SPATIAL MIMO DECISION FEEDBACK EQUALIZER OPERATING ON QUANTIZED DATA

Amine Mezghani, Mohamed-Seifeddine Khoufi and Josef A. Nossek

Institute for Circuit Theory and Signal Processing
Munich University of Technology, 80290 Munich, Germany

ABSTRACT

We study the joint optimization of the quantizer and the spatial Decision Feedback Equalizer (DFE) for the flat multi-input multi-output (MIMO) channel with quantized outputs. Our design is based on a *minimum mean square error* (MMSE) approach, taking into account the effects of quantization. Our derivation does not make use of the assumption of uncorrelated white quantization errors and considers the correlations of the quantization error with the other signals of the system. Through simulation, we compare the new DFE to the conventional spatial DFE operating on quantized data in terms of uncoded BER.

***Index Terms*—** MIMO systems, Decision feedback equalizers, Quantization

1. INTRODUCTION

It is well known that the use of multiple antennas at both sides of the transmission link (MIMO systems) can improve the communication performance dramatically. Many detection schemes have been proposed to enable reliable communications in such systems. A popular suboptimal approach with low complexity is the spatial DFE approach combined with ordering, known also as Vertical Bell Labs Layered Space-Time (V-BLAST) system [1, 2]. However, most of these contributions on receiver design for MIMO systems assume that the receiver has access to the channel data with infinite precision. In practice, however, a quantizer is applied to the receive signal, so that the channel measurements can be processed in the digital domain. The reliance on high-resolution analog-to-digital converters (ADCs) easily becomes unjustified as soon as we have to do with high speed MIMO channels [3]. In this case, the needed high resolution ADCs are expensive and even no more feasible. In fact, in order to reduce circuit complexity and save power and area, low resolution ADCs have to be employed [4]. Therefore, the proposed receiver designs do not necessarily have good performance when operating on quantized data in a real system. In [3, 5], we study the effects of quantization from an information theoretical point of view for MIMO systems. Motivated by the same approach as in our recent work [6], which concerns the linear MMSE receiver operating on quantized data, we modify the well-known spatial DFE combined with ordering for the quantized flat MIMO channel (later denoted by DFEQ), taking into account the presence of the quantizer. Under the choice of an optimal uniform/non-uniform scalar quantizer we evaluate the resulting MSE between the estimated and the transmitted symbols and we minimize it subject to the DFE detector. In our model we assume perfect channel state information (CSI) at the receiver, which can be obtained even with coarse quantization [7].

Our paper is organized as follows. First we introduce the system model and some notational issues. Section 3 reviews the conventional MMSE-DFE approach. In section 4 we discuss the properties of the chosen quantizer, then we derive the modified DFE receiver in section 5. Finally, we present some simulation results in section 6.

2. SYSTEM MODEL AND NOTATION

We consider a point-to-point MIMO Gaussian channel where the transmitter employs M antennas and the receiver has N antennas. Fig. 1 shows the general form of a quantized MIMO system, where $\boldsymbol{H} \in \mathbb{C}^{N\times M}$ is the channel matrix. The vector $\boldsymbol{x} \in \mathbb{C}^{M}$ comprises the M transmitted symbols, which are uncorrelated and have zero-mean and covariance matrix $\boldsymbol{R}_{xx} = \mathrm{E}[\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}] = \sigma_x^2\mathbf{I}$. The vector $\boldsymbol{\eta}$ refers to zero-mean complex circular Gaussian noise with covariance $\boldsymbol{R}_{\eta\eta} = \mathrm{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}]$. $\boldsymbol{y} \in \mathbb{C}^{N}$ is the unquantized channel output:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\eta}. \tag{1}$$

In our system, the real parts $y_{i,R}$ and the imaginary parts $y_{i,I}$ of the receive signals y_i, $1 \leq i \leq N$, are each quantized by a b-bit resolution uniform/non-uniform scalar quantizer. Thus, the resulting quantized signals read as:

$$r_{i,l} = Q(y_{i,l}) = y_{i,l} + q_{i,l},\ l \in \{R, I\},\ 1 \leq i \leq N, \tag{2}$$

where $Q(\cdot)$ denotes the quantization operation and $q_{i,l}$ is the resulting quantization error. In the DFE architecture of Fig. 1, the matrix $\boldsymbol{G} \in \mathbb{C}^{M\times N}$ represents the forward filter and $\boldsymbol{F} \in \mathbb{C}^{M\times M}$ the feedback filter with the following structures

$$\boldsymbol{F} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ f_{2,1} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ f_{M,1} & \dots & f_{M,M-1} & 0 \end{bmatrix} \in \mathbb{C}^{M\times M}, \tag{3}$$

