PERFORMANCE ANALYSIS OF A RAYLEIGH-PRODUCT MIMO CHANNEL WITH RECEIVER CORRELATION AND COCHANNEL INTERFERENCE

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ABSTRACT

This paper studies the analytical performance of a Rayleigh-product multiple-input multiple-output (RP-MIMO) channel in the presence of spatial correlation and cochannel interferences (CCI) at the receiver side. With optimal combining, we first derive a closed-form expression for the cumulative distribution function (C.D.F.) of the maximum eigenvalue of the resultant channel matrix. This result permits us to analyze the outage probability of the RP-MIMO channel. In addition, the ergodic capacity of a keyhole MIMO channel with cochannel interferences is also derived from these new statistical results.

Index Terms— Cochannel interference, Information rates, MIMO systems.

1. INTRODUCTION

Pioneering work by Foschini *et al.* [1] has revealed the substantial increase in capacity and performance improvement by employing multiple antennas at both the transmitter and the receiver [well known as multiple-input multiple-output (MIMO) antenna] over a single-antenna system. Nevertheless, in practice, the performance of MIMO systems is destined to be degraded due to the presence of cochannel interference (CCI) and various physical channel phenomena such as keyhole, spatial correlation and so on. For this reason, a great deal of research has been carried out on controlling and managing CCI in a wireless network and characterizing the impacts of the channel impairments on the capacity.

It has been very well known that multiple antennas at the receiver can effectively suppress CCI by maximizing the received signal-to-interference plus noise ratio (SINR) using a minimum-mean-square-error (MMSE) decorrelator [2]. This optimal combining has also been extended to MIMO antenna systems [3] and extensively investigated for Rayleigh [4–6] and Rician fading channels [7].

While most of the prior studies were based on the rich-scattering assumption that renders a full-rank MIMO channel matrix, it has, however, been evident from recent field measurements, e.g., [8], that the channel may in fact exhibits a reduced-rank behavior due to the lack of scatterers around the transmitter and receiver terminals. A more general channel model which embraces this aspect of channel phenomenon, as well as allowing for correlation amongst the antennas and scatterers, has recently been proposed in [9]. The model is referred to as the double-scattering (DS) model. Despite its generality and practical significance, very few analytical results are available and existing results tended to focus on the information-theoretic behavior in ergodic capacity [10, 11] and the diversity-multiplexing tradeoff [12, 13]. The performance of a DS channel with CCI is not at all understood.

This paper's aim is to study the statistical properties of a DS-MIMO channel with CCI using optimum combining. To proceed, in particular, we consider a Rayleigh-product (RP) MIMO channel which is a special case of a DS channel with an identity scattering matrix, and assume, as in [4], that the channel is "interference-limited" with equal-power interferers. Spatial correlation at the receive antennas is also considered. Our main contribution is that we derive a closed-form expression for the cumulative distribution function (C.D.F.) of the maximum eigenvalue of a RP-MIMO channel matrix, which permits to analyze the outage probability performance of the RP-MIMO channel using optimum combining in the presence of CCI and the ergodic capacity of an interference-limited keyhole channel.

2. SYSTEM MODEL

Consider a MIMO system equipped with N_t antennas at the transmitter and N_r antennas at the receiver. We assume there exist $N_{\mathcal{I}}$ cochannel interferers with $N_{\mathcal{I}} \geq N_r$. The received signals in vector form can be written as

$$\mathbf{y} = \sqrt{P_0}\tilde{\mathbf{H}}\mathbf{t}s_0 + \sum_{n=1}^{N_{\mathcal{I}}} \sqrt{P_n}\tilde{\mathbf{h}}_n s_n + \boldsymbol{\eta} \qquad (1)$$

where s_0 denotes the transmitted symbol of the desired user, $\{s_n\}_{n\geq 1}$ denotes the signals transmitted from the interferers, with $\mathrm{E}[|s_n|^2] = 1\ \forall n$, so that $\{P_n\}$ are the transmitted power of the users, $\mathbf{t} \in \mathbb{C}^{N_t}$ denotes the transmit beamforming vector of the desired user with $\|\mathbf{t}\| = 1$, $\tilde{\mathbf{h}}_n = \mathbf{\Sigma}^{\frac{1}{2}}\mathbf{h}_n \in \mathbb{C}^{N_r}$

