FULL DIVERSITY GROUP DECODABLE ORTHOGONAL LINEAR DISPERSION CODES FOR MISO FLAT FADING CHANNELS

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ABSTRACT

We consider a flat fading wireless link having multiple M transmitter antennas and a single receiver antenna (MISO). This system is often useful in mobile downlink communications for which the mobile receiver may not be able to support multiple antennas. For such a system, we propose a novel and very simple design of full diversity two-group and four-group decodable block diagonal linear dispersion codes with rate one for any number of the transmitter antennas. For $M = 2^n$ and $M = 2^n - 1$, we also prove that for K-ary Quadrature Amplitude Modulation (QAM) transmission equipped with a maximum likelihood (ML) detector, our proposed code minimizes the worst case average pair-wise error probability, i.e., it achieve optimal coding gain.

Index Terms— Multiple-input single-output systems, space-time block codes, full diversity, group-decodable, Maximum likelihood detection.

1. INTRODUCTION

Over the past several years, various space-time block coding (STBC) schemes have been developed to take advantage of the MIMO communication channel. In this paper, we consider a coherent communication system equipped with multiple transmitter antennas and a single receiver antenna, i.e., a MISO system. These systems are often encountered in mobile downlink communications for which the mobile receiver may not be able to support multiple antennas. For such a system, orthogonal STBCs [1–4] are attractive, since they can provide the maximum diversity using a linear processing maximum likelihood detector. However, they have a limited transmission rate [5, 6] and thus, do not achieve full MIMO channel capacity [7] if the number of the transmitter antennas is greater than two. To improve the low rate, quasi-STBCs [8–10] with constellation rotation or linear transformations and the diagonal STBC [11] have been proposed. In order to simplify the complexity of maximum likelihood detection, multi-group decodable STBCs [12–14] have been developed. Recently, Toeplitz STBCs [15] for a MISO system equipped with a zero-forcing (ZF) receiver have been designed enabling the optimal tradeoff of diversity and multiplexing gains [16]. In this paper, we propose a novel and very simple design of two-group and four group decodable block diagonal linear dispersion codes by taking advantage of the Alamouti code [1] and the full diversity rotation matrices.

Notation: Throughout this paper, we use the following notation: Matrices are denoted by uppercase boldface characters (e.g., $\mathbf{A}$), and column vectors are denoted by lowercase boldface characters (e.g., $\mathbf{b}$). The i-th entry of $\mathbf{b}$ is denoted by b_i. The columns of an $M \times N$ matrix $\mathbf{A}$ are denoted by $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_N$. The transpose and Hermitian transpose of $\mathbf{A}$ are denoted by $\mathbf{A}^T$ and $\mathbf{A}^H$ respectively. The (i, j)-th entry of $\mathbf{A}$ is denoted by $[\mathbf{A}]_{i,j}$. $\mathbf{A}_{\rm re}$ and $\mathbf{A}_{\rm im}$ denote matrices consisting of the real and imaginary parts of $\mathbf{A}$, respectively. M denote the number of transmitter antennas; Notation $\mathbf{I}_K$ denotes the $K \times K$ identity matrix.

2. CHANNEL MODEL WITH LINEAR DISPERSION CODING

Consider a coherent flat fading multiple input single output (MISO) wireless communication system having M transmitter antennas and a single receiver antenna. For each time slot (usually called a "channel use"), each of the M transmitter antennas is fed a coded symbol for transmission. Each of these transmitter antennas is linked to the receiver antenna through a channel $h_m,\ m = 1, \cdots, M$. At the receiver of such a system, for time slots $n = 1, \cdots, N$, we receive an N-dimensional signal vector $\mathbf{y} = [y_1\ y_2\ \cdots\ y_N]^T$ which, according to the input-output model of the system, can then be written as $\mathbf{y} = \sqrt{\frac{\rho}{M}}\mathbf{X}(\mathbf{s})\mathbf{h} + \boldsymbol{\xi}$ where $\mathbf{X}$ is the $N \times M$ linear dispersion (LD) coding matrix each row of which consists of coded symbols fed to the M transmitter antennas during a particular time slot, $\mathbf{h}$ is an $M \times 1$ channel vector, and $\boldsymbol{\xi}$ is an $N \times 1$ complex noise vector. Throughout this paper, we make the following assumptions: 1) The channel $\mathbf{h}$ is circularly-symmetric complex Gaussian distributed, with zero-mean and covariance matrix $\mathbf{I}_M$; 2) $\boldsymbol{\xi}$ is a circularly-symmetric complex Gaussian noise vector with variance $\mathbf{I}_N$.