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g}_1^{\mathrm{T}} \\ \vdots \\ \boldsymbol{g}_M^{\mathrm{T}} \end{bmatrix} \in \mathbb{C}^{M\times N}. \tag{4}$$

In other words, $\boldsymbol{g}_k^{\mathrm{T}}$ is the k-th row of the receiver filter $\boldsymbol{G}$ and $f_{k,j}$ is the entry at the k-th row and j-th column ($k > j$) of the feedback matrix $\boldsymbol{F}$. The DFE feeds back the already detected symbols as shown in Fig. 1 and subtracts the interference caused by these symbols from the next input of the decision module, using a feedback matrix $\boldsymbol{F}$. The decoding order is denoted by π, i.e. stream x_{π_k} (with decoding order k) sees the interference caused by the streams $x_{\pi_{k+1}} \ldots x_{\pi_K}$. Therefore an inverse permutation π^{-1} is needed after the detection module in order to get the streams in the original order. The matrices $\boldsymbol{F}$ and $\boldsymbol{G}$, together, delivers the estimate $\hat{x}_{\pi_k}$ as

$$\hat{x}_{\pi_k} = \boldsymbol{g}_k^{\mathrm{T}}\boldsymbol{r} - \sum_{j=1}^{k-1} f_{k,j}\hat{x}_{\pi_j}. \tag{5}$$

Our aim is to choose the quantizer, the receive matrix $\boldsymbol{G}$, and the feedback matrix $\boldsymbol{F}$, minimizing the MSE$=\mathrm{E}[\|\hat{\boldsymbol{x}}-\boldsymbol{x}\|_2^2]$, taking into account the quantization effect. Throughout this paper, $r_{\alpha\beta}$ denotes $\mathrm{E}[\alpha\beta^*]$. The operators $(\bullet)^{\mathrm{T}}$, $(\bullet)^{\mathrm{H}}$, $(\bullet)^*$, $\mathrm{Re}(\bullet)$, $\mathrm{Im}(\bullet)$ stand for transpose, Hermitian transpose, complex conjugate, real and imaginary parts of a complex number, respectively.

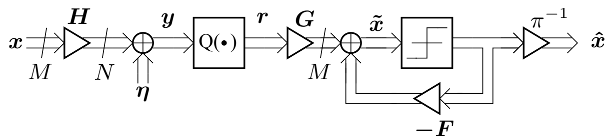

Fig. 1. Decision Feedback Equalizer on a Quantized MIMO Channel

3. REVIEW OF THE SPATIAL MMSE-DFE

We review the MMSE-DFE (or MMSE V-BLAST) algorithm in this section while ignoring the quantization (i.e. $\boldsymbol{r} \equiv \boldsymbol{y}$). Let us first consider the error signal at the k-th decoding step assuming that the previous streams were correctly decoded

$$e_{\pi_k} = x_{\pi_k} - \boldsymbol{g}_k^{\mathrm{T}}(\boldsymbol{H}\boldsymbol{x} + \boldsymbol{\eta}) + \sum_{j=1}^{k-1} f_{k,j} x_{\pi_j}. \tag{6}$$

Under these assumptions, it is easy to show by the KKT conditions that, for given ordering π, the optimal forward and feedback matrices minimizing each $\mathrm{MSE}_k = \mathrm{E}[|e_k|^2]$ individually (and thus the sum-MSE) reads as

$$f_{k,j} = \boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{h}_{\pi_j}, \text{ and } \quad \boldsymbol{g}_k^{\mathrm{T}} = \sigma_x^2 \boldsymbol{h}_{\pi_k}^{\mathrm{H}} \bar{\boldsymbol{T}}_k, \tag{7}$$

where $\boldsymbol{h}_{\pi_k}$ is the π_k-th column of the channel matrix $\boldsymbol{H}$, and

$$\bar{\boldsymbol{T}}_k = \left(\sigma_x^2 \boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} - \sigma_x^2 \sum_{l=1}^{k-1} \boldsymbol{h}_{\pi_l} \boldsymbol{h}_{\pi_l}^{\mathrm{H}} + \boldsymbol{R}_{\eta\eta} \right)^{-1} \in \mathbb{C}^{N \times N}. \tag{8}$$