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is the complex channel vector of the nth interferer with $\mathbf{\Sigma} \in \mathbb{C}^{N_r \times N_r}$ representing the correlation matrix at the receiver,

$$\tilde{\mathbf{H}} = \frac{1}{\sqrt{N_s}} \mathbf{\Sigma}^{\frac{1}{2}} \mathbf{H}_1 \mathbf{H}_2 \tag{2}$$

in which $\mathbf{H}_1 \in \mathbb{C}^{N_r \times N_s}$ and $\mathbf{H}_2 \in \mathbb{C}^{N_s \times N_t}$ are random matrices, is the Rayleigh-product MIMO channel between the desired transmitter and receiver with N_s being the number of scatterers in the environment, and $\boldsymbol{\eta}$ is the complex Gaussian noise vector with independent elements following $\mathcal{CN}(0, \sigma^2)$. In addition, The random entries of $\mathbf{H}_1, \mathbf{H}_2$ and $\{\mathbf{h}_n\}$ are independent and follow $\mathcal{CN}(0, 1)$. Now, define $\mathbf{H}_{\mathcal{I}} \triangleq [\mathbf{h}_1 \cdots \mathbf{h}_n]$ so that $\tilde{\mathbf{H}}_{\mathcal{I}} \triangleq \mathbf{\Sigma}^{\frac{1}{2}} \mathbf{H}_{\mathcal{I}}$. Then, (1) can be re-expressed as

$$\mathbf{y} = \sqrt{P_0} \tilde{\mathbf{H}} \mathbf{t} s_0 + \tilde{\mathbf{H}}_{\mathcal{I}} \mathbf{P}_{\mathcal{I}}^{\frac{1}{2}} \mathbf{s}_{\mathcal{I}} + \boldsymbol{\eta} \tag{3}$$

where $\mathbf{P}_{\mathcal{I}} = \mathtt{diag}\{P_1, \ldots, P_n\}$, and $\mathbf{s}_{\mathcal{I}} = [s_1 \cdots s_{N_{\mathcal{I}}}]^T$.

At the receiver, the signals, $\mathbf{y}$, are combined to produce the estimate of the transmitted symbol of the desired user by multiplying a vector $\mathbf{r}^\dagger$. In [3], it is shown that an MMSE decorrelator is optimal in maximizing the output SINR, i.e.,

$$\mathbf{r} = \mu \left(\tilde{\mathbf{H}}_{\mathcal{I}} \mathbf{P}_{\mathcal{I}} \tilde{\mathbf{H}}_{\mathcal{I}}^\dagger + \sigma^2 \mathbf{I} \right)^{-1} \tilde{\mathbf{H}} \mathbf{t} \tag{4}$$

where μ is an arbitrary constant which does not contribute to the SINR. The optimal $\mathbf{t}$ is also found to be the principal eigenvector of the channel, $\mathbf{\Xi} \triangleq \tilde{\mathbf{H}}^\dagger (\tilde{\mathbf{H}}_{\mathcal{I}} \mathbf{P}_{\mathcal{I}} \tilde{\mathbf{H}}_{\mathcal{I}}^\dagger + \sigma^2 \mathbf{I})^{-1} \tilde{\mathbf{H}}$, which as a result gives $\mathsf{SINR}_{\max} = P_0 \xi_{\max}$, where $\xi_{\max}$ denotes the maximum eigenvalue of $\mathbf{\Xi}$.

The statistical property of $\mathsf{SINR}_{\max}$ is important in characterizing the performance of the channel, which can be studied through the statistical distribution of $\xi_{\max}$. To do so, we consider an interference-limited environment where the noise is neglected, and that the interferers have an equal-power, i.e., $P_{\mathcal{I}} \triangleq P_1 = \cdots = P_{N_{\mathcal{I}}}$. As a consequence, we have

$$\mathsf{SIR}_{\max} \triangleq \rho_{\max} = \frac{P_0}{P_{\mathcal{I}}} \lambda_{\max} \tag{5}$$

where $\lambda_{\max}$ is the largest eigenvalue of

$$\mathbf{F} \triangleq \frac{1}{N_s} \mathbf{H}_2^\dagger \mathbf{H}_1^\dagger \left(\mathbf{H}_{\mathcal{I}} \mathbf{H}_{\mathcal{I}}^\dagger \right)^{-1} \mathbf{H}_1 \mathbf{H}_2. \tag{6}$$

3. C.D.F. OF λ_{MAX}

In this section, we present the exact expression for the C.D.F. of $\lambda_{\max}$ in closed-form, which will be useful in deriving and analyzing the outage probability and the ergodic capacity of the channel, as we shall do in Section 4.

