LONG-TERM ADAPTIVE PRECODING FOR DECISION FEEDBACK EQUALIZATION

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ABSTRACT

We consider the communication of digital signals over a multiple-input multiple-output wireless channel, using a linear precoder at the transmitter and a non-linear decision feedback equalizer at the receiver. This receiver structure can exploit the signal constellation properties by using successive quantization and interference cancellation. Recently, optimal precoder designs have been found for a wide range of performance measures assuming that perfect channel-state information (CSI) is available. Herein, we propose a design taking CSI uncertainty into account by utilizing the first and second order statistics of the channel. The resulting precoder exhibits improved performance compared to similar methods based on long-term statistics.

Index Terms— MIMO systems, Communication systems, Decision feedback equalizers, Fading channels, Channel coding

1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems arise in digital communications when modeling a link with multiple receive and transmit antennas, or systems with dispersive channels. With the appropriate signal processing, MIMO systems allow us to operate at very high data rates by multiplexing data over parallel sub-channels. An optimal design of codebooks for MIMO communications was presented in [1]. These codewords are infinitely long and the design requires perfect channel-state information (CSI) at the receiver as well as the transmitter. The design of practical and efficient codebooks and transmission schemes for MIMO systems has received much interest in recent years [2, 3, 4].

Decision feedback equalization (DFE) in conjunction with linear precoding is a promising technology that combines high performance with low complexity. Recently, a DFE design was presented [5, 6, 7] – the uniform channel decomposition (UCD) – that is optimal for many well-known performance measures simultaneously, including the average mean squared error (MSE) and bit error rate (BER). The scheme relies on perfect channel-state information at both the transmitter and receiver.

Channel-state information (CSI) can be estimated at the receiver using training sequences, pilot symbols, or decision feedback estimation. For slow-fading channels the receiver-side CSI (RX-CSI) is commonly assumed perfect. Transmitter-side CSI (TX-CSI), on the other hand, is often difficult to acquire with high accuracy. TX-CSI may be obtained by means of feedback from the receiver – for which errors due to delay and quantization are introduced – or, in a frequency/time division-duplex system, by local estimation using the reciprocity principle. The latter is problematic since uplink and downlink cannot occupy the same frequency and time slots. Consequently, it is reasonable to assume that the TX-CSI is of lower accuracy than the RX-CSI. A practical transmission scenario of interest assumes long-term channel statistics as TX-CSI, with a non-zero mean if there is a dominant path – e.g. line-of-sight – between the transmitter and receiver. This particular scenario has been studied previously in [8].

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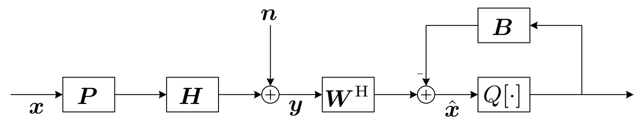

Fig. 1. Decision Feedback System.

In this paper, we extend the previous results on the minimum MSE DFE [5, 6, 7] to account for long-term TX-CSI. This calls for a random modeling of the channel at the transmitter. By shifting focus to the expected performance with respect to the random channel, the uncertainty of the channel is brought into the joint transmitter-receiver design. We derive an adaptive design that applies to the wide range of performance measures previously considered for the perfect-CSI UCD. The performance of this design is examined by means of numerical simulations and compared against similar, long-term CSI designs [8, 9].

