CHANNEL ESTIMATION USING DPSS BASED FRAMES

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ABSTRACT

Accurate and sparse representation of a moderately fast fading channel using bases functions is achievable when both channel and bases bands align. If a mismatch exists, usually a larger number of bases functions is needed to achieve the same accuracy. In this paper, we propose a novel approach for channel estimation based on frames, which preserves sparsity and improves estimation accuracy. Members of the frame are formed by modulating and varying the bandwidth of discrete prolate spheroidal sequences (DPSS) in order to reflect various scattering scenarios. To achieve the sparsity of the proposed representation, a matching pursuit approach is employed. The estimation accuracy of the scheme is evaluated and compared with the accuracy of a Slepian basis expansion estimator based on DPSS for a variety of mobile channel parameters. The results clearly indicate that for the same number of atoms, a significantly higher estimation accuracy is achievable with the proposed scheme when compared to the DPSS estimator.

***Index Terms*—** Time-varying channels, channel estimation, discrete prolate spheroidal sequences, frames.

1. INTRODUCTION

Estimation and interpolation of a moderately fast fading Rayleigh/Rice channel is an important problem in modern communications. If the channel characteristics are known, i.e. the channel autocorrelation function, then an approach based on the Wiener filter provides the optimum solution [1]. However, such an ideal case is rare in real-life applications, and we require a more universal approach. In general, basis expansions are used in such situations and several different basis functions including Fourier bases and discrete prolate spheroidal sequences (DPSS) have been adopted for such problems [2]-[5]. Previous studies have found that accurate and sparse representations are usually obtained when both the bases and the channel under investigation occupy the same band [2]. However, when the bandwidth of the basis function is mismatched and larger than that of the signal, a larger number of bases is required to approximate the channel with the same accuracy. To resolve this particular problem, it was suggested to use a bank of bases with different bandwidths [6]. However, such a representation again ignores the fact that in some cases the band occupied by the channel is not necessarily centered around DC, but rather at some frequency different from zero. Hence, a larger number of bases is again needed for accurate and sparse representation.

A need clearly exists for some type of overcomplete, redundant bases which accounts for a variety of scenarios. Therefore, in this paper we propose an overcomplete set of bases called Modulated Discrete Prolate Spheroidal Sequences (MDPSS). Such a set of bases is also known as a frame [7], [8]. The bases within the frame are obtained by modulation and variation of the bandwidth of DPSSs in such a way as to reflect various scattering scenarios. During construction of multiple bases it is assumed that at least an upper bound of the maximum Doppler frequency is known. Furthermore, in order to obtain a sparse representation of the channel using the MDPSS frame, a matching pursuit approach is employed [9]. The proposed scheme is tested using the channel model presented in Section 2 for various scattering scenarios. The results demonstrate that the MDPSS frame provides superior estimation accuracy compared to a Slepian basis expansion DPSS approach [2].

This paper is organized as follows. In Section 2, relevant properties of a mobile channel are reviewed. Details of the MDPSS frame and its implementation are covered in Section 3. Numerical study of the proposed scheme and the results of this analysis are covered in Section 4. Conclusions are drawn in Section 5.

2. PROPERTIES OF THE SPECTRUM OF A MOBILE CHANNEL

The covariance function, $\rho(\tau)$, of a SISO frequency flat mobile communication channel and its associated power spectral density $S(\omega)$ are related to the distribution of angle of arrival (AoA) $p(\theta)$ as [10]

$$\rho(\tau) = P\int_{-\pi}^{\pi} \exp\left(j2\pi f_D\tau\,\cos\theta\right)p(\theta)d\theta, \tag{1}$$

$$S(f) = \frac{P}{\sqrt{1-(f/f_D)^2}}p\left(\arccos\frac{f}{f_D}\right) + \frac{P}{\sqrt{1-(f/f_D)^2}}p\left(\pi-\arccos\frac{f}{f_D}\right). \tag{2}$$

Here, P is the total power of the received signal, $f_D = f_0 v/c$ is the maximum Doppler shift of the carrier frequency f_0 corresponding to the velocity v of the mobile; c is the speed of light. Jakes spectrum,

$$S(f) = \frac{Pf_D}{\pi\sqrt{1-(f/f_D)^2}}, \tag{3}$$

which is a widely used model by communication researchers, corresponds to the uniform distribution $p(\theta) = 1/2\pi$ of the AoA [10]. This spectrum closely resembles a uniform spectrum for most of the frequencies, and therefore, its Karhunen-Loeve basis [1] is close to the one defined by sinc type covariance function. Thus, it seems natural that representation in terms of DPSS suggested in [2] produces very good results.

However, real-world measurements reveal that the AoA can deviate significantly from that of a uniform distribution on $[-\pi:\pi]$. Often, the received signal is represented well as a sum of signals arriving from a narrow band of angles, corresponding to individual clusters [11], [12]. In this case, the PSD of the received signal could be well approximated by a piece-constant distribution of power in frequency domain. Conversely, the covariance function could be well represented by a sum of sinc type functions in the time domain [12]. In addition, in some radio environments such as dense urban environments [13], the received signal could be dominated by multiple specular components, therefore creating a mixed spectrum of the received signal. In these conditions, the use of simple DPSS may not be optimal, since proper expansions will require a significant number of higher order DPSS with small eigenvalues which, in turn will result in ill-posed problem. Therefore, modifications to Zemen's approach in [2] are required.