3.1. C.D.F. of $\lambda_{\max}$ when $N_t \le N_r$

Before presenting our main results, we find it useful to define the following notations: $m = \min(N_r, N_s)$, $n = \max(N_r, N_s)$, $p = \max(0, m - N_t)$, and $q = \max(m, N_t)$.

Theorem 1 *If $N_t \le N_r$, the C.D.F. of $\lambda_{\max}$ is given by*

$$\mathcal{F}_{\lambda_{\max}}(x) = C_1 \det \mathbf{\Delta}(x) \tag{7}$$

where C_1 is given by (8) (see top of the next page) and $\mathbf{\Delta}(x)$ is an $m \times m$ matrix function of x with

$$[\mathbf{\Delta}(x)]_{i,j} = \begin{cases} (-1)^{m-N_t-i} V_1 & \text{for } i \le p, \\ V_2 - R(x) & \text{for } i > p, \end{cases} \tag{9}$$

in which V_1, V_2 and $R(x)$ are defined in (10) (see top of the next page) where $B(\cdot,\cdot)$ is the beta function [14, (8.380.1)], and $U(\cdot,\cdot,\cdot)$ is the confluent hypergeometric function of second kind [14, (9.210.2)].

Proof: We sketch the proof when $N_s \le N_t \le N_r$ and the proofs for the cases $N_t \le N_s \le N_r$ and $N_t \le N_r \le N_s$ will be similar. Define $\mathbf{W} \triangleq \mathbf{H}_1^\dagger (\mathbf{H}_{\mathcal{I}} \mathbf{H}_{\mathcal{I}}^\dagger)^{-1} \mathbf{H}_1$ with eigenvalues $0 < \phi_1 < \cdots < \phi_{N_s} < \infty$. The maximum eigenvalue of $\mathbf{F}$ conditioned on $\mathbf{W}$ is given in [15] as

$$\mathcal{F}_{\lambda_{\max}}(x|\mathbf{W}) = \frac{\det \mathbf{\Psi}(x)}{\det \mathbf{V} \prod_{i=1}^{N_s} \Gamma(N_t - i + 1)}, \tag{11}$$

where $\mathbf{V}$ is an $N_s \times N_s$ matrix and

$$\det \mathbf{V} = \left(\prod_{i=1}^{N_s} \phi_i^{N_t} \right) \prod_{1 \le l \le k \le N_s} \left(\frac{1}{\phi_k} - \frac{1}{\phi_l} \right). \tag{12}$$

Also, $\mathbf{\Psi}(x)$ is an $N_s \times N_s$ matrix with entries

$$[\mathbf{\Psi}(x)]_{i,j} = \phi_j^{N_t - i + 1} \gamma \left(N_t - i + 1, \frac{x N_s}{\phi_j} \right), \tag{13}$$

where $\gamma(.,.)$ is the lower incomplete gamma function. To obtain the unconditional C.D.F. of $\lambda_{\max}$, we average (11) over the joint P.D.F. of $\phi_1, \ldots, \phi_{N_s}$ which is given by [16]

$$f(\mathbf{W}) = C_1 \prod_{j=1}^{N_s} \phi_j^{N_r - N_s} (1 + \phi_j)^{-N_{\mathcal{I}} - N_s} \prod_{1 \le l \le k \le N_s} (\phi_l - \phi_k)^2. \tag{14}$$

As a result, we get

$$\mathcal{F}_{\lambda_{\max}}(x) = \frac{C_1}{\prod_{j=1}^{N_s} \Gamma(N_t - i + 1)} L(x) \tag{15}$$

where

$$\begin{aligned} L(x) &= \int_{\mathbf{W}} \det \mathbf{\Psi}(x) \det \left[\phi_j^{N_s - i} \right] \\ &\quad \prod_{j=1}^{N_s} \phi_j^{N_r - N_t - 1} (1 + \phi_j)^{-N_{\mathcal{I}} - N_s} d\mathbf{W} \\ &= \det \left[\int_0^\infty g_{i,j}(t) dt \right] \end{aligned} \tag{16}$$

where $g_{i,j}(t) = t^{N_s + N_r - i - j} (1+t)^{-N_{\mathcal{I}} - N_s} \gamma(N_t - i + 1, \frac{x N_s}{t})$. Finally, integrating using [14, (3.383.5)] and after some mathematical manipulation, the desired result can be proved. $\square$

$$C_1 = \frac{\prod_{i=1}^{m}(-1)^{pN_t}\Gamma(N_{\mathcal{I}}+N_s-i+1)}{\prod_{i=1}^{m}\Gamma(N_{\mathcal{I}}-N_r+m-i+1)\Gamma(m-i+1)\Gamma(n-i+1)} \tag{8}$$

