IMPACT OF CHANNEL ESTIMATION ERROR ON PERFORMANCE OF ADAPTIVE MIMO SYSTEMS

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ABSTRACT

Adaptive modulation scheme has been widely used in multiple-input multiple-output (MIMO) systems to enhance the spectral efficiency while maintaining the bit-error-rate (BER) under a target level. In this work, we investigate the performance of adaptive modulation in the presence of imperfect channel estimation and the impact of estimation noise on the spectral efficiency. The closed-form expressions for the average spectral efficiency are derived. Two MIMO schemes are considered, i.e., orthogonal space-time block codes (OSTBC) and spatial multiplexing with zero-forcing receiver (SM-ZF), and a low complexity method to enable the transmitter to switch between OSTBC and SM-ZF is utilized to achieve higher spectral efficiency than adaptive OSTBC and adaptive SM-ZF.

***Index Terms*—** MIMO systems, Communication system operations and management, Communication system performance

1. INTRODUCTION

Channel-adaptive transmission that adjusts the transmitter to enhance the spectral efficiency [1] has been receiving a great deal of attention in the past few years. The idea of channel-adaptive transmission is to feed in the transmitter with Channel State Information (CSI) so that the transmitter can adjust itself accordingly to ensure a robust and spectrum efficient transmission. The application of channel-adaptive scheme in multiple-input multiple-output (MIMO) systems has exhibited substantial benefits on spectral efficiency and link reliability.

Adaptive modulation and transmit power allocation were suggested as means to improve the spectral efficiency in [1]. On the basis of previous work, a new kind of adaptive scheme was suggested in [2, 3] that switches between different MIMO schemes to achieve even higher spectral efficiency. In contrast to the work of [2], a new approach was proposed in [4] which selects the right MIMO scheme based on the average spectral efficiency. The average spectral efficiencies of the candidate schemes are evaluated by using closed-form expressions, the so called discrete-rate spectral efficiency (DRSE). However, all the discussions in [4] are based on the assumption that perfect CSI is available at the receiver. In this paper, we investigate the system performance in case of channel estimation noise. The influence of imperfect channel estimation is taken into account in the DRSE to evaluate the average spectral efficiency. To be able to switch between OSTBC and SM-ZF, we evaluate the performance of the candidate schemes by using the DRSE and the one with a higher spectral efficiency is selected. The scheme selection information as well as the modulation order are conveyed to the transmitter through a noise-free feedback channel without any feedback delay. The channel environment is assumed to be independent and identically distributed (i.i.d.) Rayleigh fading.

The remainder of this paper is organized as follows. In section 2, we present the candidate MIMO schemes and channel estimation model. In section 3, the SNR thresholds for adaptive modulation are derived. In section 4, the DRSEs are derived for OSTBC and SM-ZF with channel estimation nosie. In section 5, the low complexity switching method is described. In section 6, numerical results from computer simulations are presented for different antenna setups and estimation noise. Conclusions are drawn in section 7.

2. SYSTEM MODEL

We use the following notations in this literature: $\mathbf{X}$ for matrix, $\mathbf{x}$ for vector, $[\mathbf{X}]_{ij}$ for one element in $\mathbf{X}$ and $\mathbf{X}^*$ for the Hermitian of $\mathbf{X}$. We consider a point to point MIMO system with N_t transmit antennas and N_r receive antennas. The wireless channel is assumed to be i.i.d. Rayleigh fading, denoted as $\mathbf{H}$. Without loss of generality, the entries of $\mathbf{H}$ are assumed to be independent Gaussian random variables, i.e., $[\mathbf{H}]_{ij} \sim \mathcal{CN}(0,1)$. The discrete-time baseband equivalent signal model can be written as:

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{x}(n) + \mathbf{z}(n) \tag{1}$$

where $\mathbf{x}(n)$ is an $N_t \times 1$ transmitted vector and $\mathbf{y}(n)$ is an $N_r \times 1$ received vector at time n. $\mathbf{z}(n)$ is the vector of additive white Gaussian noise with covariance $\sigma_n^2\mathbf{I}$. The channel is assumed to be quasi-static (channel coefficients remain constant during one time interval, and changes independently in the next interval). In case of channel estimation error, the estimated channel, denoted as $\hat{\mathbf{H}}$, can be modeled as [5]:

$$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{\Xi}, \tag{2}$$

where Ξ represents the estimation error with $[\Xi]_{ij} \sim \mathcal{CN}(0, \sigma_\epsilon^2)$. From (2), the original channel can be rewritten as:

$$\mathbf{H} = \rho\hat{\mathbf{H}} + \sqrt{1-\rho}\mathbf{W}, \tag{3}$$

where $\rho = \dfrac{1}{1+\sigma_\epsilon^2}$ and $\mathbf{W}$ represents i.i.d. Gaussian white noise with each element $[\mathbf{W}]_{ij} \sim \mathcal{CN}(0,1)$. This is the so-called ZF channel estimation.

