ADAPTIVE RESOLUTION-CONSTRAINED SCALAR MULTIPLE-DESCRIPTION CODING

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ABSTRACT

We consider adaptive two-channel multiple-description coding. We provide an analytical method for designing a resolution-constrained symmetrical multiple-description coder that uses an index assignment matrix. We use existing index assignment algorithms that are known for their good properties within our adaptive multiple-description coding architecture. These existing index assignment algorithms are parameterized and the coefficient of quantization of the side coders is described by a rational function. This leads to an analytical solution for the design problem, facilitating real-time adaptation based on information provided by feedback channels or rate conditions imposed by network management. Our experimental results show that the practical performance closely approximates theoretically obtained optimal behavior.

Index Terms— Scalar quantization, high-rate quantization, multiple description coding (MDC), feedback channels

1. INTRODUCTION

Multiple description coding (MDC) addresses the ubiquitous problem of packet loss in packet networks by exploiting diversity. It assumes that two or more independent channels are available for transmission. Such independence is commonplace, even if a single pathway is employed. MDC algorithms create two or more descriptions of a source facilitating the reconstruction of useful estimates of the source from any subset of descriptions. Receiving all descriptions leads to an accurate reconstruction and the quality progressively decreases with the number of descriptions lost.

The capacity and packet loss rate of modern communication networks is highly variable, both in time and between networks. Existing MDC methods generally do not reflect this variability: the design of an MDC for a particular rate and packet loss environment requires off-line optimization. In this paper, we address this shortcoming of MDC by introducing an MDC algorithm that facilitates redesign of the algorithm in real time, based on information provided by a feedback channel (or network management).

Existing MDC design methods are commonly iterative. This is illustrated by the classic work on MDC for the two-channel scalar MDC by Vaishampayan [1]. The method applies to the level-constrained (resolution-constrained) case and is a generalization of the iterative Lloyd algorithm. A related iterative design procedure for entropy-constrained scalar MDC was presented in [2]. An analytical lattice-based design method of entropy constrained MDC was recently posed and solved by Østergaard for a general K-channel vector case of MDC, where the probability of packet erasure is given [3]. This method includes a step of searching for the best index assignment, which is computationally expensive. A limitation of lattice based approach is that it cannot easily be generalized to the resolution constrained case.

Effective methods for real-time adaptation of MDC can be based on analytical solutions for the design problem. In this paper, we propose an analytical solution to the resolution-constrained case of two-channel scalar MDC. We selected scalar design because it facilitates low computational complexity and is the only choice when a low processing delay is required. Scalar design can approach optimality if signal modeling is used to remove sample dependencies (for example by using Gaussian mixture models [4]).

The MDC system considered follows the architecture that is most common [1][2]: two side encoders process a sequence of indices that is produced by a "central" quantizer operating on a sequence of signal samples. The output of the side encoders are two separate sequences of indices that correspond to the two descriptions. The receiver contains three decoders: a central decoder for the case that both descriptions are received and two side decoders, one for each description. The side decoders are used if only one description is used.

A key part of an MDC system with the described common architecture is an index assignment (IA) algorithm that assigns two indices to the "central" index obtained from the quantizer. This assignment process is fully specified by a so-called IA matrix. The definition of the IA matrix is an obstacle to analytic solutions for the MDC design problem. Determining the matrix may involve combinatorial problems and complex training procedures, which are not suitable for real-time implementation. Instead, we make use of a set of existing IA algorithms that are known for their superior properties. We parameterize this set of IA algorithms and use the parameterized approximation to fill in and optimize the IA matrix. The proposed parametrization can be used for any IA algorithm that generates a periodical pattern of indices within the IA matrix.

In Section 2 we discuss the approximation of the distortion associated with the side coders by a rational function. The result is then applied to solve the MDC design problem in Section 3. We evaluate the performance of our method in Section 5.

