I}_B, \boldsymbol{0}_{N_t-B}]$. AS9) $\gamma \gg 1$.

Under these simplifying assumptions, we would like to analyze how the number B of streams affects the capacity $C^{(l)}$ for given N_t, N_r and L, under various signaling conditions expressed in terms of the SNR γ and INR η.

III. Analysis of the capacity

Under the simplified model considered in Section II-B, all the links have identical capacity. To analyze the links' capacities, it is sufficient to analyze one of them. Define

$$\boldsymbol{D}_1 = \mathrm{diag}[(\gamma/B)\boldsymbol{I}_B, (\eta/B)\boldsymbol{I}_{B(L-1)}], \quad (7)$$
$$\boldsymbol{D}_2 = (\eta/B)\boldsymbol{I}_{B(L-1)}. \quad (8)$$

Let $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ denote two random matrices of i.i.d. entries $\mathcal{CN}(0,1)$ distributed with sizes $N_r \times BL$ and $N_r \times B(L-1)$, respectively. We can obtain that $\forall l$, $C^{(l)} = C_1 - C_2$, where

$$C_i = \mathrm{E}\left\{\log_2\left[\det\left(\boldsymbol{I}_{N_r} + \boldsymbol{H}_i\boldsymbol{D}_i\boldsymbol{H}_i^{\dagger}\right)\right]\right\}, \quad i = 1,2. \quad (9)$$

Both C_1 and C_2 can be viewed as the capacity of some point-to-point MIMO system. Such single-link MIMO capacity has been relatively well understood, e.g., [4]. Closed-form expressions for computing C_1 and C_2 can be derived based on results in [5], [6]. The results there do not allow for equal diagonal entries in the $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$, but modifications can be made to deal with such cases. We have used such modified results in our numerical computations of the capacities.

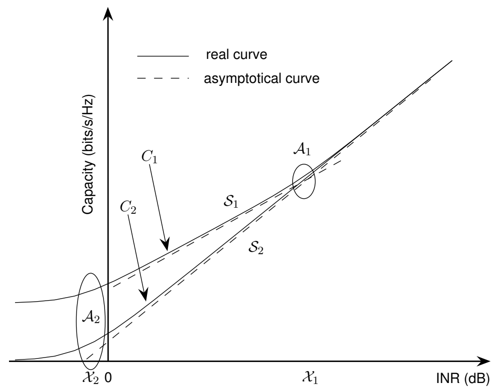

Fig. 1. Illusion of asymptotic analysis.

The numerical expressions, even though providing an exact value of the capacity, do not offer insights about how the system should be optimized. In particular, it does not provide an answer to the question: What should be the optimal number of streams per link? We will discuss this problem with an asymptotic analysis.

A. General behavior of C_1 and C_2

In our system, the *number d_r of receive degrees of freedom (DOF)* is the number of receive antennas N_r. Viewing each of C_1 and C_2 as the capacity of a point-to-point MIMO system, we can define the *number of transmit DOF* for C_i, $i = 1,2$, as the number of diagonal entries that are significantly larger than the remaining ones. And they are denoted as d_t and d_i, respectively, as the total number of transmit DOF and the number of DOF from interference. There are three cases regarding the relationship between γ and η. We will use $\gamma^{(\mathrm{dB})}$ and $\eta^{(\mathrm{dB})}$ to denote the SNR and INR in dB.