With these optimum matrices, the MSE of the symbol with decoding order k reads as:

$$\mathrm{MSE}_{\pi_k} = \sigma_x^2 - \sigma_x^4 \boldsymbol{h}_{\pi_k}^{\mathrm{H}} \bar{\boldsymbol{T}}_k \boldsymbol{h}_{\pi_k}, k \in \{1, \ldots, M\}. \tag{9}$$

Now, the problem of finding the optimal ordering π minimizing the sum-MSE ($\sum_k \mathrm{MSE}_k$) is NP hard, since we must check all $M!$ possible permutations. To reduce the complexity, V-BLAST minimizes each summand successively, i. e., π_k is chosen under the assumption that $\pi_1, \ldots, \pi_{k-1}$ are fixed

$$\pi_k = \arg \min_{i \notin \{\pi_1, \ldots, \pi_{k-1}\}} 1 - \sigma_x^2 \boldsymbol{h}_i^{\mathrm{H}} \bar{\boldsymbol{T}}_k \boldsymbol{h}_i. \tag{10}$$

4. OPTIMAL QUANTIZER

Each quantization process is given a distortion factor $\rho_q^{(i,l)}$ to indicate the relative amount of quantization noise generated, which is defined as follows

$$\rho_q^{(i,l)} = \frac{\mathrm{E}[q_{i,l}^2]}{r_{y_{i,l} y_{i,l}}}, \tag{11}$$

where $r_{y_{i,l} y_{i,l}} = \mathrm{E}[y_{i,l}^2]$ is the variance of $y_{i,l}$ and the distortion factor $\rho_q^{(i,l)}$ depends on the number of quantization bits b, the quantizer type (uniform or non-uniform) and the probability density function of $y_{i,l}$. Note that the signal-to-quantization noise ratio (SQNR) has an inverse relationship with regard to the distortion factor

$$\mathrm{SQNR}^{(i,l)} = \frac{1}{\rho_q^{(i,l)}}. \tag{12}$$

Similar to our work [6] concerning the Wiener filter for quantized data, the uniform/non-uniform quantizer design is based on minimizing the *mean square error* (distortion) between the input $y_{i,l}$ and the output $r_{i,l}$ of each quantizer. In other words, the SQNR values are maximized. Under this optimal design of the scalar finite resolution quantizer, whether uniform or not, the following equations hold for all $0 \leq i \leq N, l \in \{R, I\}$ [8, 9]:

$$\mathrm{E}[q_{i,l}] = 0 \tag{13}$$

$$\mathrm{E}[r_{i,l} q_{i,l}] = 0 \tag{14}$$

$$\mathrm{E}[y_{i,l} q_{i,l}] = -\rho_q^{(i,l)} r_{y_{i,l} y_{i,l}}. \tag{15}$$

Obviously, Eq. (15) follows from Eqs (11) and (14). For the uniform quantizer case, Eq. (13) holds only if the probability density function of $y_{i,l}$ is even.

Under multipath propagation conditions and for large number of antennas, the quantizer input signals $y_{i,l}$ are approximately Gaussian distributed and thus, they undergo nearly the same distortion factor ρ_q, i.e., $\rho_q^{(i,l)} = \rho_q \ \forall i \forall l$. Furthermore, the optimal parameters of the uniform as well as the non-uniform quantizer and the resulting distortion factor ρ_q for Gaussian distributed signal are tabulated in [8] for different bit resolutions b. Recent research work on optimally quantizing the Gaussian source can be found in [10, 11].

