APPLYING FREQUENCY DOMAIN EQUALIZATION TO PRECODED CPM

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ABSTRACT

We show how to apply Frequency Domain Equalization (FDE) to precoded Continuous Phase Modulation (CPM) systems. It is well known that differential precoding can be applied to the specific, popular class of CPM schemes with modulation index $h = 1/2Q$, where Q is any integer. This precoding halves the bit error rate (BER) compared to nonprecoded CPM without any overhead or complexity increase. We apply FDE to a block-based precoded CPM system. Therefore, we show that in addition to a cyclic prefix, two subblocks of data-dependent symbols have to be inserted in each block to cope with the memory in the CPM signal and to enable correct decoding by the receiver. We explain how to calculate these subblocks. Simulation results in a 60 GHz environment confirm that the BER is halved by precoding, and that this precoding is compatible with FDE using our new technique.

***Index Terms*—** Continuous Phase Modulation (CPM), Frequency Domain Equalization (FDE), precoding, Laurent decomposition, 60 GHz

1. INTRODUCTION

We are witnessing an explosive growth in the demand for wireless connectivity. Short range wireless links will soon be expected to deliver bit rates of over 2 Gbits/s. Worldwide, recent regulation assigned a 3 GHz or wider frequency band at 60 GHz to this kind of applications [1].

Chips for mobile devices need to be power efficient. Therefore, a suitable modulation technique for 60 GHz transceivers should allow an efficient operation of the power amplifier (PA) to deal with the high path loss. Moreover, these chips need to be cheap so the modulation technique should have a high level of immunity to analog front-end nonidealities. Continuous Phase Modulated (CPM) signals possess these properties. They have a perfectly constant envelope which makes them much more favorable than Orthogonal Frequency Division Multiplexing (OFDM) as cheap, power efficient nonlinear PA's can be used instead of expensive, power inefficient linear ones [2]. They are also more robust against other front-end imperfections such as phase noise and analog-to-digital converter clipping and quantization [3]. Moreover, they combine attractive spectral properties with excellent Bit Error Rate (BER) performance [4].

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The typical 60 GHz channel is severely frequency-selective for the targeted signal bandwidth. Equalizing such channels in the frequency domain (FD) rather than in the time domain (TD) can significantly lower the computational complexity [2]. Therefore, we perform frequency domain equalization (FDE) of CPM signals as described in [5]. It is known that just the insertion of a cyclic prefix (CP) is not sufficient to make the convolution of a CPM signal with a linear channel appear to be a cyclic convolution, enabling FDE with one complex multiplication per sample [6]. An extra subblock, below called *intrafix*, has to be inserted in each block to deal with the memory in the CPM signal.

In this paper, we apply FDE to differentially precoded CPM signals. This precoding can be used with the specific, popular class of CPM schemes with modulation index $h = 1/2Q$, where Q is any integer [7]. It approximately halves the bit error rate compared to nonprecoded CPM without any extra overhead or complexity increase. We show that two intrafixes are needed to enable the correct decoding of precoded blocks of CPM symbols. One intrafix has to be inserted in the CP, and another one in the remainder of the block, similar to what is done in [8] for nonprecoded CPM. It is shown how these intrafixes have to be calculated.

The paper is organised as follows. In Section 2, CPM is briefly introduced. Precoding of CPM signals is explained in Section 3. Our approach for combining precoded CPM with FDE is presented in Section 4. Simulation results are discussed in Section 5 and conclusions are drawn in Section 6.

Below, vectors are represented by boldface letters $\boldsymbol{x}$. The n^{th} element of a vector $\boldsymbol{x}$ is x_n.