1) *Strong interference level*: $\eta \gg \gamma$. The entries of $\boldsymbol{D}_1$ containing γ are negligible, so $d_t = d_i = B(L-1)$. Thus, the slope of C_1 and C_2 as functions of $\eta^{(\mathrm{dB})}$ at large $\eta^{(\mathrm{dB})}$ are the same and can be determined as $\min(N_r, B(L-1))$. If $B(L-1) < N_r$, which means $d_r > d_i$, the receiver still has additional DOF to support communication, which leads to a positive difference between C_1 and C_2. If $B(L-1) \geq N_r$, there will not be enough DOF at the receiver, the difference between C_1 and C_2 goes to zero as η goes to infinity.
2) *Weak interference level:* $\eta \ll \gamma$. In this case, $d_t = B$ and $d_i = 0$. As a result, C_2 is much smaller compared to C_1, and C_1 is the capacity of an $N_r \times B$ MIMO system, which depends on $\gamma^{(\mathrm{dB})}$ linearly for large $\gamma^{(\mathrm{dB})}$ and the slope is $\min(B, N_r)$.
3) *Moderate interference level*. In this case, $d_t = BL$, and $d_i = B(L-1)$. There may be a mismatch between the slope of C_1, which is $\min(N_r, BL)$, and the slope of C_2, which is $\min[N_r, B(L-1)]$, both viewed as functions of $\eta^{(\mathrm{dB})}$. The difference will affect the link capacities $C^{(l)}$.

Fig. 1 depicts the general relationship between C_1 and C_2 for fixed SNR γ and varying INR η, as well as their asymptotic straight line approximations. We can see the areas $\mathcal{A}_1$ and $\mathcal{A}_2$ can be viewed as separating the $\eta^{(\mathrm{dB})}$ axis into three regions, corresponding to the three previously discussed cases. The threshold INR levels are denoted as $\mathcal{X}_1$ and $\mathcal{X}_2$. In the middle region, the slope of C_i is denoted as $\mathcal{S}_i$, $i = 1, 2$. The slopes $\mathcal{S}_1$ and $\mathcal{S}_2$ are the same if $B(L-1) \geq N_r$, or different, otherwise. This middle region $\mathcal{X}_2 < \eta^{(\mathrm{dB})} < \mathcal{X}_1$ is more interesting, as the capacity $C^{(l)}$ outside it does not change much.

B. Approximation of $\mathcal{X}_2$

From Fig. 1, we can see that the approximate value of $\mathcal{X}_2$ can be obtained by the intercept of the asymptotic line, which can be calculated from the following lemma.

Lemma 1: [7, Equation (15)] In an i.i.d. Rayleigh-faded $N \times M$ MIMO system, the intercept or *power offset* of the linear approximation to the capacity for large SNR is

$$\mathcal{L}(N, M) = \log_2 N + \left(\hat{\gamma} - \sum_{l=1}^{N_{\Uparrow}-N_{\Downarrow}} \frac{1}{l} - \frac{N_{\Uparrow}}{N_{\Downarrow}} \sum_{l=N_{\Uparrow}-N_{\Downarrow}+1}^{N_{\Uparrow}} \frac{1}{l} + 1\right) \log_2 e \quad (3\mathrm{dB}) \quad (10)$$

where 3dB=$10\log_{10} 2$, $\hat{\gamma} \approx 0.5772$ is Euler-Mascheroni constant, $N_{\Uparrow} = \max(M, N)$, and $N_{\Downarrow} = \min(M, N)$. The asymptotic slope $\mathcal{S}$ in bits/s/Hz/(3dB) is $N_{\Downarrow}$. □

The equivalent SNR of C_2 equals $(L-1)\eta$, and hence using the lemma we have

$$\mathcal{X}_2 = (10\log_{10} 2)\mathcal{L}(N_r, B(L-1)) - 10\log_{10}(L-1) \text{ (dB)} \quad (11)$$

C. Approximation of $\mathcal{X}_1$:

The value $\mathcal{X}_1$ indicates an approximate interference level that the capacity becomes DOF limited, which could be investigated from the changes of C_1's slope. It is shown that when the smallest eigenvalue is significantly smaller than 1/SNR, the corresponding virtual channel is negligible [7], [8], and thus the slope changes. We will need the following two lemmas.