Now, let $q_i = q_{i,R} + \mathrm{j} q_{i,I}$ be the complex quantization error. Under the assumption of uncorrelated real and imaginary part of y_i, we easily obtain:

$$r_{q_i q_i} = \mathrm{E}[q_i q_i^*] = \rho_q r_{y_i y_i}, \text{ and } r_{y_i q_i} = \mathrm{E}[y_i q_i^*] = -\rho_q r_{y_i y_i}. \tag{16}$$

For the uniform quantizer case, it was shown in [11], that the optimal quantization step Δ for a Gaussian source decreases as $\sqrt{b}2^{-b}$ and that ρ_q is asymptotically well approximated by $\frac{\Delta^2}{12}$ and decreases as $b2^{-2b}$. On the other hand, the optimal non-uniform quantizer achieves, under high-resolution assumption, approximately the following distortion [12]

$$\rho_q \approx \frac{\pi\sqrt{3}}{2} 2^{-2b}. \tag{17}$$

This particular choice of the (non-)uniform scalar quantizer minimizing the distortion between $\boldsymbol{r}$ and $\boldsymbol{y}$, combined with the receiver of the next section, is also optimal with respect to the total MSE between the transmitted symbol vector $\boldsymbol{x}$ and the estimated symbol vector $\hat{\boldsymbol{x}}$, as we will see later.

5. NEAR OPTIMAL DFE RECEIVER FOR THE QUANTIZED SYSTEM

In this section, we optimize the receive filter $\boldsymbol{G}$ and the feedback matrix $\boldsymbol{F}$ based on the MMSE criterion, taking into account the quantization process. To this end, we evaluate the MSE between each sent symbol x_{π_k} and detected one $\hat{x}_{\pi_k}$ $(1 \leq k \leq M)$

$$\mathrm{MSE}_{\pi_k} = \varepsilon_{\pi_k} = \mathrm{E}[\| x_{\pi_k} - (\boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{r} - \sum_{j=1}^{k-1} f_{k,j} x_{\pi_j}) \|_2^2]. \tag{18}$$

Under the assumptions of uncorrelated symbols ($\boldsymbol{R}_{xx} = \sigma_x^2 \mathbf{I}_M$), Eq. (18) becomes

$$\begin{aligned} \varepsilon_{\pi_k} =& \sigma_x^2 - 2\mathrm{Re}(\boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{r}_{\boldsymbol{r}, x_{\pi_k}}) + \boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{R}_{rr} \boldsymbol{g}_k^* \\ & - 2\mathrm{Re}(\sum_{j=1}^{k-1} \boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{r}_{\boldsymbol{r}, x_{\pi_k}} f_{k,j}^*) + \sigma_x^2 \sum_{j=1}^{k-1} |f_{k,j}|^2, \end{aligned} \tag{19}$$

where $\boldsymbol{r}_{\boldsymbol{r}, x_{\pi_k}} = \mathrm{E}[\boldsymbol{r} x_{\pi_k}^*]$ is the conjugate transpose of the π_k-th row of the correlation matrix

$$\boldsymbol{R}_{xr} = \mathrm{E}[\boldsymbol{x}\boldsymbol{r}^{\mathrm{H}}] = \mathrm{E}[\boldsymbol{x}(\boldsymbol{y} + \boldsymbol{q})^{\mathrm{H}}] = \boldsymbol{R}_{xy} + \boldsymbol{R}_{xq}, \tag{20}$$

and $\boldsymbol{R}_{rr}$ the covariance matrix of the quantized signal given by

$$\boldsymbol{R}_{rr} = \mathrm{E}[(\boldsymbol{y} + \boldsymbol{q})(\boldsymbol{y} + \boldsymbol{q})^{\mathrm{H}}] = \boldsymbol{R}_{yy} + \boldsymbol{R}_{yq} + \boldsymbol{R}_{yq}^{\mathrm{H}} + \boldsymbol{R}_{qq}. \tag{21}$$

5.1. Derivation of the DFEQ Receiver

Before investigating the MMSE optimization, we first derive all needed covariance matrices by using the fact that the quantization error q_i, conditioned on y_i, is statistically independent from all other random variables of the system.
First we calculate $r_{y_i q_j} = \mathrm{E}[y_i q_j^*]$ for $i \neq j$:

$$\begin{aligned}
\mathrm{E}[y_i q_j^*] &= \mathrm{E}_{y_j}\big[\mathrm{E}[y_i q_j^* | y_j]\big] \\
&= \mathrm{E}_{y_j}\big[\mathrm{E}[y_i | y_j]\mathrm{E}[q_j^* | y_j]\big] \\
&\approx \mathrm{E}_{y_j}\Big[r_{y_i y_j} r_{y_j y_j}^{-1} y_j \mathrm{E}[q_j^* | y_j]\Big] \quad (22)\\
&= r_{y_i y_j} r_{y_j y_j}^{-1} \mathrm{E}[y_j q_j^*] \\
&= -\rho_q r_{y_i y_j}. \quad (23)
\end{aligned}$$