2.1. OSTBC with imperfect channel estimation

With perfect channel estimation, the effective SNR after maximal Ratio Combining (MRC) is:

$$\gamma = \frac{\gamma_0 \|\mathbf{H}\|_F^2}{RN_t}, \tag{4}$$

where R is the rate of STBC[1], $\|\cdot\|_F$ is the Frobenius norm, $\gamma_0 = P_T/\sigma_n^2$ is the average SNR and P_T is the total transmit power.

In the presence of estimation error, the actual effective SNR can be derived for $N_t = 2, 3, 4$ as:

$$\hat{\gamma} = \frac{\rho^2 \gamma_0 \|\hat{\mathbf{H}}\|_F^2}{(1-\rho)\gamma_0 RN_t + RN_t} \tag{5}$$

2.2. SM-ZF with imperfect channel estimation

For SM-ZF, the effective SNR of the ith data stream is provided as [6]:

$$\gamma_i = \frac{\gamma_0}{[(\mathbf{H}^*\mathbf{H})^{-1}]_{ii} N_t}, \qquad 1 \le i \le N_t \tag{6}$$

when perfect CSI is available. In case of estimation noise, the effective SNR for the ith data stream is provided as

$$\hat{\gamma}_i = \frac{\rho^2\gamma_0}{[(1-\rho)\gamma_0 N_t + N_t]\left[(\hat{\mathbf{H}}^*\hat{\mathbf{H}})^{-1}\right]_{ii}} \tag{7}$$

3. SNR THRESHOLDS FOR ADAPTIVE MODULATION

Adaptive modulation scheme [1] is widely used to improve the spectral efficiency while keeping the BER under a target level. In adaptive MQAM modulation systems, the modulation order is dynamically determined provided the effective SNR and SNR thresholds, where the SNR thresholds act as SNR boundaries that differentiate neighboring modulation orders. The relationship between BER and SNR (γ) under a certain modulation order (M_k) for QAM modulation can be approximated as:

$$P_b(\gamma, M_k) \approx c_k e^{-a_k\gamma}, \tag{8}$$

where a_k and c_k can be found numerically by a curve-fitting method. The SNR thresholds T_k for switching across the modulation orders can be solved from (8):

$$T_k = \frac{1}{a_k}\ln\frac{c_k}{\bar{P}_b}, \tag{9}$$

where $\bar{P}_b$ is the target bit error rate for an uncoded system. The modulation order is dynamically decided by comparing the effective SNR, e.g. (5), to the SNR thresholds (9).

4. DISCRETE-RATE SPECTRAL EFFICIENCY

The DRSE[2] suggested in [4] is a closed-form expression for evaluating the average spectral efficiency, which is expressed as:

$$\mathrm{DR} = \sum_i \left(\sum_{k=1}^{K} d_k \cdot \int_{T_k}^{T_{k+1}} p(\gamma_i)d\gamma_i\right), \tag{10}$$

where γ_i is the effective SNR on the ith sub-channel. $d_k = \log_2 M_k$ is the number of bits assigned to the sub-channel when the effective SNR falls in the interval: $[T_k, T_{k+1})$. There are in total K modulation orders for adaptation. $p(\gamma_i)$ is the probability density function (p.d.f.) of γ_i. In the presence of estimation error, the DRSEs, $\widehat{\mathrm{DR}}$, can be calculated by replacing γ_i with $\hat{\gamma}_i$ in (10).

4.1. DRSE of OSTBC with imperfect CSIR

In an i.i.d. Rayleigh fading channel, the distribution of the effective SNR depends on the p.d.f. of $\|\mathbf{H}_w\|_F^2$, which is Chi-square distributed with the degree of freedom[3] (d.o.f.) equal to N_tN_r. The p.d.f. of effective SNR can be derived from [6]:

$$p_{\hat{\gamma}}(\hat{\gamma}) = u^{N_tN_r}\frac{\hat{\gamma}^{N_tN_r-1}}{\Gamma(N_tN_r)}e^{-u\hat{\gamma}}, \tag{11}$$

where $u = \dfrac{RN_t[1+(1-\rho)\gamma_0]}{\rho\gamma_0}$ and $\Gamma(n) = (n-1)!$. The DRSE of OSTBC can be derived by inserting (11) into (10):

$$\begin{aligned}\widehat{\mathrm{DR}}_{ostbc}(\gamma_0,\rho) &= \frac{1}{R}\sum_{k=1}^{K} d_k \int_{T_k}^{T_{k+1}} p_{\hat{\gamma}}(\hat{\gamma})d\hat{\gamma} \\ &= \frac{1/R}{\Gamma(N_tN_r)}\sum_{k=1}^{K}\Delta d_k \Gamma\left(N_tN_r, uT_k\right)\end{aligned} \tag{12}$$

