AN APPROXIMATE EIGENMODE DECOMPOSITION FOR DOUBLY-SELECTIVE WIRELESS CHANNELS

*A. Barbieri**
Dep. of Inf. Eng., Univ. of Parma
Parma, ITALY

G. Caire, and U. Mitra
Dep. of Electr. Eng., Univ. of Southern California
Los Angeles, CA

ABSTRACT

We consider block-based transmissions over doubly-selective wireless channels. It is well-known that Orthogonal Frequency Division Multiplexing decomposes linear time-invariant channels into a set of parallel (non-interfering) channels via a Fourier basis. In contrast, for time-varying channels, inter-channel interference occurs and may have a dramatic impact on the performance. Following a mean square error criterion, we propose an approximate eigenmode decomposition for linear time-varying channels, defined directly in the discrete-time domain. We show the effectiveness of the proposed waveform design by evaluating the mutual information (with Gaussian inputs) for doubly-selective terrestrial Rayleigh wireless channels.

Index Terms— Dispersive channels, time-varying channels, interchannel interference, pulse shaping methods, Rayleigh channels.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is an effective technique to decompose a frequency selective channel into a set of independent and inter-symbol interference (ISI)-free sub-channels [1]. On the other hand, it is well known that in the presence of a time-selective channel, OFDM may fail to achieve its goal. This stems from the fact that time variations destroy the orthogonality between the carriers, thus introducing inter-channel interference (ICI) [2]. In this case, data detection at the receiver cannot be carried out independently symbol-by-symbol, and more complicated equalization techniques must be considered [2, 3].

An alternative approach to combat ICI, sometimes referred in the literature as *pulseshape* OFDM, consists of designing the transmitter and receiver signal sets, with the aim of minimizing the interference between the sub-channels (also denoted as channel *modes*), for a given channel model. A non-exhaustive list of references pursuing this direction includes [4, 5, 6, 7, 8]. Conceptually, the idea is to replace the rectangular pulse-shaping filters used by standard OFDM with suitable filters with better time concentration. Hence, pulse-shape OFDM consists of optimizing the pulse shape at the transmitter and the corresponding filter at the receiver, such as to increase the robustness of OFDM to time-variations (hence reducing the ICI). Following this intuitive reasoning, in [7] the use of Prolate Spheroidal Wave Functions (PSWF) was proposed. Following the idea of [9], where a Gabor expansion was proposed for source coding of nonstationary signals, along with a window functions optimization method, in [4] a WH function prototype was considered, the prototype pulse and its dual (denoted as analysis and synthesis pulses) were optimized by means of an iterative approximated method, for underspread wide-sense stationary with uncorrelated scattering (WSSUS) channels. One of the most notable work in the field of pulse-shape OFDM is [6], where the authors derived a WH orthogonal basis expansion and, through sphere-packing considerations, they also derived an optimized time-frequency sampling lattice (different from the uniform grid used by classical OFDM) that depends on the ratio between the maximal delay and Doppler spread of the considered channel.

In this paper we follow an analogous approach and we look for an *approximate* eigenmode decomposition for doubly-selective channels: we look for (the discrete-time representation of) sets of transmitter and receiver signals that are *approximately* biorthogonal [4], in a mean squared error (MSE) sense, for a given channel model. We consider WSSUS channel statistics for convenience, although this hypothesis is not required and the proposed algorithm can be applied to non-WSSUS channels as well. The design of these sets of signals capitalize on the second-order channel statistical characterization (*i.e.*, the scattering function) that is assumed known. We remark that the knowledge of the scattering function at the transmitter is an assumption widely accepted in the literature (see, e.g., [6]). Furthermore, we constrain the transmitter and receiver signals to form orthogonal sets. It is worth noting that, despite the presence of a residual interference among the sub-channels (due to the fact that an exact eigenmode decomposition does not in general exist, since two different channel realizations may not even share any eigenmode at all [10]), we still consider a symbol-by-symbol receiver, as is typically done with OFDM.

The novelty of the proposed method is that the signals set is not restricted to be a WH set, in contrast to [4, 6, 7], hence the degrees of freedom for the set optimization turn out to be dramatically increased. Moreover, we clearly state an optimality criterion, and optimize the signals sets accordingly. Eventually, the performance of the optimized sets, with respect to classical ZP-OFDM, are evaluated in terms of channel mutual information.