$$\begin{cases} V_1 \triangleq B(n+i-j, N_{\mathcal{I}}-N_r+m-i+j), \\ V_2 \triangleq B(m+n+p-i-j+1, N_{\mathcal{I}}-q-N_r+i+j-1), \\ R(x) = \sum_{k=0}^{q-i} \frac{(xN_s)^k}{\Gamma(k+1)} \times \Gamma(N_{\mathcal{I}}-N_r-p+i+j+k-1) \\ \qquad \times U(N_{\mathcal{I}}-N_r-p+i+j+k-1, i+j+k-p-n-m, xN_s), \end{cases} \tag{10}$$

3.2. C.D.F. of $\lambda_{\max}$ when $N_t \geq N_r$

Theorem 2 *If $N_t \geq N_s \geq N_r$ or $N_s \geq N_t \geq N_r$, the C.D.F. of $\lambda_{\max}$ is given by*

$$\mathcal{F}_{\lambda_{\max}}(x) = \frac{C_2 \det \hat{\mathbf{\Delta}}(x)}{\prod_{i=1}^{N_t}\Gamma(N_t-i+1)} \tag{17}$$

where

$$C_2 = \frac{\prod_{j=1}^{N_r}\Gamma(N_{\mathcal{I}}+N_s-j+1)}{\prod_{j=1}^{N_r}\Gamma(N_{\mathcal{I}}-j+1)\Gamma(N_s-j+1)\Gamma(N_r-j+1)} \tag{18}$$

and $\hat{\mathbf{\Delta}}(x)$ is an $N_r \times N_r$ matrix with entries

$$\left[\hat{\mathbf{\Delta}}(x)\right]_{i,j} = \Gamma(N_t-i+1)(V_4 - U_1(x)) \tag{19}$$

where V_3 and $U_1(x)$ are defined in (20) (see top of the next page). Note that surprisingly, the result for the case $N_t \geq N_r \geq N_s$ fits into the case $N_t \leq N_r$ (given in Theorem 1).

4. PERFORMANCE ANALYSIS

4.1. Outage probability

Outage probability is an important performance measure, it is defined by the probability of system failing to achieve an acceptable signal-to-interference ratio (SIR), γ_{th}. That is,

$$\mathcal{P}_{\text{out}} = \Pr(\rho_{\max} \leq \rho_{\text{th}}) = \mathcal{F}_{\lambda_{\max}}\left(\frac{P_{\mathcal{I}}\rho_{\text{th}}}{P_0}\right). \tag{21}$$

Fig. 1 provides numerical results $\mathcal{P}_{\text{out}}$ versus $P_0/(P_{\mathcal{I}}\rho_{\text{th}})$ for various number of scatterers N_s when $N_t = 3$, $N_r = 5$, and $N_{\mathcal{I}} = 6$. Results indicate that the number of scatterers has significant impacts on the system performance and the less the number of scatterers, the higher the outage probability. Fig. 2 validates the accuracy of the analytical results even when there are non-negligible noise and unequal-power CCIs.

4.2. Ergodic capacity of keyhole channels

In this subsection, we present the ergodic capacity of keyhole channels, which is an important special case of RP-MIMO channels when $N_s = 1$, with CCI in the following theorem.

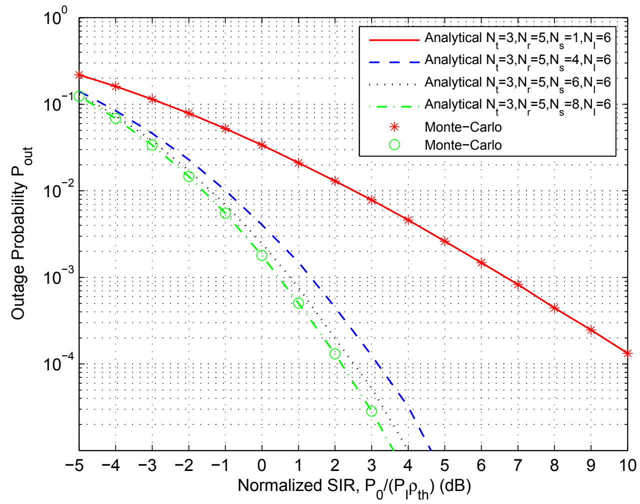

Fig. 1: Outage probability of RP-MIMO channels for various $P_0/(P_{\mathcal{I}}\rho_{\text{th}})$ without noise.