2.1. Code design

In the following, we suggest two constructions of codes for complex signals: the first design applies a complex rotation matrix directly to the complex signal, and the second applies real rotation matrices separately to the real and imaginary parts of the signal.

2.1.1. Block-diagonal complex orthogonal (BDCO) code design

(1) The number of the transmitter antennas is even; i.e., $M = 2L$. Let $\mathbf{C}_{\rm e}$ be an $L \times L$ full diversity rotation matrix [17–19], and

$$\begin{pmatrix} x_1 & x_{L+1} \\ x_2 & x_{L+2} \\ \vdots & \vdots \\ x_L & x_{2L} \end{pmatrix} = \mathbf{C}_{\rm e} \begin{pmatrix} s_1 & s_{L+1} \\ s_2 & s_{L+2} \\ \vdots & \vdots \\ s_L & s_{2L} \end{pmatrix}. \tag{1}$$

If we use $\mathcal{A}(x_i, x_{L+i})$ to denote the Alamouti space-time code formed by two symbols x_i and x_{L+i} for $i = 1, 2, \cdots, L$, then, our code is constructed by

$$\mathbf{X}(\mathbf{s}) = \mathrm{diag}(\mathcal{A}(x_1, x_{L+1}), \mathcal{A}(x_2, x_{L+2}), \cdots, \mathcal{A}(x_L, x_{2L})) \tag{2}$$

(2) The number of the transmitter antennas is odd; i.e., $M = 2L-1$. Let $\mathbf{C}_{\rm o}$ be an $L \times L$ full diversity rotation matrix [17–19] and

$$\begin{pmatrix} x_1 & x_{L+1} \\ x_2 & x_{L+3} \\ \vdots & \vdots \\ x_L & x_{2L} \end{pmatrix} = \mathbf{C}_{\rm o} \begin{pmatrix} s_1 & s_{L+2} \\ s_2 & s_{L+3} \\ \vdots & \vdots \\ s_{L+1} & s_{2(L+1)} \end{pmatrix}. \tag{3}$$

Then, in this case, our code is constructed as follows:

$$\mathbf{X}(\mathbf{s}) = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} \tag{4}$$

where $\mathbf{X}(\mathbf{s}) = \mathrm{diag}(\mathcal{A}(x_1, x_{L+1}), \cdots, \mathcal{A}(x_{L-1}, x_{2L-1}), \mathbf{0}_{2(L-1)\times 1})$ and $\mathbf{X}_2 = (\mathbf{0}_{2\times(2L-3)}, \mathcal{A}(x_L, x_{2L}))$. The codes in Eqs. (2) and (4) are BDCO codes respectively for even and odd numbers of antennas.

Example 1. For $M = N = 4$, the codeword matrix is

$$\mathbf{X} = \begin{pmatrix} x_1 & x_3 & 0 & 0 \\ -x_3^* & x_1^* & 0 & 0 \\ 0 & 0 & x_2 & x_4 \\ 0 & 0 & -x_4^* & x_2^* \end{pmatrix}$$

The symbol transmission rate of this code is one per channel use.

Example 2. For $M = 3, N = 4$, the codeword matrix is

$$\mathbf{X}(\mathbf{s}) = \begin{pmatrix} x_1 & x_3 & 0 \\ -x_3^* & x_1^* & 0 \\ 0 & x_2 & x_4 \\ 0 & -x_4^* & x_2^* \end{pmatrix}$$

The symbol transmission rate of this code is also one per channel use, but we have one time slot delay.

