Iterative Equalization with Hard and Soft Decisions for ISI-Constrained Channels

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***Abstract*— Although iterative equalizers are well-known for mitigating inter-symbol interference (ISI), recent results have shown that the schemes perform poorly in the presence of severe ISI, particularly when coding rates are high. Other equalization schemes, such as the non-iterative *canonical* decision-feedback equalizer (CDFE), have been shown to be asymptotically optimal on any channel. However, the scheme exhibits very high latency. To trade off between performance and latency, we propose an equalizer structure that iterates with both hard- and soft-decisions. In prior works, hard-decision iterative equalizers were seen to perform poorly, chiefly due to error-propagation in the feedback loop. However, the scheme proposed in this paper outperforms the linear turbo equalizer in *both* strong and weak ISI. On channels with weak ISI, the equalizer outperforms both the CDFE as well as the ideal DFE with perfect feedback. Simulation results on well-known ISI channels support the findings.**

Keywords: Intersymbol interference, decision feedback equalizers, magnetic recording, channel coding, iterative methods.

I. Introduction

Linear minimum mean square error (MMSE) turbo equalizers are well-known for mitigating inter-symbol or adjacent-channel interference with low complexity (cf. [7], [8]). Intuitively, the idea is to to exploit extrinsic information from a soft (probabilistic) channel-decoder to improve symbol estimates at the equalizer output. The equalizer, in turn, supplies soft information to the channel decoder, resulting in an iterative loop where the extrinsic information is "amplified" every iteration. However, it is known that iterative linear equalizers perform poorly on channels with severe ISI, such as magnetic recording channels (cf. [5]). On such channels – where multi-level modulation is infeasible – the reduced performance of the equalizer necessitates a substantial increase in transmit power or a commensurate decrease in data-rate. In [4], the authors showed that the performance of iterative equalizers can be analyzed via extrinsic-information transfer (EXIT) and bit-error rate (BER) charts. On channels with severe ISI, the authors used these tools to show that convergence cannot always be reached within a practical number of iterations, depending on the channel code used.

On the other hand, a well-known method to tackle severe ISI is the classical decision-feedback equalizer (DFE), which consists of a feed-forward noise-whitening matched-filter, working in conjunction with a feedback filter operating on hard-decisions from the slicer. The feed-forward filter mitigates pre-cursor ISI to create an almost minimum-phase channel-response, while the feedback filter cancels the post-cursor ISI. If perfect decisions can be provided, the DFE has been shown to be asymptotically optimal at high SNR's [2]. It is worth recalling that, at low SNR's, the DFE is *not* optimal due to residual pre-cursor ISI. Hence, an intuitively appealing idea is to combine linear turbo equalization with the classical DFE, an idea which was first investigated in [7]. However, the authors observed that the error-prone hard-decisions from the slicer reduced performance significantly. In particular, the linear turbo equalizer was observed to outperform the turbo DFE. To minimize error-propagation in conventional DFE's, a recently-proposed technique is the *canonical* DFE of [3], so entitled because the scheme approaches the performance of an ideal DFE at high SNR's. The equalizer minimizes error propagation by performing pre-cursor and post-cursor ISI cancellation in separate stages. The pre-cursor ISI cancellation is performed without delay, while the feedback equalization is performed *after* channel-decoding and de-interleaving, whence decoded codewords can be made arbitrarily reliable. However, an entire set of interleaved codewords must be decoded and de-interleaved before feedback-equalization can begin; hence, the equalizer exhibits very high latency.

Motivated by ideas from both iterative and decision-feedback equalization, we propose here an equalizer that attempts to achieve the advantages of both techniques, while minimizing the drawbacks to some extent. The scheme exhibits significantly lower latency than the canonical DFE, while it outperforms the iterative linear equalizer in both weak and strong ISI. In summary, the scheme employs zero-latency hard-decision symbols in the feedback loop, while the pre-cursor and post-cursor filters are updated with extrinsic information from the channel decoder. To mitigate error-propagation, interleaving is employed over a small set of code-words. The idea is to treat feedback error-propagation as a form of burst-noise or "fading"; hence, the interleaver here attempts to "whiten" the burst-noise while also decorrelating extrinsic information. To prevent error-propagation into adjacent sets of interleaved codewords, a small amount of excess bandwidth is required in the form of zero-padding (as in [3]). We find that, on channels with weak ISI, the proposed equalizer out-performs the canonical-DFE of [3]. In fact, it also out-performs the classical DFE with ideal decision-feedback. On channels with severe ISI, the proposed equalizer does not perform as well as either the ideal DFE or the canonical DFE. However, it out-performs the linear turbo-equalizer while maintaining moderate latency.