The following notation is used throughout the paper. The complex field is denoted by $\mathbb{C}$. Matrices and vectors are typeset with upper-case and lower-case boldface letters, respectively. The ith column of a matrix $\boldsymbol{A}$ is denoted $\boldsymbol{a}_i$, and $\boldsymbol{A}_i$ is the matrix $[\boldsymbol{a}_1 \ \dots \ \boldsymbol{a}_i]$. $\boldsymbol{A}_{ij}$ and $[\boldsymbol{A}]_{ij}$ refer to the (i,j) element $\boldsymbol{A}$. If $\boldsymbol{A}_{ij} = f(i,j)$, then we write $\boldsymbol{A} = (f(i,j))_{ij}$. The identity matrix is $\boldsymbol{I}$, and $\boldsymbol{0}$ is a matrix of zeros. $\mathrm{vec}(\cdot)$ is the vector formed by column-stacking vectorization of a matrix, and the trace is $\mathrm{tr}(\cdot)$. Hermitian transpose, complex conjugate, and transpose are denoted $[\cdot]^{\mathrm{H}}$, $[\cdot]^*$, and $[\cdot]^{\mathrm{T}}$, respectively. The expectation of a random entity is $\mathbb{E}(\cdot)$, and the covariance matrix of a random vector $\boldsymbol{a}$ is defined as $\mathbb{E}\left((\boldsymbol{a} - \mathbb{E}(\boldsymbol{a})) \cdot (\boldsymbol{a} - \mathbb{E}(\boldsymbol{a}))^{\mathrm{H}}\right)$. $\mathcal{CN}(\boldsymbol{b}, \boldsymbol{A})$ is the multivariate complex Gaussian circularly symmetric distribution with mean $\boldsymbol{b}$ and covariance $\boldsymbol{A}$. Also, we write $(\cdot)^+$ for $\max(0, \cdot)$, and the Kronecker product of matrices is denoted $\otimes$. The matrix $\boldsymbol{A}^{1/2}$ is any matrix satisfying $(\boldsymbol{A}^{1/2})^{\mathrm{H}} \boldsymbol{A}^{1/2} = \boldsymbol{A}$. Finally, the function $[\mathcal{F} \circ \exp](x)$ is defined as $\mathcal{F}(e^x)$.

2. SYSTEM AND CHANNEL MODEL

Consider a MIMO communication system with n_T transmitting antennas and n_R receiving antennas over a flat-fading channel. The system is modeled using the linear-regression model

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{P}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{x} \in \mathbb{C}^{L}$ is a vector of data symbols, $\boldsymbol{P} \in \mathbb{C}^{n_T \times L}$ is a linear precoding matrix, $\boldsymbol{H} \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, $\boldsymbol{n} \in \mathbb{C}^{n_R}$ is a vector with additive noise, and $\boldsymbol{y} \in \mathbb{C}^{n_R}$ is the vector of received signals. Without loss of generality, we normalize the input signal as $\mathbb{E}\left(\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}\right) = \boldsymbol{I}$. The additive noise is zero mean and has covariance matrix $\boldsymbol{R}_{\mathrm{n}}$ according to $\boldsymbol{n} \sim \mathcal{CN}\left(\boldsymbol{0}, \boldsymbol{R}_{\mathrm{n}}\right)$, and we further assume that $\boldsymbol{x}$ and $\boldsymbol{n}$ are uncorrelated, i.e. $\mathbb{E}\left(\boldsymbol{x}\boldsymbol{n}^{\mathrm{H}}\right) = \boldsymbol{0}$.

Using a DFE receiver as shown in Fig. 1, and assuming no error propagation [5, 6, 7], the signal estimate at the receiver, $\hat{\boldsymbol{x}}$, can be written as

$$\begin{aligned} \hat{\boldsymbol{x}} &= \boldsymbol{W}^{\mathrm{H}}\boldsymbol{y} - \boldsymbol{B}\boldsymbol{x} \\ &= \left[\boldsymbol{W}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{P} - \boldsymbol{B}\right]\boldsymbol{x} + \boldsymbol{W}^{\mathrm{H}}\boldsymbol{n}, \end{aligned} \tag{2}$$

where $\boldsymbol{W}^{\mathrm{H}} \in \mathbb{C}^{L \times n_R}$ is the equalizing filter, and $\boldsymbol{B} \in \mathbb{C}^{L \times L}$ is an upper triangular feedback matrix with zero diagonal.