2.1.2. Block-diagonal real orthogonal (BDRO) code design

(1) The number of the transmitter antennas is even; i.e., $M = 2L$. Let $\mathbf{R}_{\rm e}$ be an $L \times L$ full diversity real rotation matrix [20] and

$$\begin{pmatrix} x_1 & x_{L+1} \\ x_2 & x_{L+2} \\ \vdots & \vdots \\ x_L & x_{2L} \end{pmatrix}_{\rm re} = \mathbf{R}_{\rm e} \begin{pmatrix} s_1 & s_{L+1} \\ s_2 & s_{L+2} \\ \vdots & \vdots \\ s_L & s_{2L} \end{pmatrix}_{\rm re} \tag{5}$$

$$\begin{pmatrix} x_1 & x_{L+1} \\ x_2 & x_{L+2} \\ \vdots & \vdots \\ x_L & x_{2L} \end{pmatrix}_{\rm im} = \mathbf{R}_{\rm e} \begin{pmatrix} s_1 & s_{L+1} \\ s_2 & s_{L+2} \\ \vdots & \vdots \\ s_L & s_{2L} \end{pmatrix}_{\rm im} \tag{6}$$

Then, our code is constructed by

$$\mathbf{X}(\mathbf{s}) = \mathrm{diag}(\mathcal{A}(x_1, x_{L+1}), \mathcal{A}(x_2, x_{L+2}), \cdots, \mathcal{A}(x_L, x_{2L})) \tag{7}$$

(2) The number of the transmitter antennas is odd; i.e., $M = 2L-1$. Let $\mathbf{R}_{\rm o}$ be an $L \times L$ full diversity real rotation matrix [20] and

$$\begin{pmatrix} x_1 & x_{L+1} \\ x_2 & x_{L+2} \\ \vdots & \vdots \\ x_L & x_{2L} \end{pmatrix}_{\rm re} = \mathbf{R}_{\rm o} \begin{pmatrix} s_1 & s_{L+1} \\ s_2 & s_{L+2} \\ \vdots & \vdots \\ s_L & s_{2L} \end{pmatrix}_{\rm re} \tag{8}$$

$$\begin{pmatrix} x_1 & x_{L+1} \\ x_2 & x_{L+2} \\ \vdots & \vdots \\ x_L & x_{2L} \end{pmatrix}_{\rm im} = \mathbf{R}_{\rm o} \begin{pmatrix} s_1 & s_{L+1} \\ s_2 & s_{L+2} \\ \vdots & \vdots \\ s_L & s_{2L} \end{pmatrix}_{\rm im} \tag{9}$$

Then, in this case, our codeword matrix is constructed as follows:

$$\mathbf{X}(\mathbf{s}) = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} \tag{10}$$

where $\mathbf{X}(\mathbf{s}) = \mathrm{diag}(\mathcal{A}(x_1, x_{L+1}), \cdots, \mathcal{A}(x_{L-1}, x_{2L-1}), \mathbf{0}_{2(L-1)\times 1})$ and $\mathbf{X}_2 = (\mathbf{0}_{2\times(2L-3)}, \mathcal{A}(x_L, x_{2L}))$. The codes in Eqs. (7) and (10) are BDRO codes respectively for even and odd numbers of antennas.

Definition 1 *A STBC is said to be g-group decodable [12–14] if the objective function for the ML receiver can be expressed as a sum of g sub-factions; i.e., $\|\mathbf{y} - \mathbf{X}\mathbf{h}\|_2^2 = \sum_{i=1}^{g} F_i$, where each F_i consists of the symbols from only one group.*

Definition 2 *Let $\mathbf{T}$ be $K \times K$ rotation matrix and $\mathbf{b} = \mathbf{T}\mathbf{x}$ for $\mathbf{x} \in \mathcal{X}$. $\mathbf{T}$ is said to be a full diversity rotation matrix if $\prod_{k=1}^{K}(b_i - b_i') \neq 0$ for any $\mathbf{x} \neq \mathbf{x}' \in \mathcal{X}$ for any $\mathbf{x} \neq \mathbf{x}' \in \mathcal{X}$.*

3. CODE PROPERTIES AND PERFORMANCE ANALYSIS

Our principal purpose in this section is to discuss some properties and analyze the error performance of the code design proposed in Subsection 2.1.