The remainder of the paper is organized as follows. The transmission model is outlined in Section II. Section III briefly summarizes the ideas behind turbo and decision-feedback equalization. Section IV details the proposed equalizer structure. Numerical results on well-known channels are presented in Section V. Section VI concludes the paper.

II. TRANSMITTER AND CHANNEL MODEL

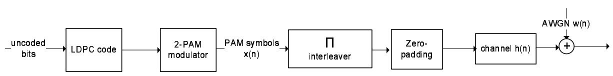


Fig. 1. Transmitter and channel.

The general transmitter model is depicted in Fig. 1. We assume bi-level pulse-amplitude modulation (2-PAM), as in magnetic re-coding channels. As a channel-code, simple regular LDPC codes are employed. As will be explained in Section V, we investigate well-known ISI channels in this paper. Let l_{ch} denote the channel length, in terms of number of symbols, and let N similarly denote the codeword length.

The interleaver, depicted in Fig. 1, is a key element that varies widely depending on the equalization scheme. In linear turbo equalizers, a random interleaver is necessary to decorrelate extrinsic information *within* a single N-element vector. For other schemes, a block interleaver spanning a number of codewords, say M codewords, is required. Depending on the equalizer, the transmitter also pads a small number of zero-amplitude symbols between codewords, which can be viewed as an excess-bandwidth factor. After interleaving, zero-padding, and parallel-to-serial conversion, let $\mathbf{x}(n)$ denote a vector of contiguous 2-PAM symbols (not necessarily N symbols long) at symbol interval n. Then, the system model can be expressed as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{w}(n) \ , \tag{1}$$

where $\mathbf{H}$ denotes the convolutional channel matrix, and $\mathbf{w}(n)$ represents a vector of i.i.d samples from a white Gaussian process with zero-mean and variance σ_w^2.

At the receiver, the equalizer estimates each symbol $x(n)$ with $\hat{x}(n)$. The soft-output equalizer of [7], [8] expresses the estimate as an *a posteriori* probability metric, taking the form

$$L_E(x(n)) = \frac{p(x(n) = +1 \mid \hat{x}(n))}{p(x(n) = -1 \mid \hat{x}(n))} \ . \tag{2}$$

Similarly, the soft channel-decoder computes a soft output for each symbol, say $L_D(x(n))$, which can then be used to estimate the *a priori* probabilities $p(x_n = +1)$ and $p(x_n = -1)$ for the equalizer.

III. PREVIOUS EQUALIZATION SCHEMES

In this section, we briefly summarize the well-known linear turbo equalizer and the DFE structure as they motivate the design of the equalizer proposed in this paper.

A. Linear MMSE Turbo Equalization

As depicted in Fig. 2, the linear MMSE turbo equalizer is a time-varying transversal filter $\mathbf{c}(n)$, updated with extrinsic information from the channel decoder (cf. [7], [8]). In particular, the equalizer employs *a priori* probabilities to compute $\bar{x}(n)$ and $v(n)$, which represent the mean and variance of each symbol. Suppose the equalizer operates on the set of contiguous received symbols $\{y(n - l_1) \dots y(n + l_2)\}$. Let us denote the span of the filter as $l \triangleq l_1 + l_2 + 1$. As shown in [7], the equalizer can be derived to be

$$\mathbf{c}(n) = \left[\sigma_w^2 \mathbf{I}_{l\times l} + \mathbf{H}\mathbf{V}(n)\mathbf{H}^T + (1 - v(n))\mathbf{s}\mathbf{s}^T\right]^{-1} \mathbf{s} \ , \tag{3}$$

where $\mathbf{s} = \mathbf{H}\left[\mathbf{0}_{1\times(l_2+l_{ch}+1)} \ \ 1 \ \ \mathbf{0}_{1\times l_1}\right]$. The matrix $\mathbf{V}(n)$ is diagonal, containing the terms $v(n - l_{ch} - l_2 + 1) \dots v(n + l_1)$. The equalizer output can be written as

$$\hat{x}(n) = \mathbf{c}^T(n)\left[\mathbf{y}(n) - \mathbf{H}\bar{\mathbf{x}}(n) + \bar{x}(n)\mathbf{s}\right] \ . \tag{4}$$

As its name implies, a salient feature of the equalizer is that pre-cursor and post-cursor ISI are minimized *jointly* via an MMSE criterion. As depicted in Fig. 2, the equalizer proceeds by decoding a codeword at a time, iteratively cancelling ISI by exchanging soft information with the channel decoder via an interleaver and deinterleaver. The mean and variance estimated are used to update $\mathbf{c}(n)$. A variation of this idea is to minimize only the pre-cursor ISI, aided with soft information, while the post-cursor ISI is cancelled via a conventional feedback filter [7]. This notion is discussed in more detail in Section IV.