This paper is organized as follows. In Section 2 the system model and the problem formulation are given, while in Section 3 a waveform optimization algorithm is proposed. The performance results are shown in Section 4, and in Section 5 some conclusions and directions for future works are presented.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a block-based transmission, where each block has a duration of $N \cdot T$ seconds, being N the number of symbols and T the symbol period. We assume that two consecutive blocks are separated by a guard time larger than the channel impulse response (CIR) length, in order to avoid any inter-block interference (IBI). Hence, the block index will be omitted. By denoting with $\{x_n\}, n = 0, \ldots, N-1$, the transmitted samples (belonging to a complex constellation), in the continuous-time domain the transmitted signal corresponding to a single block yields

$$x(t) = \sqrt{\frac{M}{T}} \sum_{n=0}^{N-1} x_n \sum_{j=0}^{NM-1} u_n(j) \sqcap \left(\frac{t}{T/M} - j \right) \qquad (1)$$

*The author performed the work while at the University of Southern California, Los Angeles.

where $\sqcap(x) = 1$ if $x \in [0,1]$ and 0 otherwise, and M is the oversampling factor (a rectangular sample-and-hold has been assumed). For $j = 0, \ldots, NM-1$, $u_n(j)$ are the samples of the n-th transmitter filter. Due to the transmit power constraint, it must be $\sum_{j=0}^{NM-1} |u_n(j)|^2 = 1$ for each n. The received continuous-time signal is given by

$$y(t) = \int h(t,\tau)x(t-\tau)d\tau + w(t) \qquad (2)$$

where $h(t,\tau)$ is the time-variant CIR, assumed zero-mean Gaussian, and $w(t)$ is a complex white Gaussian process with power spectral density N_0. We assume that $h(t,\tau) = 0$ if $\tau < 0$ or $\tau > LT$, being L a positive integer. Hence the useful signal at the receiver spans $(N+L)T$ seconds. The received signal is filtered by a set of optimized filters with expressions

$$v_k(t) = \sqrt{\frac{M}{T}} \sum_{i=0}^{(N+L)M-1} v_k^*(i) \sqcap \left(\frac{t}{T/M} - i\right)$$

where again for each k, $\sum_i |v_k(i)|^2 = 1$. The output of the k-th receiver filter is given by

$$y_k = \sqrt{\frac{M}{T}} \sum_{i=0}^{(N+L)M-1} v_k^*(i) \int y(t) \sqcap \left(\frac{t}{T/M} - i\right) dt\,. \qquad (3)$$

By joining (1), (2), and (3) we obtain

$$y_k = \sum_{n=0}^{N-1} x_n \sum_{i,j} v_k^*(i) u_n(j) H_{i,j} + w_k \qquad (4)$$

where

$$H_{i,j} = \frac{M}{T} \iint h(t,\tau) \sqcap \left(\frac{t-\tau}{T/M} - j\right) \sqcap \left(\frac{t}{T/M} - i\right) dt d\tau$$

$$w_k = \sqrt{\frac{M}{T}} \sum_i v_k^*(i) \int w(t) \sqcap \left(\frac{t}{T/M} - i\right) dt\,.$$

By collecting the N transmitted symbols in a column vector $\mathbf{x}$, and the N received samples (given by eq. (4)) in a column vector $\mathbf{y}$, (4) can be written as

$$\mathbf{y} = \underbrace{V^H H U}_{\Gamma} \mathbf{x} + \mathbf{w} \qquad (5)$$

where U is NM-by-N and V is $(N+L)M$-by-N. The noise covariance matrix is given by $E\{\mathbf{w}\mathbf{w}^H\} = N_0 V^H V$.

Our aim is to design the receiver matrix V and the transmitter shaping matrix U that *approximately* diagonalize the overall matrix Γ, and such that U is unitary[1], *i.e.* $U^H U = I_N$, and the noise components are independent and identically distributed, *i.e.* $V^H V = I_N$. The first constraint ensures that the same energy is allocated to each eigenmode (power allocation can be carried out separately).