The objective is to jointly design the precoder $\boldsymbol{P}$, equalizer matrix $\boldsymbol{W}^{\mathrm{H}}$, and feedback matrix $\boldsymbol{B}$, given long-term TX-CSI and perfect RX-CSI. Irrespective of the transmission scenario, we assume that the first and second-order statistics of the channel comprise the CSI available at the transmitter. Assuming further that the distribution of the channel, $\boldsymbol{H}$, is a complex Gaussian results in the following channel model,

$$\operatorname{vec}\left(\boldsymbol{H}\right) \sim \mathcal{CN}\left(\operatorname{vec}\left(\hat{\boldsymbol{H}}\right), \boldsymbol{R}_{\mathrm{H}}\right). \tag{3}$$

This is the well-known MIMO Ricean-fading channel model. The first-order statistic is often interpreted as a low-rank line-of-sight component. In the special case when $\hat{\boldsymbol{H}} = \boldsymbol{0}$, the model reduces to a Rayleigh-fading channel.

Various measurement campaigns, e.g. [10, 11], have revealed that in some cases, irrespective of the indices, $\mathbb{E}\left(\boldsymbol{h}_i\boldsymbol{h}_j^{\mathrm{H}}\right) \propto \mathbb{E}\left(\boldsymbol{h}_k\boldsymbol{h}_l^{\mathrm{H}}\right)$, approximately, and likewise for the rows of $\boldsymbol{H}$. In matrix notation, this implies that the channel covariance matrix has a Kronecker structure according to

$$\boldsymbol{R}_{\mathrm{H}} = \boldsymbol{R}_{\mathrm{Tx}} \otimes \boldsymbol{R}_{\mathrm{Rx}}. \tag{4}$$

The matrices $\boldsymbol{R}_{\mathrm{Rx}} \in \mathbb{C}^{n_R \times n_R}$ and $\boldsymbol{R}_{\mathrm{Tx}} \in \mathbb{C}^{n_T \times n_T}$ represent the channel covariance seen from the receiver side and the transmitter side, respectively.

3. PROBLEM FORMULATION

Designing a DFE system amounts to selecting a precoder $\boldsymbol{P}$, an equalizer $\boldsymbol{W}^{\mathrm{H}}$, and a feedback matrix $\boldsymbol{B}$. The choice of these matrices should – in some sense – be optimal in terms of the transmit power used and the quality of the received signal $\hat{\boldsymbol{x}}$.

One common approach to jointly design the transmitter and receiver is to consider the constrained optimization problem

$$\min_{\boldsymbol{P},\boldsymbol{W},\boldsymbol{B}} \mathcal{F}(\boldsymbol{P}, \boldsymbol{W}, \boldsymbol{B}), \quad \operatorname{tr}\left(\boldsymbol{P}\boldsymbol{P}^{\mathrm{H}}\right) \leq P_{\max}, \tag{5}$$

where $\mathcal{F}$ is a cost function measuring the degradation of the signal as it is sent through the channel. The constraint is a bound on the transmitted power $P = \mathbb{E}\left(\|\boldsymbol{P}\boldsymbol{x}\|^2\right) = \operatorname{tr}\left(\boldsymbol{P}\boldsymbol{P}^{\mathrm{H}}\right)$.

When perfect TX-CSI is available, $\mathcal{F}$ is commonly set to be a function of the MSEs of the individual data streams [5, 7], i.e. $\mathcal{F} = \mathcal{F}(\boldsymbol{E}_{11}, \ldots, \boldsymbol{E}_{LL})$. The MSEs are identified as the diagonal elements of the MSE matrix

$$\begin{aligned} \boldsymbol{E} &= \mathbb{E}\left(\left(\hat{\boldsymbol{x}} - \boldsymbol{x}\right)\left(\hat{\boldsymbol{x}} - \boldsymbol{x}\right)^{\mathrm{H}}\right) \\ &= \left[\boldsymbol{W}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{P} - (\boldsymbol{B} + \boldsymbol{I})\right]\left[\boldsymbol{W}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{P} - (\boldsymbol{B} + \boldsymbol{I})\right]^{\mathrm{H}} \\ &\quad + \boldsymbol{W}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}\boldsymbol{W} \end{aligned} \tag{6}$$

with the expectation taken over the signal $\boldsymbol{x}$ and the noise $\boldsymbol{n}$.