2. CONTINUOUS PHASE MODULATION

A transmitted CPM signal has the form:

$$s(t, \boldsymbol{a}) = \sqrt{\frac{2E_S}{T}} e^{j\,\phi(t, \boldsymbol{a})} \quad (1)$$

where $\boldsymbol{a}$ contains the sequence of M-ary data symbols $a_n \in \{\pm 1, \pm 3, ..., \pm(M-1)\}$ [4]. The symbol duration is T and E_S is the energy per symbol, here normalized to $E_S = 1$. The transmitted information is contained in the phase:

$$\phi(t, \boldsymbol{a}) = 2\pi h \sum_n a_n \cdot q(t - nT) \tag{2}$$

where h is the *modulation index* and $q(t)$ is the *phase response*, related to the *frequency response* $f(t)$ by the relationship $q(t) = \int_{-\infty}^{t} f(\tau)\,\mathrm{d}\tau$. The pulse $f(t)$ is a smooth pulse shape over a finite time interval $0 \le t \le LT$ and zero outside, where L is an integer. The function $f(t)$ is normalized such that $\int_{-\infty}^{\infty} f(t)\,\mathrm{d}t = \frac{1}{2}$. The phase $\phi(t, \boldsymbol{a})$ during interval $nT \le t \le (n+1)T$ can then also be written as:

$$\phi(t, \boldsymbol{a}) = h\pi \sum_{i=0}^{n-L} a_i + 2\pi h \sum_{i=n-L+1}^{n} a_i \cdot q(t - iT). \tag{3}$$

In this expression, we distinguish two types of memory in the CPM signal: the *phase state* $\theta_n = h\pi \sum_{i=0}^{n-L} a_i \mod 2\pi$, and the *correlative state* $\sigma_n = (a_{n-1}, a_{n-2}, \ldots, a_{n-L+1})$. Together, they form the *state* of the CPM signal $\chi_n = (\theta_n, \sigma_n)$ which captures all the memory. This memory has to be taken into account to enable FDE, as we will see below.

Exploiting the Laurent decomposition [9], we can write (1) as a sum of $P = 2^{L-1}$ linearly modulated signals:

$$s(t) = \sum_{p=0}^{P-1} \sum_n b_{p,n}\, l_p(t - nT) \tag{4}$$

where the *pseudocoefficients* $b_{p,n}$ are given by

$$b_{p,n} = \exp\left[j\pi h \left(\sum_{i=0}^{n} a_i - \sum_{i=1}^{L-1} a_{n-i}\, \beta_{p,i} \right) \right] \tag{5}$$

with $\beta_{p,i}$ the i^{th} bit in the binary representation of p ($p = \sum_{i=1}^{L-1} 2^{i-1} \beta_{p,i}$). The Laurent pulses $l_p(t), p = 0, ..., P-1$ are real, with $l_p(t) = 0$ for $t < 0$ and $t > (L+1)T$. We will use the linear representation of CPM (4) to construct the receiver.

3. RECEIVER FOR PRECODED CPM

In this section, we explain how the sent symbols $\boldsymbol{a}$ can be extracted from the received CPM waveform. Using (4), a Viterbi processor can generate an estimate of $\boldsymbol{b}_0$ [10]. We then show how an estimate of $\boldsymbol{a}$ can be extracted from this estimate $\hat{\boldsymbol{b}}_0$, both in the nonprecoded and in the precoded case [7].

3.1. Demodulator

It is well known [10] that a Viterbi receiver can be used to generate a maximum likelihood estimate of the first pseudo-coefficient

$$\hat{b}_{0,n} = \exp\left(j\pi h \sum_{i=0}^{n} \hat{a}_i \right). \tag{6}$$

As stated in [7], $\boldsymbol{a}$ can be uniquely determined by $\boldsymbol{b}_0$. We can therefore extract the estimate of the input symbols $\hat{\boldsymbol{a}}$ from the estimate $\hat{\boldsymbol{b}}_0$ without losing any information.