2. INDEX ASSIGNMENT ALGORITHMS

Index assignment plays an important role in the design of MDC. It maps the indices of the central quantizer to indices used by the side decoders as shown in Fig. 2. Some popular designs of IA algorithms are known to have good properties. Two such algorithms, nested IA and linear IA, were proposed by Vaishampayan [1]. A herringbone IA was presented in [5] with an application to K-channel MDC. A

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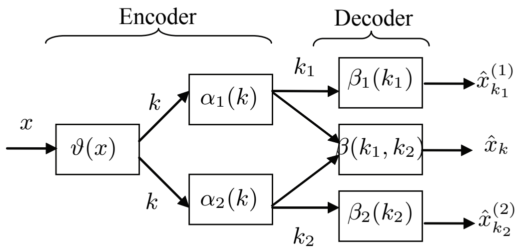

Fig. 1. Two-channel scalar MDC scheme.

relatively obvious way to fill in the IA matrix is by using staggered IA, where only two diagonals of the IA matrix are used. This algorithm has been extended to the many-diagonals case in [6].

It is convenient to consider an MDC encoder as two consecutive mappings, as is shown in Fig. 1: one from a source sample to a central qantizer index and a second from the central quantizer index to side indices. We consider the scalar resolution-constrained case and denote the number of cells by r. In the first step a source sample x is quantized using the central quantizer. Then the scalar input $x \in \mathbb{R}$ is mapped to the index k if it falls inside the interval $V_k \equiv [t_k, t_{k+1}), k \in \{1, ..., r\}$. Let us denote this mapping as ϑ: $k = \vartheta(x)$. In the second step the index k is mapped to the pair of indices k_1 and k_2 and we denote these mappings as α_1 and α_2. We consider the symmetrical case with M reconstruction points for each side quantizer. Thus, we have $k_1 \in \{1, ..., M\}$ and $k_2 \in \{1, ..., M\}$. The mapping from the central quantizer k to the side indices k_1 and k_2 is done using the IA matrix. All of the described IA algorithms result in a band IA matrix. The algorithms can each be parameterized in terms of the number of elements in each row and column of the IA matrix, v. Thus, we can generate an IA matrix for a given v.

Regularities in the IA matrix can be used to derive a rational function that corresponds to the *coefficient of quantization* for a side coder. Such a rational function of v represents the contribution of the central coder cells to the side-coder cell distortion. It depends on the geometry of the side-coder cell. The function will be derived in next section for the case of linear index assignment.

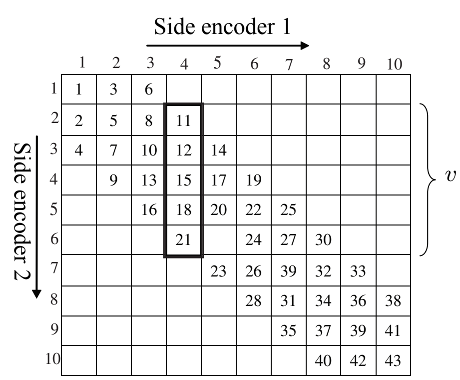

Fig. 2. An example of a linear IA with $M = 10$ and $v = 5$. Side coder cell $\alpha_1^{-1}(4)$ is marked. The central quantizer has 43 cells and each side decoder has 10 cells.

3. THE INDEX ASSIGNMENT DESIGN PROBLEM

We formulate the MDC design problem as the minimization of the mean distortion, given the probability of packet erasure for a fixed side coder rate. We consider a scalar stochastic variable X with known probability density function (pdf) $p(x)$ and make the following assumptions:

1. The source pdf $p(x)$ can be approximated as constant within the side coder cell extent;
2. The central coder cell size can be approximated as constant inside the side coder cell.

An expression for the distortion associated with side coder j, $j \in \{1, 2\}$ can be derived using the standard high-rate approach:

$$d_s^{(j)} = \int_{t_1}^{t_{r+1}} p(x) \left(x - \hat{x}^{(j)}_{\alpha_j(\alpha(x))}\right)^2 dx \tag{1}$$

$$= \sum_{i=1}^{M} p(\hat{x}_i^{(j)}) \sum_{k \in \alpha_j^{-1}(i)} \int_{V_k} \left(x - \hat{x}_i^{(j)}\right)^2 dx.$$