When channel uncertainty enters the picture, it would be desirable to consider the channel expecation $\mathbb{E}_{\mathrm{H}}\left(\mathcal{F}\right)$ of the cost function. Striving to keep $\mathcal{F}$ unspecified – in order to render the resulting design applicable to a large class of cost functions – we shall assume that the function is linear in the region of interest to achieve tractability. Thus, the problem formulation that we shall adopt is

$$\min_{\boldsymbol{P},\boldsymbol{W},\boldsymbol{B}} \mathcal{F}(\mathbb{E}_{\mathrm{H}}\left(\boldsymbol{E}_{11}\right), \ldots, \mathbb{E}_{\mathrm{H}}\left(\boldsymbol{E}_{LL}\right)), \quad \operatorname{tr}\left(\boldsymbol{P}\boldsymbol{P}^{\mathrm{H}}\right) \leq P_{\max}. \tag{7}$$

Note that the choice of an MSE-based cost function $\mathcal{F}$ is not to be regarded as a restriction. In many cases, both the bit error rate (BER) and the signal to noise ratio (SNR) can be expressed as functions of the MSEs, see e.g. [5]. Since the cost function $\mathcal{F}$ is arbitrary, functions based on BER and SNR are already incorporated into the framework.

3.1. DFE with Perfect TX-CSI

We shall briefly review some recently published results on how the optimal transmitter and receiver matrices can be determined when perfect TX-CSI is available. In this setting, the optimization problem (7) considered is reduced to

$$\min_{\boldsymbol{P},\boldsymbol{W},\boldsymbol{B}} \mathcal{F}(\boldsymbol{E}_{11}, \ldots, \boldsymbol{E}_{LL}), \quad \operatorname{tr}\left(\boldsymbol{P}\boldsymbol{P}^{\mathrm{H}}\right) \leq P_{\max}. \tag{8}$$

These results will be used for solving the corresponding problem with long-term TX-CSI. We refer to [5] for a more complete presentation.

3.1.1. DFE Receiver

Since $\mathcal{F}$ is a function of the MSEs, we shall naturally assume that it is increasing in each argument. It turns out that this condition provides sufficient information to express the minimum MSE-DFE receiver in terms of the precoding matrix $\boldsymbol{P}$ and the fixed channel $\boldsymbol{H}$. By inspection of (6), it is clear that the decision feedback matrix $\boldsymbol{B}$ can be chosen to simultaneously minimize each $\boldsymbol{E}_{ii}$. Explicitly, the non-trivial elements of the strictly upper triangular matrix $\boldsymbol{B}$ coincide with those of the matrix $\boldsymbol{W}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{P}$, i.e.,

$$\boldsymbol{B} = \mathcal{U}\left(\boldsymbol{W}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{P}\right), \tag{9}$$

rendering the matrix $\boldsymbol{W}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{P} - (\boldsymbol{B} + \boldsymbol{I})$ in (6) lower triangular. Updating the MSEs by substituting accordingly yields

$$\begin{aligned} \boldsymbol{E}_{ii} &= {\boldsymbol{w}_i}^{\mathrm{H}}\left\{\boldsymbol{H}\boldsymbol{P}_i\boldsymbol{P}_i^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}} + \boldsymbol{R}_{\mathrm{n}}\right\}\boldsymbol{w}_i \\ &\quad - {\boldsymbol{w}_i}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{p}_i - \boldsymbol{p}_i^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{w}_i + 1. \end{aligned} \tag{10}$$

Note that the dependence of $\boldsymbol{E}_{ii}$ on $\boldsymbol{W}$ is constrained to its ith column $\boldsymbol{w}_i$. Setting the gradient of $\boldsymbol{E}_{ii}$ with respect to ${\boldsymbol{w}_i}^*$ to zero then provides a closed-form expression for the equalizer

$$\boldsymbol{w}_i = \left\{\boldsymbol{H}\boldsymbol{P}_i\boldsymbol{P}_i^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}} + \boldsymbol{R}_{\mathrm{n}}\right\}^{-1}\boldsymbol{H}\boldsymbol{p}_i. \tag{11}$$

Thus, the minimum MSE receiver depends on the specific choice of cost function $\mathcal{F}$ only via the linear precoder $\boldsymbol{P}$. Assuming that this matrix is known at the receiver, either by direct computation or by transmitting the information over the channel, the ith column of $\boldsymbol{W}$ and ith row of $\boldsymbol{B}$ are completely determined by minimizing the MSE of the corresponding data stream.