3.2. Detector without Precoding

If we do not apply any precoding, the detection goes as follows. Equation (6) can be written as:

$$\begin{aligned} \hat{b}_{0,n} &= \exp\left(j\pi h \sum_{i=0}^{n-1} \hat{a}_i \right) \cdot \exp\left(j\pi h \hat{a}_n\right) \\ &= \hat{b}_{0,n-1} \cdot \exp\left(j\pi h \hat{a}_n\right) \end{aligned} \tag{7}$$

and as $\left|\hat{b}_{0,n-1}\right| = 1$, $\hat{b}_{0,n-1}^{-1} = \hat{b}_{0,n-1}^{*}$ so

$$\exp\left(j\pi h \hat{a}_n\right) = \hat{b}_{0,n} \cdot \hat{b}_{0,n-1}^{*}. \tag{8}$$

This equation allows us to extract $\hat{\boldsymbol{a}}$ from $\hat{\boldsymbol{b}}_0$. In the very popular case of $h = 1/2$ for instance, the input symbols can be estimated as

$$\hat{a}_n = -j \cdot \hat{b}_{0,n} \cdot \hat{b}_{0,n-1}^{*}. \tag{9}$$

One error in $\hat{\boldsymbol{b}}_0$ will on average cause two errors in $\hat{\boldsymbol{a}}$. We want to avoid this by applying a precoding.

3.3. Detector with Precoding

In [7] it is shown that differential encoding can be applied to the class of binary CPM signals with

$$h = 1/(2Q) \tag{10}$$

where Q is any integer, to annihilate the inherent differential decoding of CPM signals and thus improve error performance for coherent detection. In the precoder, the information bits $\boldsymbol{i}$, $i_n \in \{0, 1\}$ are first differentially encoded to obtain the sequence $\boldsymbol{p}$, $p_n \in \{0, 1\}$:

$$p_n = i_n \oplus i_{n-1} \tag{11}$$

where $\oplus$ represents the modulo 2 addition. This sequence is then mapped on the CPM symbols $\boldsymbol{a}$, $a_n \in \{-1, 1\}$:

$$a_n = 2p_n - 1 = 2(i_n \oplus i_{n-1}) - 1. \tag{12}$$

Substituting this in (6) yields

$$\begin{aligned} \hat{b}_{0,n} &= \exp\left(j\frac{\pi}{2Q} \sum_{i=0}^{n} 2(\hat{i}_i \oplus \hat{i}_{i-1}) - 1 \right) \\ &= \exp\left(-j\frac{\pi}{2Q} n \right) \cdot \exp\left(j\frac{\pi}{Q} \sum_{i=0}^{n} (\hat{i}_i \oplus \hat{i}_{i-1}) \right). \end{aligned} \tag{13}$$

If we assume (explained in the next section)

$$\hat{i}_{-1} = 0 \tag{14}$$

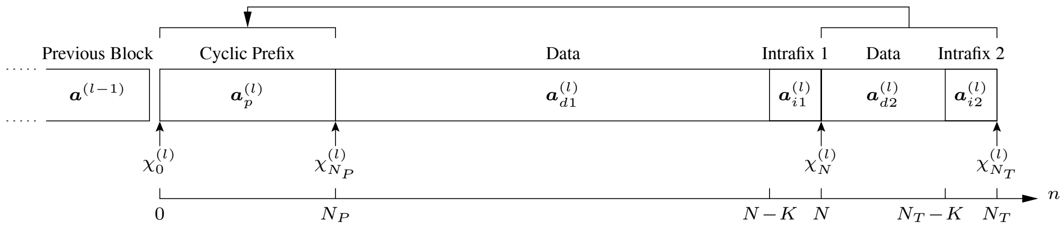

Fig. 1. Structure of an overall data block $\boldsymbol{a}^{(l)}$.

and as $\hat{i}_i \oplus \hat{i}_i = 0$, (13) becomes:

$$\hat{b}_{0,n} = \exp\left(-j\frac{\pi}{2Q}n\right) \cdot \exp\left(j\frac{\pi}{Q}\hat{i}_n\right). \tag{15}$$

This equation allows us to extract the estimate of the information bits $\hat{\boldsymbol{i}}$ from $\hat{\boldsymbol{b}}_0$. Contrary to (9), one error in $\hat{\boldsymbol{b}}_0$ will now on average cause only one error in $\hat{\boldsymbol{i}}$. This is what we wanted to obtain by precoding the information bits. For $h = 1/2$ for instance, it can be seen that

$$\hat{i}_n = \frac{\hat{b}_{0,n} \cdot e^{j\frac{\pi}{2}n} - 1}{2}. \tag{16}$$

In the next section, we investigate the compatibility of this precoding with FDE. The FDE is done as described in [5].