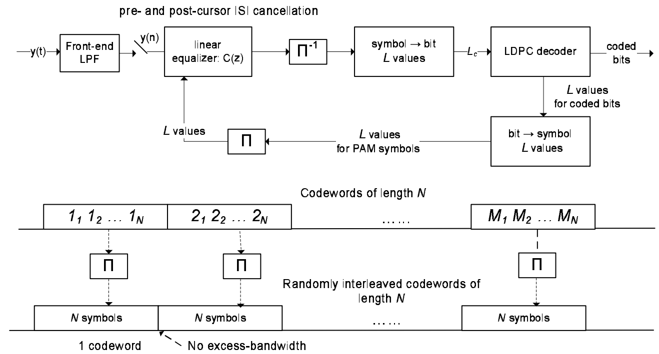


Fig. 2. Iterative linear equalizer from [7].

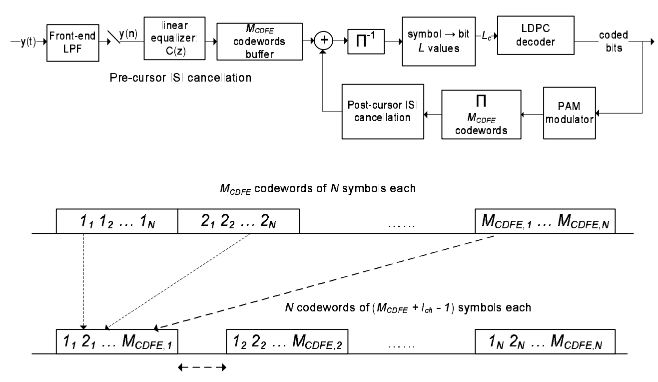


Fig. 3. Canonical DFE from [3].

B. *Classical and Canonical Decision Feedback Equalization*

Rather than cancelling pre-cursor and post-cursor ISI jointly, the classical DFE first attempts to convert the channel into a minimum-phase response (i.e., a causal and monic channel). The ISI resulting from the effective channel is cancelled using the feedback filter. But for the fact that the tentative decisions from the slicer are error-prone, the DFE would be asymptotically capacity-achieving at high SNR's (cf. [2]). However, it is worth recalling that the DFE is not optimal at low SNR's – even with ideal feedback decisions – due to residual pre-cursor ISI at the slicer input.

The canonical DFE of [3] overcomes the problem of error-propagation in classical DFE's by performing feedforward and feedback equalization *separately*, as depicted in Fig. 3. At the transmitter, M_{CDFE} codewords are block-interleaved and zero-padded, employing the format shown in the lower half of the figure. At the receiver, feedforward equalization is performed as in a classical DFE. However, the outputs of the feedforward equalizer are stored in a buffer until M_{CDFE} codewords (in this case, LDPC codewords) are received. Feedback equalization can begin only *after* the first codeword has been block-deinterleaved and decoded. The format ensures that, as each codeword is decoded, the only impairments at the input to the channel decoder are white Gaussian channel noise and residual pre-cursor ISI (cf. [3]). In effect, the channel-decoder assumes the role of the slicer in the classical DFE, albeit with vastly increased accuracy since "slicing" is performed on N-symbol codewords. However, since each interleaved set of codewords is $N \times (M_{\text{CDFE}} + l_{ch} - 1)$ symbols long, the latency of the scheme becomes high.

The zero-padding ensures that each interleaved codeword experiences no post-cursor ISI. However, the zero-padding must be viewed as an excess-bandwidth factor, viz.,

$$\eta_{\text{CDFE}} = \frac{l_{ch} - 1}{M_{\text{CDFE}}} , \tag{5}$$

where l_{ch} is the channel length. If we desire $\eta_{\text{CDFE}} \to 0$, it is clear that $M_{\text{CDFE}} \to \infty$, which implies that bandwidth efficiency comes at the expense of increased latency.