Lemma 2: (Cauchy-Binet formula) Let $\boldsymbol{A} \in \mathcal{C}^{m\times n}$, $\boldsymbol{B} \in \mathcal{C}^{n\times q}$ and $\boldsymbol{C} \in \mathcal{C}^{q\times m}$ where $m \leq n$ and $m \leq q$. Then

$$\det(\boldsymbol{ABC}) = \textstyle\sum_{(s)} \sum_{(p)} \det \boldsymbol{A}_{(s)} \cdot \det \boldsymbol{B}_{(p)}^{(s)} \cdot \det \boldsymbol{C}^{(p)}$$

where (s) is an increasingly ordered subset of $\{1, ..., n\}$ of size m; (p) is an increasingly ordered subset of $\{1, \ldots, q\}$; $\boldsymbol{B}_{(p)}^{(s)}$ is a submatrix of $\boldsymbol{B}$ where the rows and columns are those given by (s) and (p), respectively; $\boldsymbol{A}_{(s)}$ is a submatrix of $\boldsymbol{A}$ with columns given by (s); and $\boldsymbol{C}^{(p)}$ is a submatrix of $\boldsymbol{C}$ with rows given by (p). □

Lemma 3: [9, Lemma II.1] If $\boldsymbol{A}$ is a random matrix of size $m \times n$, with i.i.d. entries $\mathcal{CN}(0, 1)$, and $k \leq \min(m, n)$, then

$$\mathrm{E}\left[\det(\boldsymbol{A}_{(s)}^{(p)} (\boldsymbol{A}^{\dagger})_{(u)}^{(v)})\right] = \begin{cases} k!, & \text{if } (p) = (u), (s) = (v) \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

where (p) and (u) are increasingly ordered subsets of $\{1, \ldots, m\}$, and (s) and (v) are increasingly ordered subsets of $\{1, \ldots, n\}$, all of size k. □

Using these lemmas, we next evaluate C_1 in two cases:

1) Case 1: $BL \geq N_r$. Define $T = N_r - B(L-1)$. Using Jensen's inequality, C_1 can be approximated as

$$\begin{aligned} C_1 \approx & \mathrm{E}\left\{\log_2 \left[\det\left(\boldsymbol{H}_1 \boldsymbol{D}_1 \boldsymbol{H}_1^{\dagger}\right)\right]\right\} \\ \leq & \log_2 \left\{\mathrm{E}\left[\det\left(\boldsymbol{H}_1 \boldsymbol{D}_1 \boldsymbol{H}_1^{\dagger}\right)\right]\right\} \end{aligned}$$

There are two sub-cases. If $BL - B \geq N_r$, then

$$\begin{aligned} C_1 \approx & \log_2 \left\{\sum_{i=0}^{B} \binom{B}{i} \left(\frac{\gamma}{B}\right)^i \binom{BL-B}{N_r - i} \left(\frac{\eta}{B}\right)^{N_r - i} N_r!\right\} \\ = & \log_2 \left\{e_0 \eta^{N_r} \left[1 + \sum_{i=1}^{B} e_i \left(\frac{\gamma}{\eta}\right)^i\right]\right\} \end{aligned} \quad (13)$$

If $BL - B < N_r$, then

$$\begin{aligned} C_1 \approx & \log_2 \left\{\sum_{j=0}^{BL-Nr} \binom{B}{T+j} \left(\frac{\gamma}{B}\right)^{T+j} \binom{BL-B}{j} \left(\frac{\eta}{B}\right)^{N_r-T-j} N_r!\right\} \\ = & \log_2 \left\{f_0 \eta^{BL-B} \left[1 + \sum_{j=1}^{BL-Nr} f_j \gamma^{N_r} \left(\frac{\gamma}{\eta}\right)^j\right]\right\} \end{aligned} \quad (14)$$

where e_i's, f_j's, $i = 0, \ldots, B, j = 0, \ldots, BL - N_r$ are certain coefficients not related to γ and η. When $\gamma \ll \eta$, the terms inside the summations in (13) and (14) are negligible, so the slope with respect to $\eta^{(\mathrm{dB})}$ will be equal to $\min[B(L-1), N_r]$. When η is close to or smaller than γ, these terms will be comparable to 1 and the corresponding slope of C_1 will be smaller.