4. FDE FOR PRECODED CPM

We now consider a block-based CPM system. Parameters with respect to the l^{th} block are denoted with a superscript (l). The input data stream is cut in blocks $\left[\boldsymbol{a}_{d1}^{(l)}; \boldsymbol{a}_{d2}^{(l)}\right]$ where $\boldsymbol{a}_{d1}^{(l)}$ and $\boldsymbol{a}_{d2}^{(l)}$ have length $N - K - N_P$ and $N_P - K$ respectively. Here, N is the size of the blocks to be equalized (chosen as a power of 2 for efficient transformation into the frequency domain), N_P is the length of the CP and K is the intrafix length (further explained below). To avoid inter-block interference, N_P is chosen such that $N_P > L_C$ where $L_C T$ is the length of the channel delay spread.

For CPM, attaching a CP is not sufficient [8]. To construct a data block which yields a cyclic CPM signal we also have to take the memory introduced by the CPM modulation into account. This memory is reflected by the state $\chi_n^{(l)}$ of the modulator at symbol interval n in block l [6]. From Fig. 1 it can be seen that the condition to get a cyclic CPM signal with period NT after discarding the CP is

$$\chi_{N_P}^{(l)} = \chi_{N_T}^{(l)}. \tag{17}$$

An extra constraint is imposed by the precoding. At the beginning of each block (i.e. at $n = 0$), we initialize the encoder (11) by setting $i_{-1} = 0$, such that assumption (14) is valid. To be able to start decoding correctly at the receiver after the deletion of the CP (i.e. at $n = N_P$), we have to bring the transmitter in this same state at $n = N_P$. We therefore also have to satisfy

$$\chi_0^{(l)} = \chi_{N_P}^{(l)}. \tag{18}$$

As stated in [8], the memory of a CPM modulator can be flushed by inserting an intrafix of K data-dependent symbols, where $K \geq \max\left\{L, \left\lceil \frac{p-1}{M-1} \right\rceil\right\}$, with $h = m/p$ and m and p are relatively prime integers. In other words, we can force the modulator into a certain state at a certain point by inserting a correctly calculated intrafix. To satisfy (18), the intrafix $\boldsymbol{a}_{i2}^{(l)}$ has to be calculated such that

$$h\pi\left(\sum_{n=0}^{N_P-K-1} a_{d2,n}^{(l)} + \sum_{n=0}^{K-1} a_{i2,n}^{(l)}\right) = 0 \mod 2\pi. \tag{19}$$

The intrafix (second term in (19)) is thus calculated such that it undoes the phase rotation caused by $\boldsymbol{a}_{d2}^{(l)}$ (first term in (19)). Therefore, $\chi_0^{(l)} = \chi_{N_P}^{(l)}$ and also $\chi_N^{(l)} = \chi_{N_T}^{(l)}$. Inserting the latter equation in (17) yields the condition $\chi_{N_P}^{(l)} = \chi_N^{(l)}$, which can be satisfied by choosing the first intrafix $\boldsymbol{a}_{i1}^{(l)}$ to satisfy

$$h\pi\left(\sum_{n=0}^{N-N_P-2K-1} a_{d1,n}^{(l)} + \sum_{n=0}^{K-1} a_{i1,n}^{(l)}\right) = 0 \mod 2\pi. \tag{20}$$

Finally, the CP is inserted. The last N_P symbols $\left[\boldsymbol{a}_{d2}^{(l)}; \boldsymbol{a}_{i2}^{(l)}\right]$ of each block are copied in front of the block, creating blocks of length $N_T = N + N_P$ at the transmitter. The complete system model is shown in Fig. 2.