Under Assumption 1, given a channel realization $\mathbf{h}$ and a maximum likelihood detector (MLD), the probability $P(\mathbf{s} \to \mathbf{s}'|\mathbf{h})$ of transmitting $\mathbf{s}$ and deciding in favor of $\mathbf{s}' \neq \mathbf{s}$ at the decoder is given by [21]

$$P(\mathbf{s} \to \mathbf{s}'|\mathbf{h}) = Q\left(d(\mathbf{s}, \mathbf{s}')\right), \tag{11}$$

where $d^2(\mathbf{s}, \mathbf{s}') = \frac{\rho}{M}\mathbf{h}^H \mathbf{X}(\mathbf{e}) \mathbf{X}^H(\mathbf{e})\, \mathbf{h}$ with $\mathbf{e} = \mathbf{s} - \mathbf{s}'$ and $Q(t) = (1/\sqrt{\pi}) \int_t^\infty e^{-t^2/2} dt$. When the SNR is high, the union bound can be used [21] to construct the following "snug" bound on the average block error probability P_{ble}, $P_{ble} \leq \sum_{\mathbf{s}\neq\mathbf{s}'} P(\mathbf{s}) P(\mathbf{s} \to \mathbf{s}'|\mathbf{h}) = \sum_{\mathbf{s}\neq\mathbf{s}'} P(\mathbf{s}) Q(d(\mathbf{s}, \mathbf{s}'))$. We will find it convenient to use the following alternative expression for the Q function [22] $Q(t) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{t^2}{2\sin^2\theta}\right) d\theta$. By taking the average of Eq. (11) over the random vector $\mathbf{h}$, the average pair-wise error probability can be written as

$$P(\mathbf{s} \to \mathbf{s}') = \frac{1}{\pi}\int_0^{\pi/2} \frac{d\theta}{\det\left(\mathbf{I} + \frac{\rho \mathbf{X}^H(\mathbf{e})\mathbf{X}(\mathbf{e})}{2M\sin^2\theta}\right)}, \tag{12}$$

Now, we are in a position to formally state our main results.

Theorem 1 *The BDCO and the BDRO codes have the following common properties:*

1. *The symbol rate per channel use is one.*
2. *The code provides full diversity for the ML receiver.*
3. *The code with the full diversity complex rotation matrix $\mathbf{C}$ is two-group decodable, while the code with the full diversity real rotation matrix $\mathbf{R}$ is four-group decodable.*

OUTLINE OF PROOF: Statement 1 is clear.
To prove Statement 2: Consider the BDRO code for even number of transmitter antennas. In this case, we can see that $\det(\mathbf{X}^H(\mathbf{e})\mathbf{X}(\mathbf{e})) = \prod_{i=1}^{L} \det^2(\mathcal{A}(x_i, x_{L+i})) = \prod_{i=1}^{L}(|x_i|^2 + |x_{i+L}|^2)^2 = \prod_{i=1}^{L}\left((x_i)_{\rm re}^2 + (x_i)_{\rm im}^2 + (x_{i+L})_{\rm re}^2 + (x_{i+L})_{\rm im}^2\right)^2 \geq \max\{\prod_{i=1}^{L}\left((x_i)_{\rm re}^2 + (x_{i+L})_{\rm re}^2\right)^2, \prod_{i=1}^{L}\left((x_i)_{\rm im}^2 + (x_{i+L})_{\rm im}^2\right)^2\} \geq$

$2^M \max\{\prod_{i=1}^{M}(x_i)_{\rm re}^2, \prod_{i=1}^{M}(x_i)_{\rm im}^2\} > 0$ due to the full diversity real rotation matrix $\mathbf{R}_e$. Proofs for the other cases are similar.