2) Case 2: $BL < N_r$. Using the fact that $\det(\boldsymbol{I} + \boldsymbol{AB}) = \det(\boldsymbol{I} + \boldsymbol{BA})$, we have

$$\begin{aligned} C_1 = & \mathrm{E}\left\{\log_2 \left[\det\left(\boldsymbol{I}_{BL} + \boldsymbol{D}_1 \boldsymbol{H}_1^{\dagger} \boldsymbol{H}_1\right)\right]\right\} \\ \approx & \log_2 \left\{\mathrm{E}\left[\det\left(\boldsymbol{D}_1 \boldsymbol{H}_1^{\dagger} \boldsymbol{H}_1\right)\right]\right\} \\ \approx & \log_2 \left\{\left(\frac{\gamma}{B}\right)^B \left(\frac{\eta}{B}\right)^{BL-B} \binom{N_r}{BL} (BL)!\right\} \end{aligned} \quad (15)$$

The slope with respect to $\eta^{(\mathrm{dB})}$ is therefore $B(L-1)$.

Due to space limitation, detailed derivation is omitted. Based on the discussion on these two cases and the expressions in (13) and (14), we approximate $\mathcal{X}_1$ as $\gamma^{(\mathrm{dB})}$.

D. Calculation of $C^{(l)}$ in limit cases

We now calculate the difference between C_1 and C_2 under two extreme cases: $\eta^{(\mathrm{dB})} \to -\infty$ and $\eta^{(\mathrm{dB})} \to \infty$. In the first case, $C_2 \to 0$, and C_1 can be evaluated using Lemma 1 as

$$C_{1,\eta^{(\mathrm{dB})}\to-\infty}(\gamma) \simeq \min(N_r, B)\left(\frac{\gamma^{(\mathrm{dB})}}{10\log_2 10} - \mathcal{L}(N_r, B)\right) \quad (16)$$

where "$\simeq$" means that the two quantities are asymptotically the same. Similarly, we also have the capacity of C_2 when $\eta \gg \gamma$ as

$$C_{2,\eta\gg\gamma}(\eta) \simeq \min(N_r, BL-B)\left(\frac{\eta^{(\mathrm{dB})}}{10\log_2 10} - \mathcal{L}(N_r, BL-B)\right). \quad (17)$$

The corresponding capacity of C_1 can be approximated as

$$C_{1,\eta\gg\gamma}(\eta) \simeq \min(N_r, BL-B)\left(\frac{\eta^{(\mathrm{dB})}}{10\log_2 10} - \mathcal{L}_{corr}\right). \quad (18)$$

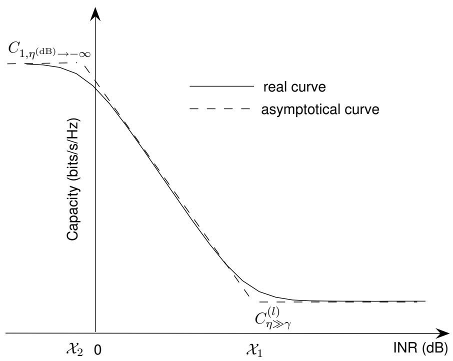

Fig. 2. Illusion of piecewise linear approximation.

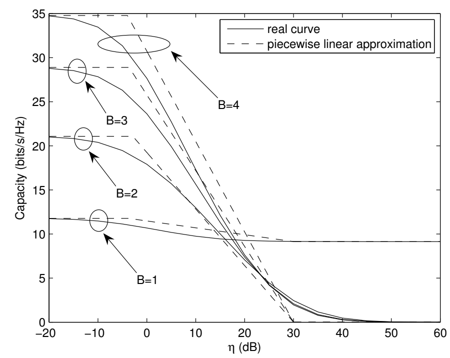

Fig. 3. Comparison between the analytical curve and piecewise linear approximation. γ=30dB, $N_r = 4$, $L = 4$.