In (22), we approximate the Bayesian estimator $\mathrm{E}[y_i|y_j]$ with the linear estimator $r_{y_i y_j} r_{y_j y_j}^{-1} y_j$, which holds with equality if the vector $\boldsymbol{y}$ is jointly Gaussian distributed. Eq. (23) follows from (16). Summarizing the results of (16) and (23), we obtain:

$$\boldsymbol{R}_{yq} \approx -\rho_q \boldsymbol{R}_{yy}. \quad (24)$$

Similarly, we evaluate $r_{q_i q_j}$ for $i \neq j$ to end up in:

$$\mathrm{E}[q_i q_j^*] = \mathrm{E}_{y_j}\big[\mathrm{E}[q_i q_j^* | y_j]\big] \approx \rho_q^2 r_{y_j y_i}^* = \rho_q^2 r_{y_i y_j}, \quad (25)$$

where we used Eqs (24) and (16). From (25) and (16) we deduce the covariance matrix of the quantization error:

$$\begin{aligned}
\boldsymbol{R}_{qq} &\approx \rho_q \mathrm{diag}(\boldsymbol{R}_{yy}) + \rho_q^2 \mathrm{nondiag}(\boldsymbol{R}_{yy}) \\
&= \rho_q \boldsymbol{R}_{yy} - (1-\rho_q)\rho_q \mathrm{nondiag}(\boldsymbol{R}_{yy}),
\end{aligned} \quad (26)$$

with diag($\boldsymbol{A}$) denotes a diagonal matrix containing only the diagonal elements of $\boldsymbol{A}$ and nondiag$(\boldsymbol{A}) = \boldsymbol{A} - \mathrm{diag}(\boldsymbol{A})$. Inserting the expressions (24) and (26) into Eq. (21), we obtain:

$$\boldsymbol{R}_{rr} \approx (1-\rho_q)(\boldsymbol{R}_{yy} - \rho_q \mathrm{nondiag}(\boldsymbol{R}_{yy})). \quad (27)$$

In a very similar way, we get the covariance matrix $\boldsymbol{R}_{xq} = \mathrm{E}[\boldsymbol{x}\boldsymbol{q}^{\mathrm{H}}]$ as

$$\mathrm{E}[\boldsymbol{x}\boldsymbol{q}^{\mathrm{H}}] \approx -\rho_q \boldsymbol{R}_{xy}. \quad (28)$$

Thus, Equation (20) becomes

$$\boldsymbol{R}_{xr} \approx (1-\rho_q)\boldsymbol{R}_{xy}. \quad (29)$$

Finally, $\boldsymbol{R}_{yy}$ and $\boldsymbol{R}_{xy}$ can be easily obtained from our system model:

$$\begin{aligned}
\boldsymbol{R}_{yy} &= \boldsymbol{R}_{\eta\eta} + \sigma_x^2 \boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}, \quad (30)\\
\boldsymbol{R}_{xy} &= \sigma_x^2 \boldsymbol{H}^{\mathrm{H}}. \quad (31)
\end{aligned}$$

Now, we return to our MMSE-DFE problem. When differentiating each MSE expression from (19) with respect to $f_{k,j}^*$, we obtain

$$\frac{\partial \varepsilon_{\pi_k}}{\partial f_{k,j}^*} = \sigma_x^2 f_{k,j} - \boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{r}_{\boldsymbol{r},x_{\pi_j}}, \quad (32)$$

which should be equal to zero in order to optimize the MSE. Thus we obtain using (29) and (31) the optimal feedback matrix

$$f_{k,j} = (1-\rho_q)\boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{h}_{\pi_j}, \forall k \in \{1,\ldots,\mathrm{M}\} \quad \text{and } j < k. \quad (33)$$

This delivers the following individual MSE

$$\begin{aligned}
\varepsilon_{\pi_k} = \sigma_x^2 &- 2\mathrm{Re}(\boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{r}_{\boldsymbol{r},x_{\pi_k}}) + \boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{R}_{rr} \boldsymbol{g}_k^* \\
&- \sigma_x^2 (1-\rho_q)^2 \boldsymbol{g}_k^{\mathrm{T}} (\sum_{j=1}^{k-1} \boldsymbol{h}_{\pi_j} \boldsymbol{h}_{\pi_j}^{\mathrm{H}}) \boldsymbol{g}_k^*.
\end{aligned} \quad (34)$$