Let us define $m_j(i) = \min(\alpha_j^{-1}(i))$, $h_j(i) = \alpha_j^{-1}(i) - m_j(i)$ and substitute $x = y + t_{m_j(i)}$. Furthermore, let $\Delta_i = t_{i+1} - t_i$. We obtain

$$d_s^{(j)} = \sum_{i=1}^{M} p(\hat{x}_i^{(j)}) \sum_{k \in h_j(i)} \int_{k\Delta_i}^{(k+1)\Delta_i} \left(y - y_i^{(j)}\right)^2 dy, \tag{2}$$

where $y_i^{(j)}$ is the side coder reconstruction point. The distortion of a single side coder cell is then

$$\sum_{k \in h_j(i)} \int_{k\Delta_i}^{(k+1)\Delta_i} \left(y - y_i^{(j)}\right)^2 dy =$$

$$= \Delta_i^3 \sum_{k \in h_j(i)} (\frac{1}{3} + k + k^2) - \Delta_i^2 y_i^{(j)} \sum_{k \in h_j(i)} (1 + 2k) +$$

$$+ |h_j(i)| \Delta_i (y_i^{(j)})^2. \tag{3}$$

Differentiation with respect to $y_i^{(j)}$ allows us to find the side-coder reconstruction point that minimizes the contribution of one side coder cell contribution to the side distortion. Substituting this result into (3) allows us to write an expression for the side-coder distortion

$$d_s^{(j)} = \sum_{i=1}^{M} \Bigg\{ p(\hat{x}_i^{(j)}) \Bigg[\sum_{k \in h_j(i)} (\frac{1}{3} + k + k^2) +$$

$$- \frac{1}{4|h_j(i)|} \Bigg(\sum_{k \in h_j(i)} (1 + 2k) \Bigg)^2 \Bigg] \Delta_i^3 \Bigg\}. \tag{4}$$

Thus, the side-coder distortions can be approximated by

$$d_s^{(j)} = \frac{f(v)}{v} \int_{t_1}^{t_{r+1}} p(x) \Delta(x, v)^2 dx, \tag{5}$$

which has a form that is typical in high-rate theory. The function $f(v) \simeq f(v, i)$ in (5), which specifies the coefficient of quantization of the side coder, is

$$f(v, i) = \sum_{k \in h_j(i)} (\frac{1}{3} + k + k^2) - \frac{1}{4|h_j(i)|} \left(\sum_{k \in h_j(i)} (1 + 2k) \right)^2. \tag{6}$$

Table 1. Rational functions $f(v)$ and polynomials corresponding to the considered IA algorithms.

IA	$f(v)$	optimization polynomial
staggered	$\frac{2}{3}$	-
herringbone (odd v)	$\frac{1}{16}-\frac{9v}{64}+\frac{5v^2}{24}-\frac{5v^3}{96}-\frac{v^4}{48}+\frac{5v^5}{192}$	$5wv^5-2wv^4-20wv^2+(35w-8)v-18w$
herringbone (even v)	$\frac{v^2}{12}-\frac{v^3}{48}-\frac{v^4}{48}+\frac{5v^5}{192}$	$5wv^4-2wv^3-8wv+8w-8$
linear	$\frac{4v^6-3v^4+18v^2-3}{192v}$	$2wv^6-(5w+4)v^2+3w$
nested	$\frac{5v^6-8v^5+14v^4-16v^3+45v^2+24v-48}{192v}$	$5wv^6-4wv^5+8wv^3-(37w+8)v^2-36wv+96w$

We recall that, in the symmetrical case, v denotes the number of diagonals of the IA matrix. We need an expression for $f(v,i)$ to optimize the side distortion analytically. Next we derive an approximation of $f(v,i)$ for the linear index assignment case; the method also applies to other index assignment algorithms. We note that for linear index assignment $|h_j(i)| = v$ (this holds also for others index assignments considered here) for almost all i (we neglect the boundary conditions, assuming $v \ll M$). With $z = \frac{1}{2}(v-1)$ and taking a typical pattern of indices for the linear index assignment we find the elements of the set $h_j(i,z)$:

$$\begin{aligned} &h_j(i,z) \simeq h(z) = ... \\ &\{(k-1)(z+1) : k = 1, ..., z+1\} \cup \\ &\cup\{z(z-1) + z(k-z-1) : k = z+2, ..., 2z+1\}, \end{aligned} \tag{7}$$