For Statement 3: Consider the BDRO code for even number of the transmitter antennas. Here, the ML detection is equivalent to finding an optimal code matrix that minimizes the function, $F(\mathbf{s}) = \|\mathbf{y} - \mathbf{X}(\mathbf{s})\mathbf{h}\|_2^2$. Since $F(\mathbf{s})$ can be rewritten as $F(\mathbf{s}) = \|\mathbf{z} - \mathbf{H}\mathbf{u}\|_2^2$, where $\mathbf{z} = (y_1, y_2^*, y_3, y_4^*, \cdots, y_{2L})^T$, $u = (x_1, -x_{L+1}, x_2, -x_{L+2}, \cdots, -x_{2L})^T$ and $\mathbf{H} = \mathrm{diag}(\mathcal{A}(h_1,h_2), \cdots, \mathcal{A}(h_{2L-1}, h_{2L}))$, then $F(\mathbf{s}) = \mathbf{z}^H\mathbf{z} - \mathbf{z}^H\mathbf{H}\mathbf{u} - \mathbf{u}^H\mathbf{H}^H\mathbf{z} + \mathbf{u}^H\mathbf{H}^H\mathbf{H}\mathbf{u}$. Let $\mathbf{w} = \mathbf{H}z = (w_1, w_2, \cdots, w_{2L})^T$, then, $F(\mathbf{s})$ can be written as $F(\mathbf{s}) = \mathbf{z}^H\mathbf{z} + F_1 + F_2 + F_3 + F_4$. where F_k for $k = 1,2,3,4$ are defined by

$$
\begin{aligned}
F_1 &= -\sum_{i=1}^{L}(w_{2i-1})_{\rm re}(x_i)_{\rm re} - (|h_{2i-1}|^2 + |h_{2i}|^2)((x_i)_{\rm re}^2)\\
F_2 &= \sum_{i=1}^{L}(w_{2i-1})_{\rm im}(x_i)_{\rm im} + (|h_{2i-1}|^2 + |h_{2i}|^2)((x_i)_{\rm im}^2)\\
F_3 &= \sum_{i=1}^{L}(w_{2i})_{\rm re}(x_{i+L})_{\rm re} + (|h_{2i-1}|^2 + |h_{2i}|^2)((x_{i+L})_{\rm re}^2)\\
F_4 &= -\sum_{i=1}^{L}(w_{2i})_{\rm im}(x_{i+L})_{\rm im} - (|h_{2i-1}|^2 + |h_{2i}|^2)((x_{i+L})_{\rm im}^2)
\end{aligned}
$$

According to our code design Eq. (7), F_1 and F_2 only involve the real and imaginary parts of s_i for $i = 1,2,\cdots,L$, respectively, while F_3 and F_4 only involve the real and imaginary parts of s_{i+L} for $i = 1,2,\cdots,L$, respectively. In other words, the original objective function $F(\mathbf{s})$ is now separated into four subfunctions, with each involving one group of independent variables. Therefore, minimizing $F(\mathbf{s})$ is equivalent to minimizing each subfunction. Proofs for other cases are similar. □

Theorem 2 *When the number of transmitter antennas is either $M = 2^n$ or $2^n - 1$, the code design with the full diversity complex rotation matrix $\mathbf{C}$ minimizes the worst case pair-wise error probability of the maximum likelihood detector for the square QAM constellation among all the linear dispersion codes with rate one.*