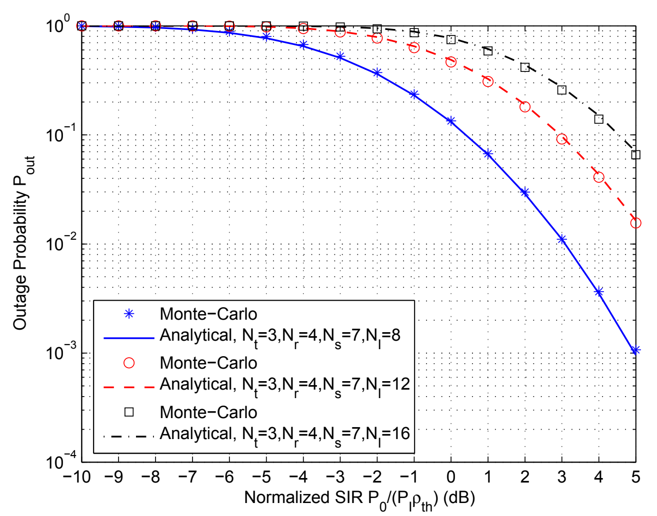

Fig. 2: Outage probability of RP-MIMO channels for various $P_0/(P_{\mathcal{I}}\rho_{\text{th}})$ with $P_I/\sigma^2 = 3$ (dB) and CCIs with unequal power. For the CCIs, we have set $\sum_{i=1}^{N_{\mathcal{I}}} P_i = N_{\mathcal{I}}$ and $P_i = \frac{2i-1}{N_{\mathcal{I}}}$, for $i = 1, \ldots, N_{\mathcal{I}}$.

$$
\begin{cases} V_3 \triangleq B(N_s+N_r-i-j+1, N_{\mathcal{I}}-N_r+i+j-1), \\ U_1(x) = \sum_{k=0}^{N_t-i} \frac{(xN_s)^k}{\Gamma(k+1)} \Gamma(N_{\mathcal{I}}-N_r+i+j+k-1) U(N_{\mathcal{I}}-N_r+i+j+k-1, i+j+k-N_r-N_s, xN_s), \end{cases} \tag{20}
$$

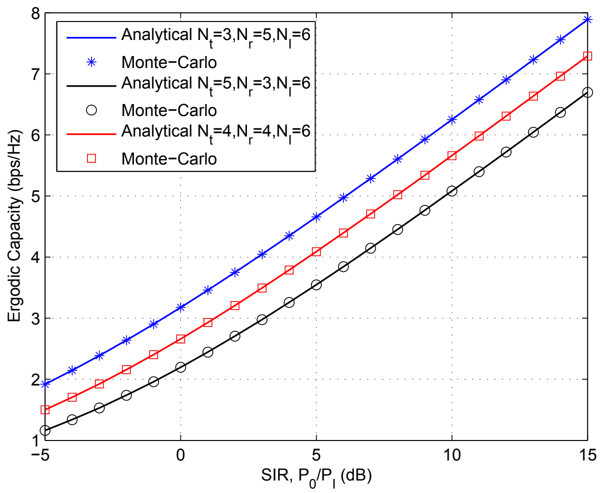

Fig. 3: Ergodic capacity of keyhole channels for various SIR $= P_0/P_{\mathcal{I}}$.

Theorem 3 *The ergodic capacity of keyhole channels with CCI under optimum combining is*

$$
C = \frac{(\log_2 e) G_{4,3}^{2,4}\left(\frac{P_0}{P_{\mathcal{I}}}\middle|\begin{smallmatrix}1,1,1-N_t,1-N_r\\1,1+N_{\mathcal{I}}-N_r,0\end{smallmatrix}\right)}{\Gamma(N_{\mathcal{I}}-N_r+1)\Gamma(N_t)\Gamma(N_r)} \tag{22}
$$

where $G_{p,q}^{m,n}(\cdots)$ is the Meijer G-function in [14, (9.301)].

From (22), we observe that when $N_r = N_{\mathcal{I}}$, ergodic capacity is a symmetric function of N_t and N_r. Therefore, they have the same impact on the ergodic capcity. The correctness of the analytical expression (22) is verified by Fig 3.

5. CONCLUSION

This paper has examined the analytical performance of MIMO optimum combining systems in RP channels (i.e., a double-scattering channel with identity correlation and scattering matrices) with CCIs. We derived the system outage probability based on the C.D.F. of the maximum eigenvalue of the resultant channel. Using our new results, we also obtained the ergodic capacity of keyhole-channels in closed-form.

6. REFERENCES

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