If this diagonalization can be carried out exactly, the overall block-based channel is transformed in a set of N parallel and independent Gaussian sub-channels, and a symbol-by-symbol receiver is optimal in the Maximum-Likelihood (ML) sense. On the other hand, since exact diagonalization is not generally possible for the class of random linear time-varying (LTV) channels considered, the resulting N sub-channels are affected by mutual interference (despite the noise samples are independent among the sub-channels, due to the assumption on V). However, for the sake of simplicity of the receiver architecture, we impose that the receiver treats the output of the linear transformation V^H as the output of N virtual parallel channels, treating residual ICI as noise[2]. Under the above assumptions, the normalized average mutual information of the considered communication system and mismatched receiver, for Gaussian inputs, is given by

$$I = \frac{1}{N+L} \sum_{n=0}^{N-1} E_H \left\{\log_2 \left(1 + SINR_n\right)\right\} \qquad (6)$$

$$SINR_n = \frac{|\Gamma_{n,n}|^2}{N_0 + \sum_{k \neq n} |\Gamma_{n,k}|^2} \qquad (7)$$

where the expectation is carried out with respect to the channel matrix H and $SINR_n$ is the signal-to-interference and noise ratio of the n-th sub-channel. In the above expressions, it has been implicitly assumed that the receiver knows exactly the matrix Γ.

It is worth noting that, if the eigendecomposition (*i.e.*, diagonalization of Γ) is not exact, the mutual information (6) is upper-bounded by a constant, *i.e.*, the system is interference limited at high SNR. The symbol-by-symbol receiver processing considered here is meaningful only when the residual ICI does not dominate with respect to the Gaussian noise. Otherwise, interference cancellation techniques have to be considered.

2.1. Exact Eigendecomposition and ZP-OFDM

If the channel matrix H is exactly known at the transmitter and the receiver (e.g., if the channel is constant or extremely slowly-varying and a reliable feedback channel is available), an exact eigendecomposition is immediately found by means of the singular value decomposition (SVD) of the channel matrix. Hence, under this hypothesis, Γ in (5) can be made perfectly diagonal for any channel matrix H.

In order to evaluate the performance of the proposed decomposition approach, Zero-padded OFDM (ZP-OFDM) will be considered in the numerical results. ZP-OFDM fits exactly within the general system model (5), where the transmitter filters are the normalized and oversampled versions of the discrete Fourier transform (DFT) signals. Several equalization techniques (*i.e.*, choices of V) for ZP-OFDM were proposed in the literature: we consider the overlap-and-add (OLA), described in [11]. The greatest advantage of OFDM is that if the channel is time-invariant the overall matrix Γ of (5) is diagonal with probability one, irrespectively of the CIR. On the contrary, if the channel is time-varying, the resulting Γ matrix is in general non-diagonal (namely, ICI appears). It is worth noting that OFDM does not take advantage of the knowledge of the channel statistical characterization.

2.2. Proposed Approach

Our aim is to design U and V such that the overall random channel matrix $\Gamma = V^H H U$ is *approximately* diagonal, following a MSE criterion. The n-th received sample is

$$y_n = \Gamma_{n,n} x_n + \underbrace{\sum_{k \neq n} \Gamma_{n,k} x_k + w_n}_{z_n}$$

where $\mathbf{z} = [z_0, \ldots, z_{N-1}]^t$ is a random vector with zero mean, but with correlated components (at the receiver this correlation is not exploited since symbol-by-symbol detection is carried out).

[1]A tall rectangular matrix A is said to be unitary if its columns form an orthonormal basis, namely if $A^H A$ is an indentity matrix.

[2]Interference cancellation techniques at the receiver are possible and are matter of current investigation. However, since the focus of this paper is on waveform design, we shall not deal with interference cancellation schemes.

Our aim is to maximize the overall average signal to interference ratio[3] (SIR), given by

$$SIR = \frac{\sum_n E\{|\Gamma_{n,n}|^2\}}{\text{trace}\, E\{\Gamma^H \Gamma\} - \sum_n E\{|\Gamma_{n,n}|^2\}} \quad (8)$$

where the expectation is made with respect to the channel matrix. It can be shown that the average overall received energy trace $E\{\Gamma^H\Gamma\}$ is independent of U and *approximately* independent of V, provided that they respect the constraints. Moreover, since $f(x) = \frac{x}{\beta - x}$ is an increasing function for $x \in [0, \beta)$, we eventually obtain that the max SIR problem is equivalent to the following optimization problem

$$\max_{U,V} \sum_n E\{|\Gamma_{n,n}|^2\}, \quad \text{s.t. } U^H U = I_N,\ V^H V = I_N\,. \quad (9)$$

Some observations are in order now. First, the proposed method can be somehow interpreted as a sort of "stochastic SVD", since the aim is to find two unitary shaping matrices that approximately diagonalize a given random matrix, in a MSE sense. Second, maximizing the average SIR (8) does not automatically lead to a maximization of the mutual information (6), since the latter depends not only on the overall SIR, but also on the SIR of every single sub-channel. The SIR maximization is therefore a suitable heuristic criterion, and it is used because of its tractability. Finally, note that the constrained optimization problem (9) is not convex, thus the method proposed in the next Section could fail to yield the globally optimal solution.