3.1.2. Linear Precoder

The problem of determining $\boldsymbol{P}$ for an arbitrary cost function $\mathcal{F}$ has not been solved. However, the application of majorization theory has resulted in solutions for two important classes of functions, incorporating many of the most common performance measures. When $\mathcal{F} \circ \exp$ is Schur concave – e.g. product of MSEs, negative sum of SNRs – then $\boldsymbol{B}$ becomes zero and the optimal solution coincides with the corresponding linear design. If $\mathcal{F} \circ \exp$ is Schur convex – e.g. average BER, sum of MSEs – then there is a precoder $\boldsymbol{P}$ that simultaneously minimizes every function in this class. It is obtained by solving

$$\boldsymbol{P} = \boldsymbol{V} \operatorname{diag}\left(\sqrt{\boldsymbol{p}}\right) \boldsymbol{V}_P{}^{\mathrm{H}}, \tag{12}$$

which uses the singular value decomposition $\tilde{\boldsymbol{H}} = \boldsymbol{U\Sigma V}^{\mathrm{H}}$, where $\tilde{\boldsymbol{H}} = \boldsymbol{R}_{\mathrm{n}}{}^{-1/2}\boldsymbol{H}$. The power allocation vector $\boldsymbol{p}$ is given by the standard waterfilling solution

$$p_i = \left(\mu - \frac{1}{\boldsymbol{\Sigma}_{ii}^2}\right)^+, \quad P_{\max} = \sum_{i=1}^{\operatorname{rank}(\tilde{\boldsymbol{H}})} p_i, \tag{13}$$

and the right singular matrix $\boldsymbol{V}_P$ is obtained via the equal-diagonal generalized triangular decomposition [12]

$$\begin{bmatrix} \boldsymbol{U\Sigma} \operatorname{diag}\left(\sqrt{\boldsymbol{p}}\right) \\ \boldsymbol{I} \end{bmatrix} = \boldsymbol{QRV}_P. \tag{14}$$

3.2. DFE with Long-term TX-CSI

We shall return to the random-channel problem formulation in (7). However, since we assume that perfect CSI is available on the receiver side, the equalizer $\boldsymbol{W}$ and feedback matrix $\boldsymbol{B}$ are given by (11) and (9), respectively, for each channel realization $\boldsymbol{H}$ and precoder $\boldsymbol{P}$. The transmitter on the other hand, considers the expectation $\mathbb{E}_{\mathrm{H}}\left(\boldsymbol{E}_{ii}\right)$ of the MSEs, which at this point are given by

$$\boldsymbol{E}_{ii} = 1 - \boldsymbol{p}_i{}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\left(\boldsymbol{H}\boldsymbol{P}_i\boldsymbol{P}_i{}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}} + \boldsymbol{R}_{\mathrm{n}}\right)^{-1}\boldsymbol{H}\boldsymbol{p}_i. \tag{15}$$

A slight rearrangement based on the matrix-inversion lemma gives the following relation

$$\boldsymbol{E}_{ii} = \left[\left(\boldsymbol{I} + \boldsymbol{P}_i{}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}{}^{-1}\boldsymbol{H}\boldsymbol{P}_i\right)^{-1}\right]_{ii}. \tag{16}$$

Computing the expectation of the MSEs above, with respect to the channel, is non-trivial for the general MIMO Ricean channel that is considered. Therefore, we shall continue the optimization by considering, instead, a lower bound on the expectation. Matrix-convexity of the matrix inverse can be utilized to find such a bound, as was done in the related linear design problem [9]. Explicitly, for any complex vector $\boldsymbol{z}$ and random matrix $\boldsymbol{A}$ with finite expectation,

$$\boldsymbol{z}^{\mathrm{H}}\left(\mathbb{E}\left((\boldsymbol{A})^{-1}\right) - \left(\mathbb{E}\left(\boldsymbol{A}\right)\right)^{-1}\right)\boldsymbol{z} \geq 0. \tag{17}$$