Differentiating this MSE expression with respect to $\boldsymbol{g}_k^{\mathrm{H}}$, we obtain

$$\frac{\partial \varepsilon_{\pi_k}}{\partial \boldsymbol{g}_k^{\mathrm{H}}} = -\boldsymbol{r}_{\boldsymbol{r},x_{\pi_k}}^{\mathrm{H}} + \boldsymbol{g}_k^{\mathrm{T}} \boldsymbol{R}_{rr} - \sigma_x^2 (1-\rho_q)^2 \boldsymbol{g}_k^{\mathrm{T}} (\sum_{j=1}^{k-1} \boldsymbol{h}_{\pi_j} \boldsymbol{h}_{\pi_j}^{\mathrm{H}}), \quad (35)$$

Setting this to zero, we get with (29) and (31)

$$\boldsymbol{g}_k^{\mathrm{T}} = \sigma_x^2 \boldsymbol{h}_{\pi_k}^{\mathrm{H}} \underbrace{\left[\frac{1}{1-\rho_q} \boldsymbol{R}_{rr} - \sigma_x^2 (1-\rho_q) \boldsymbol{H}_{\pi_{k-1}}\right]^{-1}}_{\boldsymbol{T}_k}, \quad (36)$$

where $\boldsymbol{H}_{\pi_{k-1}} = \sum_{j=1}^{k-1} \boldsymbol{h}_{\pi_j} \boldsymbol{h}_{\pi_j}^{\mathrm{H}}$, and with (27), we obtain

$$\boldsymbol{T}_k = \left[\boldsymbol{R}_{yy} - \rho_q \mathrm{nondiag}(\boldsymbol{R}_{yy}) - (1-\rho_q)\boldsymbol{H}_{\pi_{k-1}}\right]^{-1}. \quad (37)$$

Thus, using (33) the optimal feedback matrix reads as

$$[\boldsymbol{F}]_{k,j} = \sigma_x^2 (1-\rho_q) h_{\pi_k}^{\mathrm{H}} \boldsymbol{T}_k \boldsymbol{h}_{\pi_j}, \forall k \in \{1,\ldots,\mathrm{M}\} \text{ and } j < k. \quad (38)$$

Notice that, when we set $\rho_q = 0$ in the expressions of $\boldsymbol{G}^{\mathrm{DFEQ}}$ and $\boldsymbol{F}^{\mathrm{DFEQ}}$, we obtain exactly the same expressions as in (7) for the unquantized system with MMSE-DFE detection.
Using Eqs (19), (33) and (36), the MSE resulting from the optimal design of the receiver filter and feedback matrix becomes after some computations

$$\mathrm{MSE}_{\pi_k}^{\mathrm{DFEQ}} = \sigma_x^2 - \sigma_x^4 (1-\rho_q) \boldsymbol{h}_{\pi_k}^{\mathrm{H}} \boldsymbol{T}_k \boldsymbol{h}_{\pi_k}. \quad (39)$$

Finally, in order to find the optimum processing order π, we use the same approach as in V-BLAST described by (10), but we evaluate the individual MSEs according to (39), so that we take into account the quantization process.

5.2. Effects of Quantization on the MSE

In order to verify whether the chosen quantizer minimizes the MSE of our system, we examine the first derivative of each $\mathrm{MSE}_k^{\mathrm{DFEQ}}$ in (39) with respect to ρ_q:

$$\frac{\partial \mathrm{MSE}_{\pi_k}^{\mathrm{DFEQ}}}{\partial \rho_q} = \boldsymbol{h}_{\pi_k}^{\mathrm{H}} \boldsymbol{T}_k \mathrm{diag}(\boldsymbol{R}_{yy}) \boldsymbol{T}_k \boldsymbol{h}_{\pi_k} = \boldsymbol{g}_k^{\mathrm{T}} \mathrm{diag}(\boldsymbol{R}_{yy}) \boldsymbol{g}_k^*,$$

where we used (39) and (37). Obviously, the derivative of the MSE with respect to the distortion is positive. Therefore, each $\mathrm{MSE}_k^{\mathrm{DFEQ}}$ is monotonically increasing with respect to ρ_q. Since we chose the quantizer to minimize the distortion factor ρ_q, our receiver quantizer designs are jointly optimum with respect to the total MSE.