3. OPTIMIZATION ALGORITHM

We propose to solve the constrained optimization problem of (9) with a "greedy" algorithm, described below. The rationale is to maximize (in a MSE sense) each diagonal element $\Gamma_{n,n}$ separately, by optimizing each pair $(\mathbf{u}_n, \mathbf{v}_n)$ (being $\mathbf{u}_n$ and $\mathbf{v}_n$ the n-th column of U and V, respectively) separately from the other pairs, only taking into account the constraints (see eq. (9)). First of all, note that the following geometric problem

$$\operatorname*{argmax}_{\mathbf{v}} \mathbf{v}^H A \mathbf{v} \qquad \text{s.t. } \|\mathbf{v}\|^2 = 1, \mathbf{v}^H V = 0 \quad (10)$$

being $\mathbf{v}$ a N-by-1 column vector, A a square Hermitian matrix, and V a N-by-C ($C \le N$) matrix of linear constraints, can be solved easily by means of a SVD and an eigendecomposition (taking the largest eigenvalue). The details are not given here due to the lack of space. The following is the algorithm proposed to solve (9):

1. For $k = 0, 1, \ldots, K-1$ [4];
2. Initialize $\mathbf{u}_k^{(0)}$ and $\mathbf{v}_k^{(0)}$. See below for details about the initialization;
3. For $i = 1, \ldots, N_{iter}$ ($N_{iter} = 100$ in our simulations);
4. Solve the following optimization problem

$$\begin{aligned}\mathbf{v}_k^{(i)} &= \operatorname*{argmax}_{\mathbf{v}} E|\mathbf{v}^H H \mathbf{u}_k^{(i-1)}|^2 \\ &= \operatorname*{argmax}_{\mathbf{v}} \mathbf{v}^H \underbrace{E\{H\mathbf{u}_k^{(i-1)}\mathbf{u}_k^{(i-1)H} H^H\}}_{A} \mathbf{v} \\ &\text{s.t. } \|\mathbf{v}\|^2 = 1,\ \mathbf{v}^H V(k) = 0\end{aligned}$$

   that is equivalent to (10). The $(N+L)M$-by-k matrix $V(k)$ collects the previous k vectors $(\mathbf{v}_0, \ldots, \mathbf{v}_{k-1})$;

5. Solve the following optimization problem

$$\begin{aligned}\mathbf{u}_k^{(i)} &= \operatorname*{argmax}_{\mathbf{u}} E|\mathbf{u}^H H^H \mathbf{v}_k^{(i)}|^2 \\ &= \operatorname*{argmax}_{\mathbf{u}} \mathbf{u}^H E\{H^H \mathbf{v}_k^{(i)} \mathbf{v}_k^{(i)H} H\}\mathbf{u} \\ &\text{s.t. } \|\mathbf{u}\|^2 = 1,\ \mathbf{u}^H U(k) = 0\,.\end{aligned}$$

6. Return to 3
7. Return to 1

Different initializations can be conceived: the simplest (that was used in the numerical simulations of Section 4) is to initialize $\mathbf{u}_k = \mathbf{v}_k$ with the k-th column of the N-th order DFT matrix, with proper oversampling. Since $\mathbf{v}_k$ has size $(N+L)M$, its last LM symbols are a copy of the first LM. It is worth noting that, since the optimization is carried out in an oversampled domain, a further constraint on the signal bandwidth is required. We have taken into account this constraint in the form of additional linear equality constraints in the optimization (not reported here due to the lack of space).

4. NUMERICAL RESULTS

We compared the performance of the proposed eigenmode decomposition with respect to ZP-OFDM (Section 2.1), in terms of average mutual information (6). U and V are evaluated once for all, and Γ is obtained as $\Gamma = V^H H U$, for each realization of H. The average with respect to H was made by means of a Monte Carlo average. We also evaluated the following rate

$$\frac{1}{N+L} E_H\{\log_2 \det(I + H^H H)\} \quad (11)$$

(for an oversampling factor $M = 1$) that can be achieved, for instance, by means of a minimum mean square error decision feedback equalizer (MMSE-DFE), that must be adaptive because the channel is time-varying, with a sufficiently large interleaving in the time-domain [12]. This rate is an upper bound for all the schemes where the input covariance is white, *i.e.*, no spectral shaping is carried out at the transmitter.