OUTLINE OF PROOF: First, we establish a lower bound on the worst case pair-wise error probability for any LD code [23] with symbol rate one. Let $\mathbf{X}_{\mathcal{F}}(\mathbf{s})$ be an arbitrarily given code matrix, $\mathbf{X}_{\mathcal{F}}(\mathbf{s}) = \sum_{k=1}^{N}(\mathbf{F}_{1,k}s_k + \mathbf{F}_{2,k}s_k^*)$ with a power budget $\sum_{k=1}^{N}\mathrm{tr}(\mathbf{F}_{1,k}^H\mathbf{F}_{1,k} + \mathbf{F}_{2,k}^H\mathbf{F}_{2,k}) \le MN$. Consider the case when the error is such that $|e_m| = d_{\min}$ and $e_k = 0,\ k = 1,2,\cdots,N,\ k \neq m$. Now we select an integer such that $m = \arg\min_{1\le k\le N}\mathrm{tr}(\mathbf{F}_{1,k}^H\mathbf{F}_{1,k} + \mathbf{F}_{2,k}^H\mathbf{F}_{2,k})$. Therefore, we have $\mathrm{tr}(\mathbf{F}_{1,m}^H\mathbf{F}_{1,m} + \mathbf{F}_{2,m}^H\mathbf{F}_{2,m}) \le \frac{1}{N}\sum_{k=1}^{N}\mathrm{tr}(\mathbf{F}_{1,k}^H\mathbf{F}_{1,k} + \mathbf{F}_{2,k}^H\mathbf{F}_{2,k}) \le M$. Notice that the denominator in the integral of Eq. (12) can be written as $\det\left(\mathbf{I} + \frac{\rho\mathbf{X}_{\mathcal{F}}^H(\mathbf{e})\mathbf{X}_{\mathcal{F}}(\mathbf{e})}{2M\sin^2\theta}\right) = \det\left(\mathbf{I} + \frac{\rho\, d_{\min}^2(\mathbf{F}_{1,m}^H\mathbf{F}_{1,m} + \mathbf{F}_{2,m}^H\mathbf{F}_{2,m})}{2M\sin^2\theta}\right)$. Using Hardamard's inequality [24] and employing the relationship between the arithmetic mean and the geometrical mean, then $\det\left(\mathbf{I} + \frac{\rho\, d_{\min}^2(\mathbf{F}_{1,m}^H\mathbf{F}_{1,m} + \mathbf{F}_{2,m}^H\mathbf{F}_{2,m})}{2M\sin^2\theta}\right) \le \prod_{i=1}^{M}\left(1 + \frac{\rho\, d_{\min}^2[\mathbf{F}_{1,m}^H\mathbf{F}_{1,m} + \mathbf{F}_{2,m}^H\mathbf{F}_{2,m}]_{ii}}{2M\sin^2\theta}\right) \le \left(1 + \frac{\rho\, d_{\min}^2\mathrm{tr}(\mathbf{F}_{1,m}^H\mathbf{F}_{1,m} + \mathbf{F}_{2,m}^H\mathbf{F}_{2,m})}{2M^2\sin^2\theta}\right)^M \le \left(1 + \frac{\rho\, d_{\min}^2}{2M\sin^2\theta}\right)^M$. Therefore, from Eq. (12), the worst case pair-wise error probability is lower bounded by $\max_{\mathbf{s},\mathbf{s}'\in\mathcal{S}^Q,\,\mathbf{s}\neq\mathbf{s}'} P_{\mathcal{F}}(\mathbf{s}\to\mathbf{s}') \ge J\left(\frac{\rho\, d_{\min}^2}{2M}\right)$. where $J(a)$ is given by $J(a) = \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{a}{\sin^2\theta}\right)^{-M}d\theta$ for $a > 0$. Thus, we obtain

$$\min_{\mathcal{F}}\max_{\mathbf{s},\mathbf{s}'\in\mathcal{S}^Q,\,\mathbf{s}\neq\mathbf{s}'} P_{\mathcal{F}}(\mathbf{s}\to\mathbf{s}') \ge J\left(\frac{\rho\, d_{\min}^2}{2M}\right). \quad (13)$$

Now, we establish an upper bound. For our code design with $M = 2^n$, we have that $\det(\mathbf{X}^H(\mathbf{e})\mathbf{X}(\mathbf{e})) = \prod_{i=1}^{L}\det^2(\mathcal{A}(x_i, x_{L+i})) = \prod_{i=1}^{L}(|x_i|^2 + |x_{i+L}|^2)^2 \ge \max\{\prod_{i=1}^{L}|x_i|^4, \prod_{i=1}^{L}|x_{L+i}|^4\} \ge d_{\min}^{2M}$ due to the full diversity complex rotation matrix $\mathbf{C}_e$. This is also true for our code design with $M = 2^n - 1$. Now, using the Minkowski's inequality [24] we obtain that for any nonzero vector $\mathbf{e}$ and nonzero θ in the interval $[0,\pi/2]$, there is $\det\left(\mathbf{I}_M + \frac{\rho}{2M\sin^2\theta}\mathbf{X}^H(\mathbf{e})\mathbf{X}(\mathbf{e})\right)^{1/M} \ge 1 + \frac{\rho}{2M\sin^2\theta}\det\left(\mathbf{X}^H(\mathbf{e})\mathbf{X}(\mathbf{e})\right)^{1/M} \ge 1 + \frac{\rho\, d_{\min}^2}{2M\sin^2\theta}$. We conclude that $\det\left(\mathbf{I}_M + \frac{\rho}{2M\sin^2\theta}\mathbf{X}^H(\mathbf{e})\mathbf{X}(\mathbf{e})\right) \ge \left(1 + \frac{\rho\, d_{\min}^2}{2M\sin^2\theta}\right)^M$ This results in $\max_{\mathbf{s},\mathbf{s}'\in\mathcal{S}^Q,\,\mathbf{s}\neq\mathbf{s}'} P(\mathbf{s}\to\mathbf{s}') \le J\left(\frac{\rho\, d_{\min}^2}{2M}\right)$. Therefore, we have

