ADAPTIVE CHANNEL PREDICTION BASED ON POLYNOMIAL PHASE SIGNALS

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ABSTRACT

Motivated by recently published physics based scattering SISO and MIMO channel models in mobile communications [1, 2], a new adaptive channel prediction based on non-stationary polynomial phase signals is proposed. To mitigate the influence of the time-varying amplitudes and to reduce the computation complexity, an iterative estimation of the polynomial phase parameters using the Non-linear *instantaneous* LS criterion is proposed. Given the polynomial phase parameters, the time-varying amplitudes are estimated using the Kalman filter. The performance of the new predictor is evaluated by Monte Carlo simulations in SISO scenarios with multiple scattering clusters. The new predictor outperforms the classical Linear Prediction and previous prediction methods based on sinusoidal modeling.

***Index Terms* -** Radio propagation, Rayleigh channels, Nonlinear estimation, Adaptive Kalman filtering, Prediction methods

1. INTRODUCTION

Different from the power spectrum based non-parametrical methods [3], radio channel prediction based on parametric modeling attracted much interests in last several years due to its potential advantage on long range channel prediction and compression of feedback information [4, 5, 6]. Recently, i.e. in [1], a physics based scattering model was proposed to simulate narrow band SISO Rayleigh fading channels, which is similar to the recently standardized 3GPP MIMO channel models [2]. In this model, the observed multipath channel $y(t)$ from p scattering clusters is modeled as

$$y(t) = h(t) + e(t) = \sum_{i=1}^{p} h_i(t) + e(t), \tag{1}$$

$$h_i(t) = s_i(t) e^{j\phi_i(t)}, \tag{2}$$

where $e(t)$ is an additive Gaussian noise with zero mean and variance σ_e^2. By definition, $\phi_i(t) = 2\pi l_{i,c}(t)/\lambda$, λ is the wavelength, $l_{i,c}(t)$ is the length of the *virtual* propagation path from the transmitter antenna to the receiver antenna via the center of gravity of the i^{th} cluster at $(x_{i,c}, y_{i,c}) = \sum_{j=1}^{q_i}(x_{i,j}, y_{i,j})/q_i$, where $(x_{i,j}, y_{i,j})$ is the coordinate of the j^{th} scatter in the i^{th} cluster, and q_i is the number of scatters. These scatters are assumed to be uniformly distributed in a circular area with a radius of $\gamma\lambda$, where γ describes the roughness of the scattering surface, and $0 \le \gamma < 1$. Let $\Delta l_{i,j}(t) = l_{i,j}(t) - l_{i,c}(t)$, where $l_{i,j}(t)$ is the length of the propagation path from transmitter antenna to receiver antenna via the scatter at $(x_{i,j}, y_{i,j})$. The time-varying amplitude $s_i(t)$ in (2) can be expressed as

$$s_i(t) = s_i' \sum_{j=1}^{q_i} e^{j2\pi\Delta l_{i,j}(t)/\lambda}, \tag{3}$$

where s_i' is the reflected amplitude of scatters in the i^{th} cluster, which is assumed to be constant over the scattering cluster and time. In this paper, we assume that $\mathbf{y} = [y(t), y(t-1), y(t-N+1)]^T$ is observed, where ^{T} is the transpose operation. The channel $h(t+L)$ will be predicted, where $L > 0$ is the prediction horizon.

In previous parametric prediction methods based on sinusoidal modeling [4, 5, 6], $\phi_i(t)$ is assumed to be a linear function of time, i.e. $\phi_i(t) = \omega_i t$, where the Doppler frequency ω_i is assumed to be constant in the observation interval. In fact, the time-varying phases are due to the relative movement between the mobile and the scattering clusters, which are nonlinear functions of time and result in time-varying Doppler frequencies [6]. In this paper, the nonlinearity of $\phi_i(t)$ is approximated by a polynomial of time t with order M, i.e.

$$\phi_i(t) \approx \sum_{m=1}^{M} \beta_{i,m} t^m. \tag{4}$$

The pM model parameters of the polynomial phase signals (PPS) are collected in the vector $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T, \cdots, \boldsymbol{\theta}_p^T]^T$, where $\boldsymbol{\theta}_i = [\beta_{i,1}, \cdots, \beta_{i,M}]^T$.