Letting $\boldsymbol{z}$ be the columns of the identity matrix $\boldsymbol{I}$ gives rise to the inequality

$$\mathbb{E}\left(\left[\boldsymbol{A}^{-1}\right]_{ii}\right) \geq \left[\left(\mathbb{E}\left(\boldsymbol{A}\right)\right)^{-1}\right]_{ii}. \tag{18}$$

Applying this relation on the MSEs in (16) results in

$$\begin{aligned} \mathbb{E}\left(\boldsymbol{E}_{ii}\right) &= \mathbb{E}\left(\left[\left(\boldsymbol{I} + \boldsymbol{P}_i{}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}{}^{-1}\boldsymbol{H}\boldsymbol{P}_i\right)^{-1}\right]_{ii}\right) \\ &\geq \left[\left(\boldsymbol{I} + \boldsymbol{P}_i{}^{\mathrm{H}}\tilde{\boldsymbol{H}}^{\mathrm{H}}\tilde{\boldsymbol{H}}\boldsymbol{P}_i\right)^{-1}\right]_{ii}, \end{aligned} \tag{19}$$

where $\tilde{\boldsymbol{H}}$ is implicitly given by

$$\begin{aligned} \tilde{\boldsymbol{H}}^{\mathrm{H}}\tilde{\boldsymbol{H}} &= \mathbb{E}\left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}{}^{-1}\boldsymbol{H}\right) \\ &= \hat{\boldsymbol{H}}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}{}^{-1}\hat{\boldsymbol{H}} \\ &+ \left(\sum_{k,l=1}^{n_R}[\boldsymbol{R}_{\mathrm{n}}{}^{-1}]_{kl}[\boldsymbol{R}_{\mathrm{H}}]_{(l+n_R[j-1]),(k+n_R[i-1])}\right)_{ij}. \end{aligned} \tag{20}$$

In the special case with a channel covariance matrix having Kronecker structure, $\boldsymbol{R}_{\mathrm{H}} = \boldsymbol{R}_{\mathrm{Tx}} \otimes \boldsymbol{R}_{\mathrm{Rx}}$, this reduces to

$$\tilde{\boldsymbol{H}}^{\mathrm{H}}\tilde{\boldsymbol{H}} = \hat{\boldsymbol{H}}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}{}^{-1}\hat{\boldsymbol{H}} + \operatorname{tr}\left(\boldsymbol{R}_{\mathrm{Rx}}\boldsymbol{R}_{\mathrm{n}}{}^{-1}\right)\boldsymbol{R}_{\mathrm{Tx}}{}^{\mathrm{T}}. \tag{21}$$

Note that the lower bounds on the expected MSEs (19) have the same form as the MSEs with perfect TX-CSI (16). From this fact we conclude that the proposed long-term precoder given $\hat{\boldsymbol{H}}$, $\boldsymbol{R}_{\mathrm{H}}$, and $\boldsymbol{R}_{\mathrm{n}}$ coincides with the precoder with perfect TX-CSI (12) for the modified channel $\tilde{\boldsymbol{H}}$ in (20).

4. NUMERICAL RESULTS

This section presents simulation results that demonstrate the gains attained by using long-term adaptive precoding designed for DFE. Our precoder is compared with a previously proposed precoder [8], designed with minimum MSE as objective. For comparison we also include the well known V-BLAST scheme [2] (MMSE, without optimal detection ordering) that does not utilize TX-CSI, the hypothetical case when the TX-CSI is perfect [5, 6, 7], and a precoder designed for a linear MMSE receiver that (in some aspects) is similar to ours [9]. Two different TX-CSI scenarios are considered. In the first, Ricean statistics – comprised of a long term line-of-sight component together with the correlation properties of the multi-path components – are known at the transmitter. The second scenario demonstrates a highly correlated Rayleigh fading channel where the correlation matrix is known at the transmitter.