$$\min_{\mathrm{tr}(\mathcal{F}^H\mathcal{F})\le Q}\ \max_{\substack{\mathbf{s},\mathbf{s}'\in\mathcal{S}^Q\\ \mathbf{s}\neq\mathbf{s}'}} P_{\mathcal{F}}(\mathbf{s}\to\mathbf{s}') \le J\left(\frac{\rho\, d_{\min}^2}{2M}\right). \quad (14)$$

Combining (13) with (14) yields $\min_{\mathrm{tr}(\mathcal{F}^H\mathcal{F})\le Q}\max_{\substack{\mathbf{s},\mathbf{s}'\in\mathcal{S}^Q\\ \mathbf{s}\neq\mathbf{s}'}} P_{\mathcal{F}}(\mathbf{s}\to\mathbf{s}') = J\left(\frac{\rho\, d_{\min}^2}{2M}\right)$. This completes the proof of theorem 2. □

We make the following comments on Theorems 1 and 2.

1. Our code guarantees that the worst case pair-wise error probability is minimized for K-ary QAM when $M = 2^n$. As a result, optimal coding gain is achieved. Although the diagonal space-time (DAST) block code [11] also provides the optimal coding gain in this case, the number of the code matrices in the DAST code achieving the optimal coding is much larger than that of our code matrices. As a result, its performance is much worse than ours, which can be seen in the performance comparison by simulations shown in Fig. 1. Also, our code is two-group decodable, but the DAST code must be decoded jointly.
2. When the number of the transmitter antennas is either $M = 2^n$ or $M = 2^n - 1$, even though our BDRO code is four-group decodable achieving full diversity, the coding gain is less than that of our BDCO two-group decodable code. Since BDRO codes are simpler to detect due to the reduced number of symbols which are real, this results in a trade off between detection simplicity and performance.

4. CONCLUSION

A novel and very simple design of two-group and four-group decodable block diagonal linear dispersion codes with rate one for any number of the transmitter antennas was proposed in this paper. it was shown that our proposed code has the following properties: (1) It achieves full diversity for the ML receiver. (2) When the number of the transmitter antennas is equal to 2^n or $2^n - 1$, our proposed code minimizes the worst case average pair-wise error probability of the

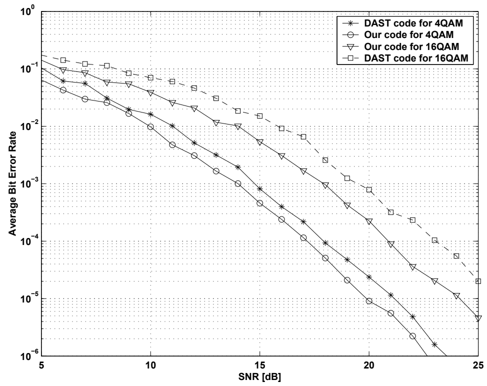

Fig. 1. Average BER comparison of our code with the DAST code for the MISO system with 4 transmitter antennas

ML detector for K-ary QAM. Therefore, in this case, it achieves optimal coding gain. (3) ML detection can be efficiently implemented by separating the original symbol group into two-subgroup or four-group. (4) When the number of the transmitter antennas is even, our code is delay-optimal. When the number of the transmitter antennas is odd, our code is one time slot delay.

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