where $\mathcal{L}_{corr}$ is the power offset for correlated Rayleigh fading channel [7, Equation (28)], which can also be approximated through various bounds, e.g. (13), (14) and (15). Subtracting $C_{2,\eta\gg\gamma}(\eta)$ from $C_{1,\eta\gg\gamma}(\eta)$, we have

$$C^{(l)}_{\eta\gg\gamma} \simeq \min(N_r, BL-B)[\mathcal{L}(N_r, BL-B)) - \mathcal{L}_{corr}]. \quad (19)$$

Base on the former analysis, the capacity of ad hoc MIMO network can be evaluated through piecewise linear approximation, shown in Fig. 2, which we summarize as follows:

Proposition 1: The average capacity of ad hoc MIMO network satisfying the assumption AS1)– AS9) can be evaluated through the following piecewise linear approximation:

$$C^{(l)} \approx \begin{cases} C_{1,\eta^{(\mathrm{dB})}\to-\infty}, & \text{if } \eta \leq \mathcal{X}_2 \\ C^{(l)}_{\eta\gg\gamma} & \text{if } \eta \geq \mathcal{X}_1 \\ \frac{\eta^{(\mathrm{dB})}-\mathcal{X}_2}{\mathcal{X}_1-\mathcal{X}_2}(C^{(l)}_{\eta\gg\gamma} - C_{1,\eta^{(\mathrm{dB})}\to-\infty}) & \text{otherwise} \end{cases}$$

where $\mathcal{X}_2$ is given in (11), $\mathcal{X}_1 = \gamma^{(\mathrm{dB})}$; and $C_{1,\eta^{(\mathrm{dB})}\to-\infty}$, $C_{2,\eta\gg\gamma}(\eta)$ and $C_{1,\eta\gg\gamma}(\eta)$ are as given in (16)–(18).

IV. Simulation and Discussion

To evaluate our analysis result, we show the comparison between the piecewise linear approximation and the exact capacity curves in Fig. 3. The exact curves were obtained using methods in [5], [6], modified to allow for equal values on the diagonals of $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$. The received SNR γ is 30dB. We used $N_t = N_r = L = 4$.

The capacity shows different characteristics when the stream number increases, as we discussed. When $\eta > \gamma$, the capacity tends to zero if the number of receive DOF N_r is less than the number of interference streams $B(L-1)$, otherwise it tends to a positive constant. Thus the stream number B in this region should not be larger than $\lfloor N_r/L \rfloor$. Notice that if $B = \lfloor N_r/(L-1) \rfloor$ and it is larger than zero, $C^{(l)}$ is not zero. However, such B is not an optimal choice as the DOF is not enough to recover all the data streams from the interference streams. When $\eta < \mathcal{X}_2$, the interference is negligible. We should use as many streams as possible, namely $B = N_t$ streams in this case [4]. As to the middle part, the capacity can be approximated as a linear function. In the left part of this region, we still favor large number of streams, but the performance becomes worse and smaller number of streams is preferred as INR increases. The transition INR for stream number change from N_t to $\lfloor N_r/L \rfloor$ can be obtained either through analytical curve or piecewise linear approximation. And the receiver will feedback the optimal number of streams to the transmitter. In Fig. 3, an accurate estimate of critical interference level from numerical calculation is η=18dB, whereas the piecewise linear approximation gives 20dB.

Finally, we point out that the analytical analysis will suffer from numerical stability when BL is very large [5], thus the proposed piecewise linear method will be good in analyzing the capacity of a large ad hoc network and can be used to determine the optimal stream number.

V. Conclusions

The capacity of a MIMO ad hoc system using stream number selection is investigated. We assumed that there is no CSI at the transmitters, and single-user detectors. We proposed a piecewise linear approximation to the link capacities in weak, strong, and moderate interference regimes. The capacity-transition points can be determined through our piecewise linear approximation. Our results provided rules for determining the optimal number of streams at various interference levels.

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