Proposed Approach

1. Estimate the PPS parameters using the Nonlinear *instantaneous* LS techniques iteratively.
2. Estimate the dynamic model for the varying amplitudes.
3. Predict channel using the Kalman filter with the estimated parameters.

The first step is presented in Section 2. The second and the third step are described in Section 3. The performance of the proposed approach is evaluated using Monte Carlo simulations in SISO scenarios with multiple scattering clusters.

2. PPS PARAMETER ESTIMATION USING NILS

Given p and M, as the first step of the predictor design, an estimation of the model parameters $\boldsymbol{\theta}$ using Nonlinear *Instantaneous* Least Square (NILS) criterion is proposed in this section [7, 8]. By assuming the slowly time-varying amplitude, $s_i(t)$, within a local time interval with length n to be constant, the estimate of $\boldsymbol{\theta}$ is, i.e.,

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_{k=t}^{t-N+n} \|\mathbf{y}(k) - \mathbf{A}_k \mathbf{s}(k)\|^2, \quad (5)$$

where

$$\begin{aligned} \mathbf{y}(k) &= [y(k), \cdots, y(k-n+1)]^T, & (6)\\ \mathbf{A}_k &= [\mathbf{a}_{1,k}, \cdots, \mathbf{a}_{p,k}], & (7)\\ \mathbf{a}_{i,k} &= [e^{j\phi_i(k)}, \cdots, e^{j\phi_i(k-n+1)}]^T, & (8)\\ \mathbf{s}(k) &= [s_1(k), \cdots, s_p(k)]^T. & (9) \end{aligned}$$

Further, the Least Square (LS) estimate of the instantaneous amplitude $\mathbf{s}(k)$ is

$$\hat{\mathbf{s}}(k) = \left(\mathbf{A}_k^H \mathbf{A}_k\right)^{-1} \mathbf{A}_k^H \mathbf{y}(k) = \mathbf{A}_k^\dagger \mathbf{y}(k), \quad (10)$$

where ^{H} is the Hermitian transpose. Note that the selection of n is a user's choice. For fast time-varying amplitudes, a small n is selected. Meanwhile, the resolution of the model parameters is reduced. When $n = N$, NILS becomes the standard Nonlinear LS (NLLS) [9, 10].

Since the criterion function (5) contains pM parameters, the full non-linear optimization is computationally intensive. An iterative parameter estimate is proposed, where the NILS problem in (5) is solved using an iterative SAGE/RELAX procedure following [11, 12]. A detailed description of the algorithm is given in the appendix in Section 6.

3. ADAPTIVE CHANNEL PREDICTION BASED ON PPS BASES

Given $\hat{\boldsymbol{\theta}}$, an adaptive channel predictor similar to [1] is proposed in this section. Let $s_{i,z}(t) = s_i(t) - \mu_{s,i}$, where $\mu_{s,i}$ is the nonzero-mean of $s_i(t)$ [1]. An AR(d) model is proposed to describe the dynamics of $s_{i,z}(t)$, i.e.

$$\begin{aligned} s_{i,z}(t+1) &= \sum_{l=1}^{d} \alpha_{i,l} s_{i,z}(t-l+1) + v_i(t), & (11)\\ &= \boldsymbol{\alpha}_i^T \mathbf{s}_{i,z}(t) + v_i(t), & (12) \end{aligned}$$