A WSSUS Rayleigh channel model was considered, whose CIR is

$$h(t,\tau) = \sum_{\ell=1}^{P} h_\ell(t)\delta(\tau - \tau_\ell)$$

where P is the number of paths[5], $h_\ell(t)$ are a set of independent zero-mean WSS Gaussian processes, characterized by

$$E\{h_\ell(t)h_{\ell'}^*(t')\} = \delta_{\ell-\ell'}\sigma_\ell^2 J_0(2\pi B_D(t-t'))$$

being B_D the Doppler bandwidth. The deterministic values σ_ℓ^2 and τ_ℓ were chosen according to the UMTS "Vehicular A" channel model [13].In our simulations we assumed a block size of $N = 64$, an oversampling factor $M = 2$, a symbol rate of 2 MHz and either $B_D = 5$ KHz or $B_D = 80$ KHz (corresponding to a normalized Doppler bandwidth of $B_D T = 2.5 \cdot 10^{-3}$ and $B_D T = 4 \cdot 10^{-2}$, respectively). This two scenarios will be denoted in the following as slow and fast fading, respectively. With the above assumptions, a guard time of $L = 6$ symbol periods must be inserted between consecutive blocks in order to avoid IBI. We remark that, in contrast to pulseshape OFDM techniques, fast Fourier transform (FFT) cannot be employed to efficiently carry out filtering at the trasmitter and the receiver. However, for the considered (very small) block length,

[3] Additive noise is not considered, since its variance does not depend on the choice of U and V, provided that $V^H V = I_N$.

[4] For generality, we allow the algorithm to look for a smaller number of eigenmodes: this reduces the spectral efficiency, but also can strongly reduce the mutual interference among the modes. Hence, K is a design parameter, that must be chosen according to the target SNR. However, in this paper we always assume $K = N$.

[5] We remark that the assumption of discrete multipath is not required by the proposed method, and a continuous delay channel can be handled in exactly the same way, provided that the scattering function is known at the transmitter.

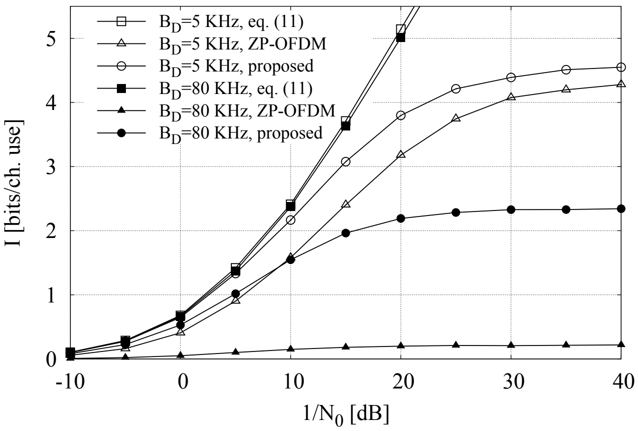

Fig. 1. Average mutual information (6) with respect to the SNR $1/N_0$.

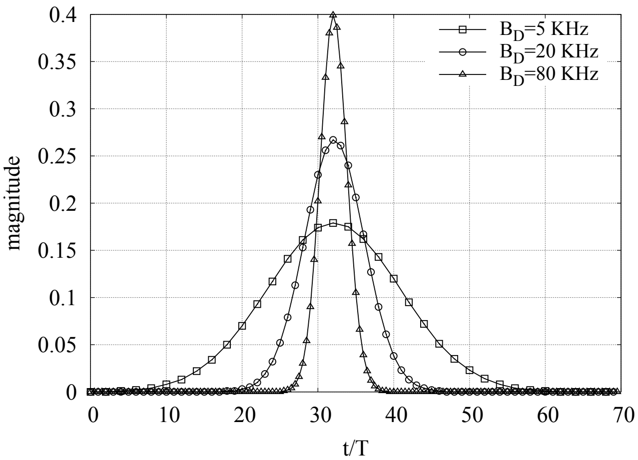

Fig. 2. The first optimized receiver filter, for three values of the Doppler bandwidth.

the difference in computational complexity between the proposed approach and those based on FFT is marginal.