5. SIMULATION RESULTS

For simulations, the binary 3-RC CPM scheme ($f(t) = (1 - \cos\frac{2\pi t}{LT})/2LT$ with $L = 3$) was chosen. Results for modulation index $h = 0.25$ and $h = 0.5$ are presented. A huge bandwidth is available at 60 GHz, so the bit rate is $R_b = 1$ Gbits/s. For this system, the channel is severely frequency-selective so a blocksize $N = 256$ and CP length $N_P = 64$ are chosen. The receiver lowpass (LP) filter is modeled as a raised cosine filter with roll-off factor $R = 0.5$. Details about the simulated 60 GHz multipath environment can be found in [5].

Fig. 3 shows the BER of the nonprecoded (solid lines) and the precoded (dashed lines) $h = 0.25$ and $h = 0.5$ CPM

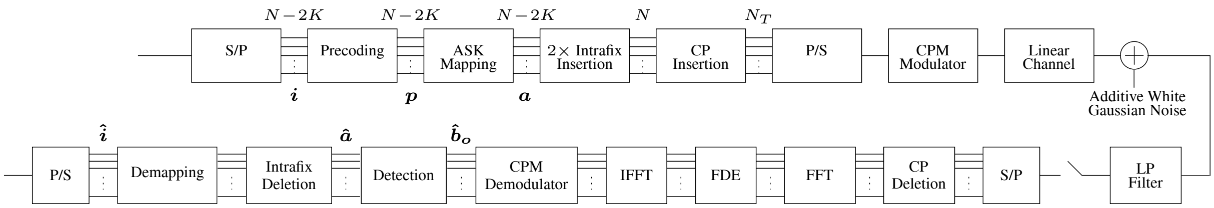

Fig. 2. System model with block sizes at different points of the transmitter mentioned. The upper part represents the transmitter and the wireless channel, the lower part is the receiver.

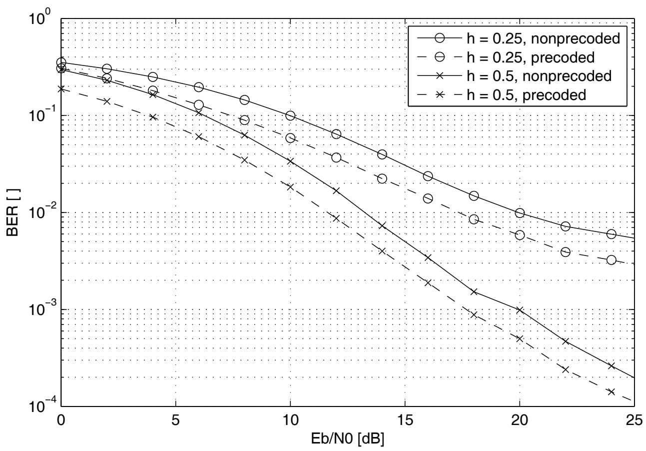

Fig. 3. BER performance of the $3RC$, $h = 0.5$ and $h = 0.25$ schemes in a 60 GHz environment, both nonprecoded (solid lines) and precoded (dashed lines).

schemes in a 60 GHz environment. The Minimum Mean Square Error (MMSE) frequency domain channel equalizer described in [5] is used. As can be seen, precoding almost exactly reduces the BER by half.

6. CONCLUSIONS

We have shown how to enable FDE for differentially precoded CPM systems. More precisely, we have developed a block construction that satisfies two requirements. First, the CPM blocks should be cyclic to enable FDE. This can be achieved by inserting, in addition to a cyclic prefix, a subblock of data-dependent symbols in each block. This subblock, called *intrafix*, copes with the memory in the CPM signal. Second, the transmitter should be forced into the same state at the beginning of the block and right after the cyclic prefix to enable correct decoding by the receiver. We have shown that this can be done by inserting a second intrafix in each block. It was also explained how these intrafixes should be calculated.

To validate our new algorithm, we have presented simulation results in a 60 GHz enviroment. These results confirm that the BER is halved by precoding, and that the precoding is compatible with FDE thanks to our new block construction.

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