where $\boldsymbol{\alpha}_i = [\alpha_{i,1}, \cdots, \alpha_{i,d}]^T$, $\mathbf{s}_{i,z}(t) = [s_{i,z}(t), \cdots, s_{i,z}(t-d+1)]^T$, and d is small. The $v_i(t)$ is a driving noise with pdf, $\mathcal{CN}(0, \sigma_{v,i}^2)$. Note that the local amplitude estimates $\hat{s}_i(k)$ can be obtained from the NILS procedure (27). These estimates can be further used to estimate $\mu_{s,i}$ and $\hat{s}_{i,z}(t)$. The AR parameters $\boldsymbol{\alpha}_{i,d}$ and $\sigma_{v,i}^2$ can then be estimated from $\hat{s}_{i,z}(t)$ using LS. The signal model (1) with the PPS bases (4) can then be expressed in a state-space structure as

$$\begin{aligned} \mathbf{x}(t+1) &= \boldsymbol{\Gamma}\mathbf{x}(t) + \mathbf{u}(t), & (13)\\ y(t) &= \mathbf{c}^T(t)\mathbf{x}(t) + e(t), & (14) \end{aligned}$$

where

$$\begin{aligned} \mathbf{x}(t) &= [\mathbf{x}_1^T(t), \cdots, \mathbf{x}_p^T(t)]^T, & (15)\\ \mathbf{x}_i(t) &= [s_{i,z}(t), \cdots, s_{i,z}(t-d+1), \mu_{s,i}]^T, & (16)\\ \boldsymbol{\Gamma} &= diag(\boldsymbol{\Gamma}_1, \cdots, \boldsymbol{\Gamma}_p), & (17) \end{aligned}$$

$$\boldsymbol{\Gamma}_i = \begin{bmatrix} \alpha_{i,1} & \alpha_{i,2} & \cdots & \alpha_{i,d-1} & \alpha_{i,d} & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots & \vdots \\ 0 & 0 & & 1 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}, \quad (18)$$

$$\begin{aligned} \mathbf{u}(t) &= [\mathbf{u}_1^T(t), \cdots, \mathbf{u}_p^T(t)]^T, & (19)\\ \mathbf{u}_i(t) &= [v_i(t), \mathbf{0}_{d-1}^T, w_i(t)]^T, & (20)\\ \mathbf{c}(t) &= [\mathbf{c}_1^T(t), \cdots, \mathbf{c}_p^T(t)]^T, & (21)\\ \mathbf{c}_i(t) &= [e^{j\hat{\phi}_i(t)}, \mathbf{0}_{d-1}^T, e^{j\hat{\phi}_i(t)}]^T. & (22) \end{aligned}$$

In the state-space model, the variance of $w_i(t)$, $\sigma_{w,i}^2$, should be set much smaller than the variance of $v_i(t)$, since $w_i(t)$ is the innovation noise for the mean amplitude which is constant. Then, the prediction of the $h(t+L)$ is

$$\hat{h}(t+L|t) = \mathbf{c}^T(t+L)\boldsymbol{\Gamma}^L \hat{\mathbf{x}}(t|t), \quad (23)$$

where $\hat{\mathbf{x}}(t|t)$ is obtained by the Kalman filter. The initial covariance matrix $\mathbf{C}_{\mathbf{X}} = diag(\mathbf{C}_{\mathbf{X},1}, \cdots, \mathbf{C}_{\mathbf{X},p})$, $\mathbf{C}_{\mathbf{X},i} = diag([\sigma_{s,i}^2 \mathbf{1}_d^T, 0])$, and the initial state $\mathbf{x}_i(0|0) = [\mathbf{0}_d^T, \hat{\mu}_i]^T$. The covariance matrix of $\mathbf{u}(t)$ is $\mathbf{Q} = diag(\mathbf{Q}_1, \cdots, \mathbf{Q}_p)$, $\mathbf{Q}_i = \mathbf{E}[\mathbf{u}_i(t)\mathbf{u}_i^H(t)] = diag([\hat{\sigma}_{v,i}^2, \mathbf{0}_{d-1}^T, \hat{\sigma}_{w,i}^2])$. The difference between the predictor in (23) and the one in [1] is in the observation vector $\mathbf{c}(t)$ in (21). The non-stationary PPS bases are used in (23), whereas [1] used single-cluster sinusoid with time-varying amplitudes.