Now, we expand the MSE expression (39) into a Taylor series around $\rho_q = 0$ up to the order one, to get an approximation of each individual MSE

$$\begin{aligned}
&\mathrm{MSE}_{\pi_k}^{\mathrm{DFEQ}} \approx \mathrm{MSE}_{\pi_k}^{\mathrm{DFE}} + \\
&\rho_q h_{\pi_k}^{\mathrm{H}} (\frac{\boldsymbol{R}_{yy}}{\sigma_x^2} - \boldsymbol{H}_{\pi_{k-1}})^{-1} \mathrm{diag}(\boldsymbol{R}_{yy}) (\frac{\boldsymbol{R}_{yy}}{\sigma_x^2} - \boldsymbol{H}_{\pi_{k-1}})^{-1} h_{\pi_k},
\end{aligned} \quad (40)$$

where $\mathrm{MSE}_{\pi_k}^{\mathrm{DFE}} = \mathrm{MSE}_{\pi_k}^{\mathrm{DFEQ}}|_{\rho_q=0}$ is the achievable MSE without quantization from (9). The second term gives the increase in each MSE_{π_k} due to the quantization as a function of ρ_q and the channel parameters. It reveals also the residual error at infinite SNR.

6. SIMULATION RESULTS

The performance of the modified DFE filter for a 4- and 5-bit quantized output MIMO system (DFEQ), in terms of BER averaged over 10^6 channel realizations, is shown in Fig. 2 for a 10×10 MIMO system (QPSK), compared with the conventional DFE receiver (DFE) and our modified DFE detector (DFEQ). The symbols and the noise are assumed to be uncorrelated, that is $\boldsymbol{R}_{xx} = \sigma_x^2\mathbf{I}$ and $\boldsymbol{R}_{\eta\eta} = \sigma_\eta^2\mathbf{I}$. Hereby, the (pseudo-)SNR (in dB) is defined as

$$\mathrm{SNR} = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_\eta^2}\right). \tag{41}$$

Furthermore, the entries of $\boldsymbol{H}$ are complex-valued realization of independent zero-mean Gaussian random variables with unit variance. Clearly, the modified DFE filter outperforms the conventional DFE filter at high SNR. This is because the effect of quantization error is more pronounced at higher SNR values when compared to the additive Gaussian noise variance. Since the conventional MMSE-DFE filter looses its regularized structure at high SNR values, its performance degrades asymptotically, when operating on quantized data. For comparison, we also plotted the BER curve for the DFE receiver, if no quantization is applied.

In Fig. 3, we consider a 4-bit quantized 4×4 MIMO configuration. In addition to the BER curves of our DFEQ detector and a conventional DFE, it shows the simulated BER with the conventional decision feedback equalization, but considering the quantization error $\boldsymbol{q}$ as an additive white noise, which is uncorrelated with the other signals of the system. For the additive quantization noise model, we take as effective noise covariance matrix (see first equality of (16)):

$$\bar{\boldsymbol{R}}_{\eta\eta} = \boldsymbol{R}_{\eta\eta} + \bar{\boldsymbol{R}}_{qq} = \boldsymbol{R}_{\eta\eta} + \mathrm{diag}(\boldsymbol{R}_{yy}). \tag{42}$$

Obviously, this model, which has been commonly used in the literature is also outperformed by the presented DFEQ at any SNR level and independently of the resolution.

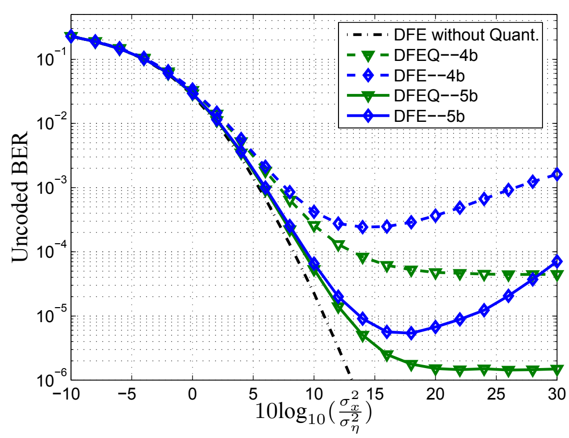

Fig. 2. DFEQ vs. the conventional DFE, QPSK modulation with $M = 10$, $N = 10$, 4- and 5-bit uniform quantizer, with $\rho_q = 0.011535$ and $\rho_q = 0.00349$, respectively.