4.1. Ricean-fading channel with correlation

We consider a MIMO system of dimensionality $n_T = n_R = L = 4$. Given the TX-CSI, ξ, the channel matrix is distributed according to

$$\operatorname{vec}\left(\boldsymbol{H}|\xi\right) = \mathcal{CN}\left(\boldsymbol{h}_{\mathrm{Rx}}\boldsymbol{h}_{\mathrm{Tx}}^{\mathrm{H}}, \boldsymbol{R}_{\mathrm{Tx}} \otimes \boldsymbol{R}_{\mathrm{Rx}}\right), \tag{22}$$

where the line-of-sight component, $\boldsymbol{h}_{\mathrm{Rx}}\boldsymbol{h}_{\mathrm{Tx}}^{\mathrm{H}}$, is rank one. The steering vectors, $\boldsymbol{h}_{\mathrm{Rx}}$ and $\boldsymbol{h}_{\mathrm{Tx}}$, are assumed to have a Vandermonde structure as

$$\begin{aligned} [\boldsymbol{h}_{\mathrm{Rx}}]_n &= \sigma \exp(i\alpha_{\mathrm{Rx}}n), \\ [\boldsymbol{h}_{\mathrm{Tx}}]_n &= \exp(i\alpha_{\mathrm{Tx}}n), \end{aligned}$$

where $\sigma^2 \in [0, 1]$ defines the power in the line-of-sight component and $\alpha_{\mathrm{Rx}}, \alpha_{\mathrm{Tx}}$ define the direction-of-arrival at the receiver and transmitter respectively. In the following example we use parameters $\sigma^2 = 0.7$, $\alpha_{\mathrm{Rx}} = 0.2$, and $\alpha_{\mathrm{Tx}} = -0.5$. The receiver-side correlation matrix is defined as

$$\boldsymbol{R}_{\mathrm{Rx}} = \boldsymbol{R}_{\mathrm{Rx}}{}^{\mathrm{H}} \;\text{ and }\; [\boldsymbol{R}_{\mathrm{Rx}}]_{nm} = \sqrt{1-\sigma^2}\rho_{\mathrm{Rx}}^{m-n} \;\text{ for }\; m \geq n.$$

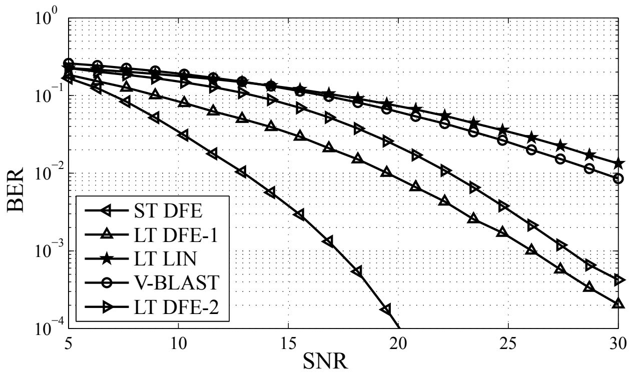

Fig. 2. Ricean-fading channel with correlation.

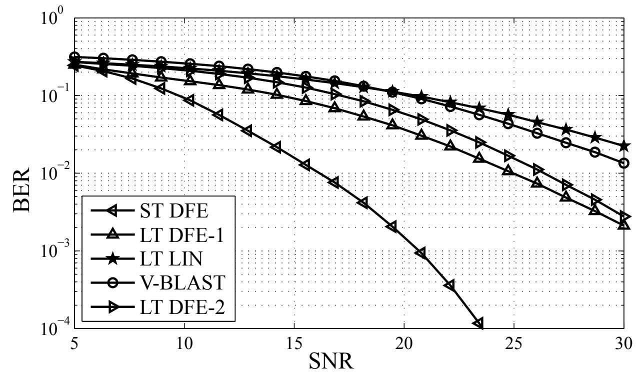

Fig. 3. Rayleigh-fading channel with correlation.

Below, we use the correlation coefficient $\rho_{\text{Rx}} = 0.8\,e^{0.4i}$. The transmitter-side correlation matrix, $\boldsymbol{R}_{\text{Tx}}$, is defined similarly using correlation coefficient $\rho_{\text{Tx}} = 0.8\,e^{i}$.