4. SIMULATIONS

In the Monte Carlo simulations, the channels are generated using the scattering model in (2). Motivated by the fundamental physical similarity, the *quadratic phase law*, $M = 2$, in Synthetic Aperture Radar [13], is adopted in this paper. More detailed discussion on model order selection can be found in [14]. Note that there is no *true* model parameters, since the scatter model does not fit our assumed data model perfectly. The proposed predictor (23) is evaluated. In the simulations, the distances from MS to BS and scattering clusters are 100 m and 10 m respectively. The mobile is moving at a constant speed of $v = 10$ m/s towards the BS. The number of clusters p is 6. Each cluster consists of 100 scatters, and they are uniformly distributed in a circular area with a radius of $\gamma = \lambda/2$. The wavelength λ is 0.15 m. The DOAs of the cluster centers are uniform in $[-\pi, \pi)$ and $s'_i \sim \mathcal{CN}(0, 1/q_i)$. The number of samples N is 500. The length of a local interval n is 100. The prediction horizon L is 5, which corresponds to approximately $\lambda/3$. The order of the AR model of the time varying amplitude d is 1. The number of Monte Carlo simulations is 200. The Normalized Mean Square Error (NMSE), i.e.,

$$\mathbf{E}[e^2_{NSE}] = \frac{N \cdot |h(t+L) - \hat{h}(t+L)|^2}{\mathbf{h}^H \mathbf{h}}, \qquad (24)$$

is used to measure the prediction accuracy.

In the iterative estimation method proposed in Section 6, the *Simplex* method is used in (26) [15]. The initial values of the parameters of the polynomial phases in Step 3 are set to be $[\omega_o, 0]$, where $\omega_o = \arg\max_{\omega} \left|\mathcal{F}\{\mathbf{y}^b_r(t)\}\right|^2$, where $\mathcal{F}\{\cdot\}$ is the DFT operator. The previously estimated parameters are used as the initial values in Step 4. The tolerance ϵ in (34) is 0.01. For comparison, the Linear Prediction (LP) and the sinusoidal modeling based Linear Minimum Mean Square Error (LMMSE) predictor, (i.e. eq. (2) and eq. (29) in [6]), are tested using the same data. The order of LP and LMMSE prediction is the same as the number of PPS components and is fixed.

The simulation results are presented in Figure 1. It can be seen that the new predictor based on PPS modeling outperforms LP. The performance of the LMMSE method is much worse than the others, and is therefore not included in the figure. At high SNR's, the performance of the new predictor decreases slowly with the increase of SNR. This is due to both the model errors and the parameter estimation errors from the suboptimal iterative procedure.

5. CONCLUSIONS

A new adaptive channel predictor based on non-stationary PPS modeling is proposed. An iterative parameter estimation using the NILS criterion is proposed to mitigate the influence of the time-varying amplitude and reduce the computation complexity. The time-varying amplitudes are estimated using the Kalman filter. The new predictor outperforms the classical LP and the stationary sinusoidal model based LMMSE channel predictors in simulations.

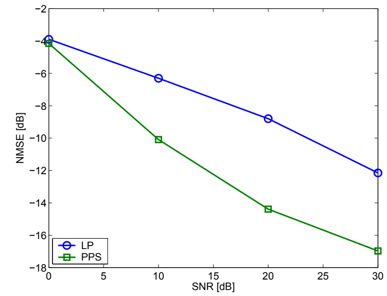

Fig. 1. Performance evaluation. The number of scattering cluster $p = 6$, the radius of the clusters is $\lambda/2$, the number of scatters in single cluster is 100, the number of channel samples $N = 500$, the mobile velocity $v = 10$ m/s, the distances from MS to the center of gravity of the clusters and the BS are 10 m and 100 m respectively.