valid for both side coders. Based on these approximations and (6), we can compute following rational function $f(v)$ (the dependency on i is neglected) for the linear index assignment:

$$f(v) = \frac{4v^6 - 3v^4 + 18v^2 - 3}{192v}. \tag{8}$$

We can make the same approximation for other index assignment algorithms. The resulting rational functions are shown in Table 1. The distortion for the central coder is expressed as

$$d_0 = \frac{1}{12}\int_{t_1}^{t_{r+1}} p(x)\Delta(x,v)^2 dx. \tag{9}$$

Because we consider the symmetric case, the side distortion for both channels d_1 and d_2 is

$$d_s = d_1 = d_2 = \frac{f(v)}{v}\int_{t_1}^{t_{r+1}} p(x)\Delta(x,v)^2 dx. \tag{10}$$

Knowing the probability of packet erasure w we can now express the total distortion as the weighted sum of the central distortion, the side distortion and the distortion for the case that no packets arrive:

$$d_t = (1-w)^2 d_0 + 2(1-w)wd_s + w^2 E\{X^2\}. \tag{11}$$

To find the best performing MDC, we must minimize the composite distortion $d(v,w) = (1-w)^2 d_0 + 2(1-w)wd_s$ over the set of mapping functions $\wp$ used for index assignment, given a rate constraint:

$$\begin{aligned} &\min_{\wp} d(v,w) \\ &\text{subject to } \int_{t_1}^{t_{r+1}} \frac{1}{\Delta(x,v)}dx = 2^R v = Mv, \end{aligned} \tag{12}$$

where R denotes the rate in bits. M is the number of the reconstruction points for each of the side coders.

Our design problem can be written in a Lagrangian formulation:

$$\begin{aligned} \eta = &\frac{(1-w)^2}{12}\int_{t_1}^{t_{r+1}} p(x)\Delta(x,v)^2 dx + \\ &+2(1-w)w\frac{f(v)}{v}\int_{t_1}^{t_{r+1}} p(x)\Delta(x,v)^2 dx + \\ &+\lambda\left(\int_{t_1}^{t_{r+1}} \frac{1}{\Delta(x,v)}dx - Mv\right). \end{aligned} \tag{13}$$

The solution for $\Delta(x,v)$ of the corresponding Euler-Lagrange equation is

$$\Delta(x,v) = \frac{1}{Mv}\frac{\int_{t_1}^{t_{r+1}}(p(x))^{\frac{1}{3}}dx}{(p(x))^{\frac{1}{3}}}. \tag{14}$$

The central coder distortion can be expressed as

$$d_0 = \frac{1}{12M^2v^2}\left(\int_{t_1}^{t_{r+1}}(p(x))^{\frac{1}{3}}dx\right)^3 \tag{15}$$

and the expression for the side coder distortion is

$$d_s = \frac{f(v)}{M^2v^3}\left(\int_{t_1}^{t_{r+1}}(p(x))^{\frac{1}{3}}dx\right)^3. \tag{16}$$

It is now possible to derive the optimal number of elements v of a column of the IA matrix for all considered index assignments. To this purpose, we differentiate the composite distortion with respect to v

$$\begin{aligned} \frac{\partial d(v,w)}{\partial v} &= \xi\big[-\frac{(1-w)^2}{6v^3} + \\ &-2(1-w)w\frac{f'(v)v^3 - 3f(v)v^2}{v^6}\big], \\ \xi &= \frac{1}{M^2}\left(\int_{t_1}^{t_{r+1}} p(x)^{1/3}dx\right)^3. \end{aligned} \tag{17}$$

After some algebra we find that the composite distortion is a univariate polynomial of v. The candidates for the optimal values of v are the real roots of the polynomials given in the Table 1. The resulting MDC design algorithm for the IA matrix is shown in Table 2.

4. RESULTS

In this section compare the proposed analytical MDC design procedure of Table 2 with known asymptotic results and with numerical results. For the numerical results we use a Gaussian source.