[1] $R = 1$ for $N_t = 2$ (Alamouti code) and $R < 1$ otherwise

[2] DRSE means the rate of every transmission is limited to a certain set of integers as the modulation order is. This is in contrast to the continuous rate suggested in [1].

[3] The number of independent real-valued random variables.

where $\Delta d_k = d_k - d_{k-1}$ and $d_0 = 0$. $\Gamma(a, x)$ is the incomplete Gamma function:

$$\Gamma(a, x) = \int_x^{\infty} t^{a-1} e^{-t} dt. \tag{13}$$

4.2. DRSE of SM-ZF with imperfect CSIR

The p.d.f. of the effective SNR on the ith stream of the ZF receiver is given in [7]. Taking into account the estimation noise, the p.d.f. can be derived as:

$$p_{\hat{\gamma}_i}(\hat{\gamma}_i) = \frac{v e^{-v\hat{\gamma}_i}}{\Gamma(N_r - N_t + 1)} (v\hat{\gamma}_i)^{N_r - N_t} \tag{14}$$

where $v = N_t[1+(1-\rho)\gamma_0]/\rho\gamma_0$. The DRSE can be obtained by inserting (14) into (10):

$$\widehat{\mathrm{DR}}_{zf}(\gamma_0, \rho) = \sum_{i=1}^{N_t} \sum_{k=1}^{K} \Delta d_k \frac{\Gamma(N_r - N_t + 1, vT_k)}{\Gamma(N_r - N_t + 1)}. \tag{15}$$

5. A LOW COMPLEXITY ADAPTATION SCHEME

In a conventional adaptive system [1], the modulation order at the transmitter is updated periodically depending on the time-varying channel. The complexity of the system mainly lies in the link adaptation, where the effective SNR is estimated and the suitable constellation size is decided and fed back to the transmitter, i.e., 3 bits (there are in total 5 choices for the modulation order: $0, 2, 4, 16, 64$) for OSTBC and $3N_t$ bits for SM-ZF. In this paper, we add a very limited overhead to the conventional adaptive modulation system to obtain dramatic improvement in the spectral efficiency.

The new adaptation scheme to switch between OSTBC and SM-ZF is based on the DRSE, which can be used to evaluate the average spectral efficiencies of the two schemes and the one with a higher spectral efficiency is selected. The selection is based on the statistical channel information– the average SNR. Since the statistical information does not change as time elapses, the selection of the optimal scheme only needs to be done once. The following operation to determine the modulation order is exactly the same as the conventional adaptive system once the scheme is determined. Therefore, only one bit (the choice for the scheme) is fed back via link adaptation to determine the scheme, which adds limited complexity to the existing adaptive modulation systems with a single MIMO scheme. Additionally, some extra computations are needed for determination of the scheme and extra space is needed to store all MIMO schemes, but only the one that is selected is activated.

6. SIMULATION AND NUMERICAL RESULTS

We consider adaptive MQAM modulation with the constellation size $M = \{0, 2, 4, 16, 64\}$. The target BER is set to 0.1%. The average spectral efficiencies are obtained from the software-defined radio workbench (SDR-WB) based on $10,000$ channel realizations.

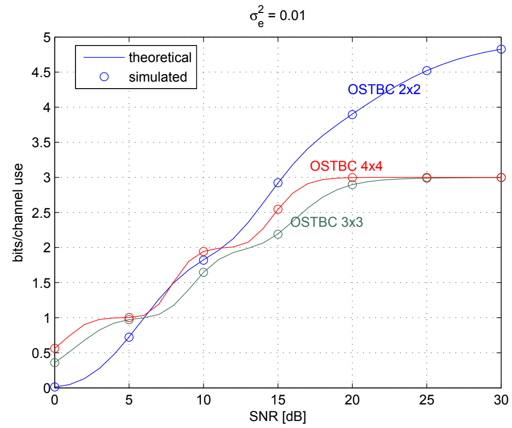

Fig. 1. *Spectral efficiencies achieved by OSTBC in an i.i.d. Rayleigh fading channel, $\sigma_\epsilon^2 = 0.01$*