6. APPENDIX: ITERATIVE PPS PARAMETER ESTIMATE

An iterative estimate of PPS parameters using NILS is proposed as follows:

1. Set $q = 0$, and let the initial residual signal be
$$\mathbf{y}_r(k) = \mathbf{y}(k), \quad k = t, t-1, \cdots, t-N+n. \qquad (25)$$
2. Let $q = q + 1$.
3. Estimate parameters associated with a new component in the residual signal $\mathbf{y}_r(k)$, i.e., $\boldsymbol{\theta}_q$, using
$$\hat{\boldsymbol{\theta}}_q = \arg\max_{\boldsymbol{\theta}_q} \sum_{k=t}^{t-N+n} \|\mathbf{a}^H_{q,k}\mathbf{y}_r(k)\|^2. \qquad (26)$$
The estimate of the instantaneous amplitude $\hat{s}^b_q(k)$ and channel $\hat{\mathbf{h}}^b_q(k)$ are given by
$$\hat{s}^b_q(k) = (\mathbf{a}^H_{q,k}\mathbf{a}_{q,k})^{-1}\mathbf{a}^H_{q,k}\mathbf{y}_r(k), \qquad (27)$$
$$\hat{\mathbf{h}}^b_q(k) = [\hat{h}^b_q(k), \cdots, \hat{h}^b_q(k-n+1)]^T, \qquad (28)$$
$$\hat{h}^b_q(k) = \hat{s}^b_q(k)e^{j\hat{\phi}^b_q(k)}, \qquad (29)$$
where ^{b} indicates *before* parameter estimate update in Step 5), and $\hat{\phi}^b_q(k)$ is calculated from (4) using $\hat{\boldsymbol{\theta}}_q$.
4. Let
$$\hat{\mathbf{H}}^b = \left[\sum_{i=1}^{q}\hat{\mathbf{h}}^b_i(t), \cdots, \sum_{i=1}^{q}\hat{\mathbf{h}}^b_i(t-N+n)\right], \qquad (30)$$

and initialize the signal estimates *after* update by

$$\hat{\mathbf{h}}_i^a(k) = \hat{\mathbf{h}}_i^b(k), \tag{31}$$

for $i = 1, \cdots, q$ and $k = t, t-1, \cdots, t-N+n$, where ^{a} indicates *after* parameter estimate update in Step 5).

5. Update $\hat{\mathbf{h}}_l^a(k)$ using (26)-(29) by setting

$$\mathbf{y}_r(k) \quad = \quad \mathbf{y}(k) - \sum_{\substack{i=1 \\ i \neq l}}^{q} \hat{\mathbf{h}}_i^a(k), \tag{32}$$

for $l = 1, \cdots, q$ and $k = t, t-1, \cdots, t-N+n$.

6. Put

$$\hat{\mathbf{H}}^a = \left[\sum_{i=1}^{q} \hat{\mathbf{h}}_i^a(t), \cdots, \sum_{i=1}^{q} \hat{\mathbf{h}}_i^a(t-N+n) \right]. \tag{33}$$

If

$$\frac{\|\hat{\mathbf{H}}^a - \hat{\mathbf{H}}^b\|_F^2}{\|\hat{\mathbf{H}}^b\|_F^2} > \epsilon, \tag{34}$$

where $\| \cdot \|_F^2$ is the Frobenius norm and ϵ is a user-selected tolerance, let $\hat{\mathbf{H}}^b = \hat{\mathbf{H}}^a$ and goto Step 5).

7. If $q < p$, let

$$\mathbf{y}_r(k) \quad = \quad \mathbf{y}(k) - \sum_{i=1}^{q} \hat{\mathbf{h}}_i^a(k), \tag{35}$$

for $k = t, t-1, \cdots, t-N+n$, and

$$\hat{\mathbf{h}}_i^b(k) \quad = \quad \hat{\mathbf{h}}_i^a(k), \tag{36}$$

for $i = 1, \cdots, q$, and $k = t, t-1, \cdots, t-N+n$, and goto Step 2) searching for a new component.

Note that the nonlinear optimization problem in (26) can be solved using grid search and/or any classical local search method, such as, *Simplex* in [15].

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