In Fig. 1 the average mutual information (6), for the case of slow and fast fading, is shown as a function of the SNR (equal to $1/N_0$). First, it can be noted that the the upper bound given by eq. (11) grows to infinity for increasing E_S/N_0, while the mutual information of ZP-OFDM saturates at disappointing values (especially for fast fading), due to the large ICI caused by time-variations. In contrast, the proposed decomposition method is able to reach much higher values of asymptotic mutual information, and also gains significantly at low and medium SNRs. Clearly, also in this case the mutual information curve is upper bounded, due to the fact that an exact eigendecomposition is actually unfeasible.

In Fig. 2 the magnitude of the first optimized discrete-time receiver filter (*i.e.*, the first column of V) is shown for three values of the Doppler bandwidth. As it can be seen, with respect to standard OFDM, the receiver filters have the energy more concentrated in time. Similar to prior work on pulse shape OFDM ([4, 6]), we see that for faster fading channels the pulses become more time-localized.

5. CONCLUSIONS

We proposed a basis waveform design method for time-varying channels that is not only very general, since it applies to very general channel models (not necessarily WSSUS), but also provides a significant improvement with respect to OFDM. For large SNR, since perfect ICI elimination is not possible, interference cancellation techniques are advocated, as for example turbo-equalization schemes. A possible improvement to the proposed scheme is to optimize the signal spaces so as to obtain a banded, rather than diagonal, overall matrix Γ, as proposed in [3], and adopting an ICI mitigation algorithm at the receiver. Also, a relevant issue is how to estimate the channel coefficients at the receiver, e.g., by using a suitable grid of pilot symbols. These issues, however, are matter of current research and shall be reported elsewhere.

6. REFERENCES

[1] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, pp. 5–14, May 1990.

[2] X. Cai and G. B. Giannakis, "Bounding performance and suppressing intercarrier interference in wireless mobile OFDM," *IEEE Trans. Commun.*, vol. 51, pp. 2047–2056, Dec. 2003.

[3] S. Hwang and P. Schniter, "Efficient communication over highly spread underwater acoustic channels," in *Proc. of the ACM Int. Work. on UnderWater Net. (WUWNet)*, Sept. 2007.

[4] W. Kozek and A. F. Molisch, "Nonorthogonal pulseshapes for multicarrier communications in doubly dispersive channels," *IEEE J. Sel. Areas Comm.*, vol. 16, pp. 1579–1589, Oct. 1998.

[5] G. Matz and F. Hlawatsch, "Time-varying communication channels: fundamentals, recent developments, and open problems," in *Proc. European Signal Processing Conference (EUSIPCO)*, Florence, Italy, Sept. 2006.

[6] T. Strohmer and S. Beaver, "Optimal OFDM design for time-frequency dispersive channels," *IEEE Trans. Commun.*, vol. 51, no. 7, pp. 1111–1122, July 2003.

[7] A. Vahlin and N. Holte, "Optimal finite duration pulses for OFDM," *IEEE Trans. Commun.*, vol. 44, pp. 10–14, Jan. 1996.

[8] M. M. Hartmann G. Matz and D. Schafhuber, "Theory and design of multipulse multicarrier systems for wireless communications," in *Proc. Asilomar Conf.*, Nov. 2003, pp. 492–496.

[9] W. Kozek, "Matched generalized Gabor expansion of nonstationary processes," in *Proc. Asilomar Conf. Signals, Systems, Comp.*, Nov. 1993, vol. 1, pp. 499–503.

[10] W. Kozek and A. F. Molisch, "On the eigenstructure of underspread WSSUS channels," in *Proc. of IEEE Work. on Sig. Proc. Adv. in Wir. Comm. (SPAWC)*, Apr. 1997, pp. 325–328.

[11] B. Muquet, W. Zhengdao, G. B. Giannakis, M. de Courville, and P. Duhamel, "Cyclic prefixing or zero padding for wireless multicarrier transmissions?," *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 2136–2148, Dec. 2002.

[12] S. Das and P. Schniter, "Noncoherent communication over the doubly selective channel via successive decoding and channel re-estimation," in *Proc. Annu. Allerton Conf. Communications, Control and Computing*, Monticello, IL, 2007.

[13] B. Yang, K. B. Letaief, R. S. Cheng, and Z. Cao, "Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling," *IEEE Trans. Commun.*, vol. 49, no. 3, pp. 467–479, Mar. 2001.