IV. Iterative DFE with Hard and Soft Decisions

The ideas behind the linear MMSE iterative equalizer were extended to the classical DFE in [7], resulting in an iterative DFE. However, the equalizer was observed to perform poorly in relation to the linear turbo equalizer. In this section, we extend the scheme to show that good results can be obtained. We first summarize the main idea of an iterative DFE based on the development in Section III-A. As with the linear equalizer, the feedforward equalizer is updated with extrinsic information from the channel decoder at each symbol interval. Naturally, the feedback filter must also be updated as the post-cursor portion of $h(n)*c(n)$. Here, "$*$" denotes time-domain convolution. With $l_2 = 0$, the feedback vector can be expressed as a combination of soft and hard decisions, viz.,

$$\bar{\mathbf{x}}(n) = \left[x'(n - l_{ch} + 1) \dots x'(n-1) \; \bar{x}(n) \dots \bar{x}(n + N - 1)\right] ,$$

where $x'(n - l_{ch} + 1) \dots x'(n-1)$ denote the hard-decisions from the slicer. If the hard-decisions can be assumed to be correct, then the matrix $\mathbf{V}(n)$ described in Section III-A is a diagonal matrix with entries $\left[\mathbf{0}_{1\times(M-1)} \; v(n) \; v(n+1) \; \dots \; v(n+N-1)\right]$ on the diagonal. As with the linear equalizer, we set $\bar{x}(n) = 0$ and $v(n) = 1$ to ensure that the $\hat{x}(n)$ and $L_D(x(n))$ are independent. The key idea of the iterative DFE is to cancel *pre-cursor* ISI by taking advantage of extrinsic information, and then to cancel the remaining *post-cursor* ISI using hard-decisions. Despite its intuitive appeal, the iterative DFE of [7] was observed to perform poorly due to error-prone decisions from the slicer.

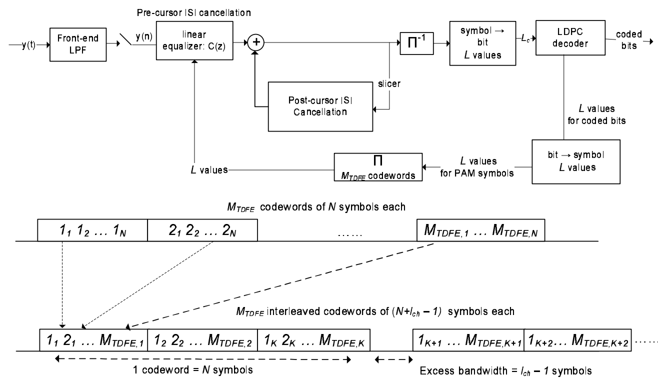


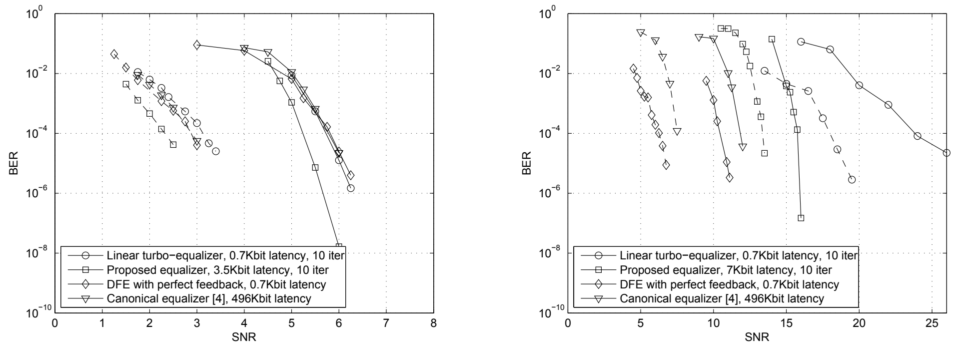
Fig. 4. Proposed hard- and soft-decision equalizer.

Consider now the equalizer shown in Fig. 4. The pre-cursor and post-cursor filters still operate as described previously; however, the extrinsic information is exchanged with the channel-decoder in a novel format. Although the feedback loop introduces error-propogation due to hard-decision errors, the interleaver treats the errors as a form of burst-noise, and "whitens" the noise into the channel decoder. Since multiple codewords are interleaved, the interleaver also assumes the more conventional role of decorrelating the extrinsic information. As in the canonical DFE, a small amount of zero-padding between sets of interleaved symbols prevents errors from propagating infinitely. Unlike the canonical DFE, the equalizer does not depend on the channel decoder for hard decisions. The latency, therefore, is much lower than that of the canonical DFE.