We first relate our results to the asymptotical results of [7]. Asymptotically with increasing rate, the design problem becomes

$$\begin{aligned} &\min_{\wp} d_0 \\ &\text{subject to } \ d_s < d_c \text{ and } \int_{t_1}^{t_{r+1}} \frac{1}{\Delta(x,v)}dx = Mv, \end{aligned}$$

Table 2. Design algorithm for a given erasure probability.

For all IA algorithms do:

1. Compute optimal v for given w using optimization polynomial from Table 1;
2. Generate matrix for the selected IA and optimal v;
3. Design central quantizer using (14);
4. Compute reconstruction points of central quantizer using

$$\hat{x}_k^{(C)} = \frac{\int_{t_k}^{t_{k+1}} xp(x)dx}{\int_{t_k}^{t_{k+1}} p(x)dx};$$

and reconstruction points for the side quantizer

$$\hat{x}_i^{(j)} = \frac{\sum_{k\in\alpha_j^{-1}(i)} \int_{t_k}^{t_{k+1}} xp(x)dx}{\sum_{k\in\alpha_j^{-1}(i)} \int_{t_k}^{t_{k+1}} p(x)dx};$$

Select the IA algorithm with lowest value for (11).

where d_c is a constraint on the side distortion. This leads to following result

$$d_c M^2 v^3 - f(v) \left(\int_{t_1}^{t_{r+1}} (p(x))^{\frac{1}{3}} dx \right)^3 \geq 0, \qquad (18)$$

which again allows for finding an optimal v. The performance of our design is compared to the asymptotical optimality results for the level-constrained two-channel scalar MDC in Fig. 3.

We evaluated the accuracy of the approximation for $f(v)$ for the case of a Gaussian source with unit variance. The total distortion obtained by using $f(v)$, based on plugging (15) and (16) into (11), was compared to the distortion computed numerically. The results are shown in Fig. 4. The approximation by $f(v)$ is seen to be accurate for rates as low as $R = 3$. As shown, the staggered index assignment always has $v = 2$, so the distortion cannot be decreased by increasing v for low probabilities of erasure, resulting in the levelling off of the corresponding curves.

5. CONCLUSION

We described an algorithm for designing resolution-constrained two-channel scalar MDC. We conclude from our results that the method is a practical and accurate method for designing two-channel MDC optimized for the probability of packet erasure. Thus, if information about network capacity and channel quality is available, then this can be used to perform real-time optimization of coders employing resolution-constrained MDC.

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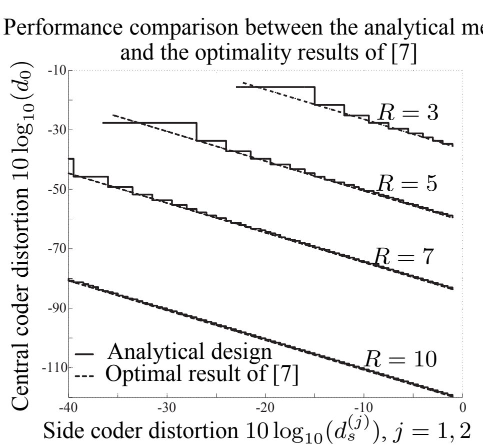

Fig. 3. Comparison to the optimality results of [7] (herringbone index assignment), $M = 2^R$.

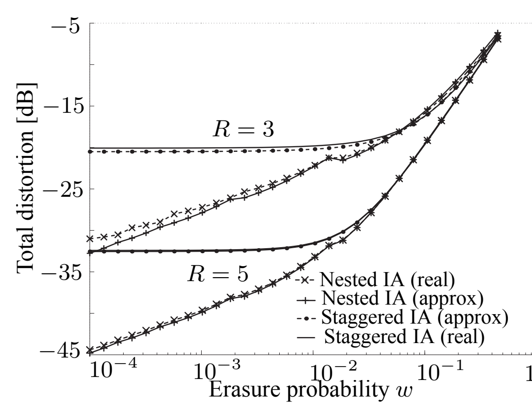

Fig. 4. Theoretically and numerically computed total distortion for a Gaussian source, $E\{X^2\} = 1$, $M = 2^R$.