7. CONCLUSION

We addressed the problem of designing a MMSE-DFE receiver for MIMO channels with quantized outputs. We provided an approximation for the mean squared error for each data stream, where the quantizer is optimized for a Gaussian input. Then, we proposed an optimized DFE receiver operating on quantized data combined with ordering, which shows better performance in terms of BER than the conventional DFE filter. An essential aspect of our derivation is that we do not make the assumption of uncorrelated white quantization error. Moreover, our receiver does not present any extra complexity from the implementation point of view.

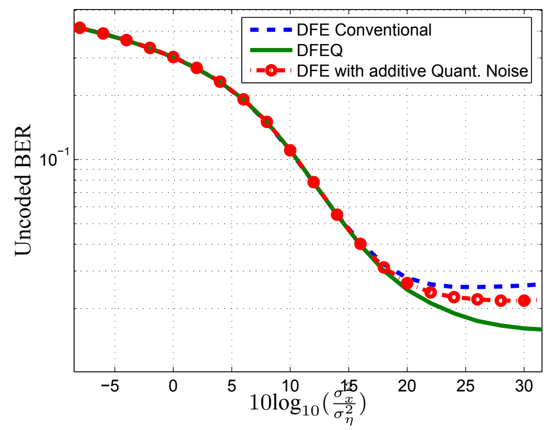

Fig. 3. DFEQ vs. the conventional DFE and DFE with additive quantization noise model, 16QAM modulation with $M = 4$, $N = 4$, 4-bit ($\rho_q = 0.011535$) uniform quantizer.

8. REFERENCES

[1] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel," in *URSI International Symposium on Signals, Systems, and Electronics*, Pisa, Italy, September 1998, pp. 295–300.

[2] K. Kusume, M. Joham, and W. Utschick, "MMSE Block Decision-Feedback Equalizer for Spatial Multiplexing with Reduced Complexity," in *Proc. Globecom 2004*, November 2004, vol. 4, pp. 2540–2544.

[3] J. A. Nossek and M. T. Ivrlač, "Capacity and coding for quantized MIMO systems," in *Intern. Wireless Commun. and Mobile Computing Conf. (IWCMC)*, Vancouver, Canada, July 2006, pp. 1387–1392, invited.

[4] R. Schreier and G. C. Temes, "Understanding Delta-Sigma Data Converters," *IEEE Computer Society Press*, 2004.

[5] A. Mezghani and J. A. Nossek, "On Ultra-Wideband MIMO Systems with 1-bit Quantized Outputs: Performance Analysis and Input Optimization," *IEEE International Symposium on Information Theory (ISIT), Nice, France*, June 2007.

[6] A. Mezghani, M. S. Khoufi, and J. A. Nossek, "A Modified MMSE Receiver for Quantized MIMO Systems," *In Proc. ITG/IEEE WSA, Vienna, Austria*, February 2007.

[7] M. T. Ivrlač and J. A. Nossek, "On Channel Estimation in Quantized MIMO Systems," *In Proc. ITG/IEEE WSA, Vienna, Austria*, February 2007.

[8] J. Max, "Quantizing for Minimum Distortion," *IEEE Trans. Inf. Theory*, vol. 6, no. 1, pp. 7–12, March 1960.

[9] J. G. Proakis, *Digital Communications*, McGraw Hill, New York, third edition, 1995.

[10] N. Al-Dhahir and J. M. Cioffi, "On the Uniform ADC Bit Precision and Clip Level Computation for a Gaussian Signal," *IEEE Trans. on Sig. Proc.*, vol. 44, no. 2, February 1996.

[11] D. Hui and D. L. Neuhoff, "Asymptotic Analysis of Optimal Fixed-Rate Uniform Scalar Quantization," *IEEE Trans. Inform. Theory*, vol. 47, no. 3, pp. 957–977, March 2001.

[12] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*, Kluwer Academic Publishers, Dordrecht, Niederlande, first edition, 1992.