The excess bandwidth required to support the equalizer can be expressed as

$$\eta_{\text{turbo-DFE}} = \frac{l_{ch} - 1}{N} , \tag{6}$$

where N is the codeword length as defined previously. Notice that, to obtain $\eta_{\text{turbo-DFE}} \to 0$, we must increase codeword length $N \to \infty$, rather than increase interleaver depth as with the canonical DFE. Although increasing N also increases latency, it also permits more powerful channel-codes to be employed. Moreover, suppose we compare the proposed equalizer and the CDFE by operating at exactly the same rate, i.e., $\eta_{\text{turbo-DFE}} = \eta_{\text{CDFE}}$, while employing the same channel-code. Then, we must have $M_{\text{CDFE}} := N$ to maintain the same data-rate. Since N is usually on the order of hundreds or thousands of bits, it is evident that the CDFE incurs very high latency compared to the proposed scheme.



(a) Performance on mild ISI channel A of [6].

(b) Performance on severe ISI channel C of [6].

Fig. 5. Continuous lines show performance with a rate-3/4 LDPC code. Dotted lines show performance with a rate-1/3 LDPC code of same length.

Compared to the classical DFE with perfect feedback, the proposed equalizer has the advantage of superior pre-cursor ISI cancellation due to the extrinsic information from the channel-decoder. The drawback of the proposed scheme is, naturally, the hard-decision errors in the feedback loop. Thus, the role of the interleaver is crucial to the operation of the entire receiver.

V. Simulation Results

To evaluate the performance of the equalizers, we investigate the channels A and C, first defined in [6] and also employed in [4], [7]. Channel A exhibits the least amount of ISI, while C exhibits the strongest as it contains a spectral-null in the mid-band region. For the channel-code, we consider two types of codes: a relatively high-rate (rate-3/4) LDPC code and a low-rate (rate-1/3) LDPC code from [1]. Both codes have same length of 704 bits. To our knowledge, this family of codes exhibits best performance among regular LDPC codes. The canonical DFE and the proposed scheme are assumed to operate at the same data-rate with $\eta_{\text{turbo-DFE}} = \eta_{\text{CDFE}}$. For channels A and C, this corresponds to excess bandwidths of 1.2% and 0.57% respectively. Naturally, the linear turbo equalizer and the classical DFE do not require excess-bandwidth for ISI cancellation. In practice, some excess-bandwidth is desirable for timing and acquisition. Fig. 5(a) depicts the performance of all the equalization schemes on channel A. The linear turbo equalizer performs well after 10 iterations, and the BER plot is almost identical to that of the idealized DFE and the canonical DFE. However, the proposed equalizer performs about 1 dB better. Although its interleaver depth of 5 results in a latency of 3.5 Kbit, this latency is far below that of the canonical DFE.

Fig. 5(b) depicts the performance of the schemes on channel C. The linear turbo equalizer exhibits relatively poor performance on this channel. Similar results have also been observed in [5] for magnetic channels. The proposed equalizer, with $M_{\text{turbo-DFE}} = 10$ (7 Kbit latency), outperforms the linear turbo equalizer by a significant margin for both low-rate and high-rate codes. However, both the ideal DFE and canonical DFE significantly outperform the proposed equalizer by several dB. On the other hand, notice that the CDFE exhibits a latency approximately 2 orders of magnitude higher than the proposed scheme. The ideal DFE cannot, of course, be realized unless precoding schemes are employed to cancel post-cursor ISI *a priori*. However, this would raise further complications, such as the need for channel knowledge at the transmitter. Moreover, schemes such as Tomlinson precoding (cf. [6]) require systems with multi-level modulation.

VI. Conclusion

We have proposed an iterative equalization scheme that employs extrinsic information to cancel pre-cursor ISI, and imperfect decision feedback to cancel post-cursor ISI. The performance on channels with weak ISI is promising, as it outperforms other well-known schemes. In severe ISI, the scheme outperforms linear turbo equalizers, although it does not perform as well as the canonical DFE or the ideal DFE. In all cases, the latency of the proposed scheme is much lower than that of the canonical DFE.

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