Fig. 2 shows the comparison between the various precoders using QPSK symbols on all sub-channels. The BER was evaluated at different SNR levels using Monte Carlo simulations. From the figure we can conclude that the adaptive DFE precoder (LT DFE-1) outperforms the previously proposed DFE precoder (LT DFE-2) as well as the V-BLAST scheme and the precoder designed for linear MMSE detection (LT LIN). When perfect TX-CSI is available (ST DFE) the performance of the DFE can be improved by an order of magnitude, although the difference is small in the low-SNR region.

4.2. Rayleigh-fading channel with correlation

As in the previous example we consider a MIMO system of dimensionality $n_T = n_R = L = 4$. This time however, the channel mean component, $\hat{\boldsymbol{H}}$, is zero. For simplicity, we assume the same Kronecker-structured covariance matrix. Given the TX-CSI, ξ, the channel matrix is distributed according to

$$\text{vec}\left(\boldsymbol{H}|\xi\right) = \mathcal{CN}\left(\boldsymbol{0}, \boldsymbol{R}_{\text{Tx}} \otimes \boldsymbol{R}_{\text{Rx}}\right), \tag{23}$$

where the transmit and receive correlation matrices are defined by the correlation coefficients $\rho_{\text{Tx}} = 0.9\,e^{0.5i}$ and $\rho_{\text{Rx}} = 0.9\,e^{1.5i}$ respectively. Fig. 3 shows the performance of the various precoders using QPSK symbols on the sub-channels. Again, we conclude that the adaptive DFE precoder outperforms the other adaptive precoders and the V-BLAST scheme.

5. CONCLUSIONS

We have addressed the problem of linear precoding using long-term channel statistics and assuming a non-linear decision feedback equalizer at the receiver. A design taking CSI uncertainty into account by utilizing the first and second order statistics of the channel has been proposed, and we demonstrated that the resulting precoder exhibits improved performance compared to similar methods based on long-term statistics.

6. REFERENCES

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Technical Memorandum, Bell Laboratories (Published in European Transactions on Telecommunications, Vol. 10, No.6, pp. 585-595, Nov/Dec 1999)*, 1995.

[2] P.W. Wolniansky, G.J. Foschini, G.D. Golden, and R.A. Valenzuela, "V-blast: An architecture for realizing very high data rates over the rich-scattering wireless channel," *URSI International Symposium on Signals, Systems, and Electronics*, pp. 295–300, 1998.

[3] A. Scaglione, S. Barbarossa, and G. B. Giannakis, "Filterbank tranceivers optimizing information rate in block transmissions over dispersive channels," *IEEE Transactions on Information Theory*, vol. 45, pp. 1019–1032, April 1999.

[4] D. P. Palomar, J. Cioffi, and M. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Transactions on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, 2003.

[5] D.P. Palomar and Y. Jiang, *MIMO Tranceiver Design via Majorization Theory*, Now Publishers Inc., 2007.

[6] F. Xu, T. N. Davidson, J. Zhang, and K. M. Wong, "Design of block transceivers with decision feedback detection," *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 965–978, 2006.

[7] M.B. Shenouda and T.N. Davidson, "A framework for designing mimo systems with decision feedback equalization or tomlinson-harashima precoding," in *International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2007.

[8] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Linear and nonlinear preequalization/equalization for mimo systems with long-term channel state information at the transmitter," *Wireless Communications, IEEE Transactions on*, vol. 3, pp. 373–378, 2004.

[9] X. Zhang, D.P. Palomar, and B. Ottersten, "Robust design of linear MIMO transceivers," *IEEE Transactions on Signal Processing*, September 2007, To Appear.

[10] K. Yu, M. Bengtsson, B. Ottersten, P. Karlsson, and M. Beach, "Second order statistics of NLOS indoor MIMO channels based on 5.2 GHz measurements," in *Proceedings IEEE Global Telecommunications Conference*, 2001, vol. 1, pp. 156–160.

[11] J.P. Kermoal, L. Schumacher, K.I. Pedersen, P.E. Mogensen, and F. Frederiksen, "A stochastic mimo radio channel model with experimental validation," *Selected Areas in Communications, IEEE Journal on*, vol. 20, no. 6, pp. 1211–1226, 2002.

[12] Y. Jiang, W. Hager, and J. Li, "The generalized triangular decomposition," *Mathematics of Computation*, 2007, To appear.