6.1. DRSEs of OSTBC and SM-ZF

The spectral efficiencies of OSTBC in i.i.d. Rayleigh fading channel are plotted in Fig. 1, where both the empirical results from computer simulation and the theoretical results from (12) are presented. In case of 3×3 and $4{\times}4$ MIMO systems, we use the orthogonal codes with $R = 1/2$. This explains that the spectral efficiencies level out at 3 *bits/channel use* when $N_t = 3, 4$. As can be seen from the figure, the theoretical results match very well with the simulation results for all cases. Another observation is that the DRSEs start to converge as SNR further increases. This is due to that the interference caused by imperfect estimation becomes the dominant part in effective SNR as channel noise become negligible.

Similarly, the performance of SM-ZF in i.i.d. Rayleigh fading channel is shown in Fig. 2, where the DRSE of SM-ZF is provided in (15).

6.2. Impact of the estimation noise

Fig. 3 presents the difference of spectral efficiencies with and without estimation noise. It is noted that the estimation noise affected the spectral efficiency more in high SNR regions. This is obvious from (5) and (7) that the estimation noise part $(1 - \rho)$ is multiplied by γ_0, where a high SNR can make the trivial estimation noise become evident. Furthermore, the degradation of the spectral efficiencies is shown in Fig. 4. It is observed that the OSTBC is more robust to estimation noise than SM-ZF. This is because ZF project the received signal, consisting of two parts–the interference and the de-

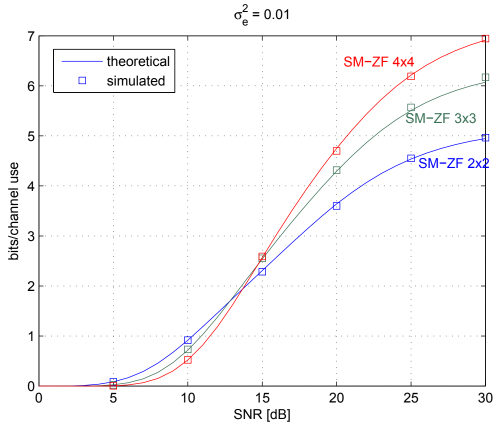

Fig. 2. *Spectral efficiencies achieved by SM-ZF in an i.i.d. Rayleigh fading channel,* $\sigma_\epsilon^2 = 0.01$

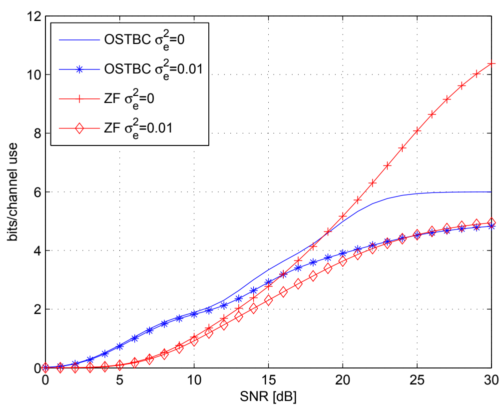

Fig. 3. *Performance of OSTBC and SM-ZF with (*$\sigma_\epsilon^2 = 0.01$*) and without channel estimation noise*

sired signal, onto a subspace that is orthogonal to the interference while keeping as much as possible of the desired signal. If there exists channel estimation noise, it means a deviation in direction and causes high degradation in the resulted SNR. The detection of OSTBC, on the other hand, combines the received signal constructively and does not amplify the noise.

7. CONCLUSIONS

In this paper, we considered an adaptive MQAM modulation system in the presence of channel estimation noise. Closed-form expressions of the average spectral efficiencies, termed as DRSE, are derived for OSTBC and SM-ZF with N_t transmit antennas and N_r receive antennas in i.i.d. Rayleigh fading channel with arbitrary estimation noise.

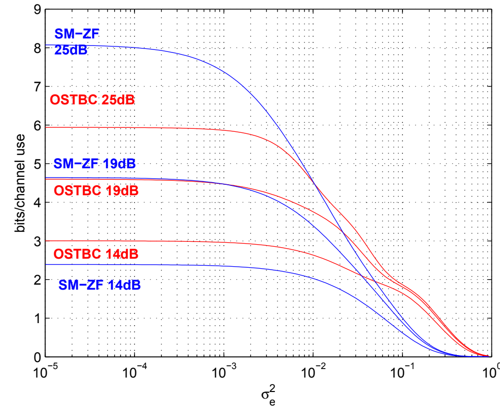

Fig. 4. *Performance degradation by having channel estimation noise in 2×2 systems*

Acknowledgment

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