3. FRAMES BASED ON DPSS

Throughout this section and the rest of the paper, it is assumed that only N discrete samples of the channel are available and that they were obtained with a sampling period T. Hence, the discrete frequency, ν, represents a continuous frequency, f, normalized with sampling period, $\nu = fT$.

3.1. Discrete Prolate Spheroidal Sequences

Given N, the kth DPSS, $v_k(n, N, W)$, for $k = 0, 1, ..., N-1$ is defined as the real solution to the system of equations [14]:

$$\sum_{m=0}^{N-1} \frac{\sin[2\pi W(n-m)]}{\pi(n-m)} v_k(m, N, W) = \lambda_k(N, W) v_k(n, N, W) \tag{4}$$

with $\lambda_k(N, W)$ being the ordered non-zero eigenvalues of (4)

$$\lambda_0(N, W) > \lambda_1(N, W), ..., \lambda_{N-1}(N, W) > 0. \tag{5}$$

The first $2NW$ eigenvalues are very close to 1 while the rest rapidly decay to zero [14]. Interestingly enough, it has been observed that these quantities are also the eigenvalues of an $N \times N$ matrix $C(m, n)$ [14], where the elements of such a matrix are

$$C(m, n) = \frac{\sin[2\pi W(n-m)]}{\pi(n-m)} \quad m, n = 0, 1, ..., N-1, \tag{6}$$

and the vector obtained by time-limiting the DPSS, $v_k(n, N, W)$, is an eigenvector of $C(m, n)$. The DPSS are doubly orthogonal, that is, they are orthogonal on the infinite set $\{-\infty, ..., \infty\}$ and orthonormal on the finite set $\{0, 1, ..., N-1\}$, that is,

$$\sum_{-\infty}^{\infty} v_i(n, N, W) v_j(n, N, W) = \lambda_i \delta_{ij} \tag{7}$$

$$\sum_{n=0}^{N-1} v_i(n, N, W) v_j(n, N, W) = \delta_{ij}, \tag{8}$$

where $i, j = 0, 1, ..., N-1$.

3.2. Modulated Discrete Prolate Spheroidal Sequences

If the DPSS are used for channel estimation, then usually accurate and sparse representations are obtained when both the DPSS and the channel under investigation occupy the same frequency band [2]. However, problems arise when the channel is centered around some frequency $|\nu_o| > 0$ and the occupied bandwidth is smaller than $2W$, as shown in Fig. 1.

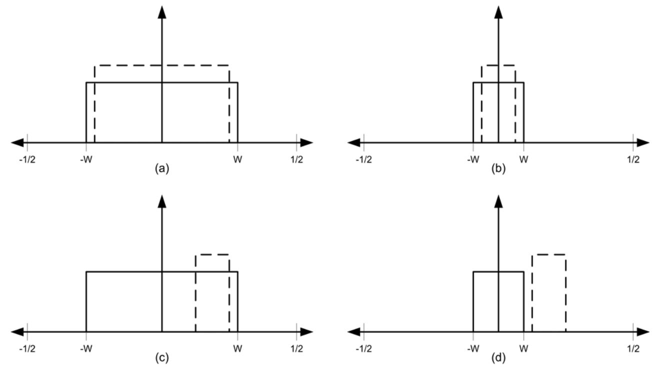

Fig. 1. Comparison of the bandwidth for a DPSS (solid line) and a channel (dashed line): (a) both have a wide bandwidth; (b) both have narrow bandwidth; (c) a DPSS has a wide bandwidth, while the channel's bandwidth is narrow and centered around $\nu_o > 0$; (d) both have narrow bandwidth, but centered at different frequencies.

In such situations, a larger number of DPSS is required to approximate the channel with the same accuracy despite the fact that such narrowband channel is more predictable than a wider band channel [15]. In order to find a better basis we consider so-called Modulated Discrete Prolate Spheroidal Sequences (MDPSS), defined as

$$M_k(N, W, \omega_m; n) = \exp(j\omega_m n) v_k(N, W; n), \tag{9}$$

where $\omega_m = 2\pi\nu_m$ is the modulating frequency. It is easy to see that MDPSS are also doubly orthogonal, obey the same equation (4) and are bandlimited to the frequency band $[-W + \nu : W + \nu]$.

The next question which needs to be answered is how to properly choose the modulation frequency ν. In the simplest case when the spectrum $S(\nu)$ of the channel is confined to a known band $[\nu_1; \nu_2]$, *i.e.*

$$S(\nu) = \begin{cases} \gg 0 & \forall \nu \in [\nu_1, \nu_2] \text{ and } |\nu_1| < |\nu_2| \\ \approx 0 & \text{elsewhere} \end{cases}, \tag{10}$$

the modulating frequency, ν_m, and the bandwidth of the DPSSs are naturally defined by

$$\nu_m = \frac{\nu_1 + \nu_2}{2} \tag{11}$$

$$W = \left| \frac{\nu_2 - \nu_1}{2} \right|, \tag{12}$$

as long as both satisfy:

$$|\nu_m| + W < \frac{1}{2}. \tag{13}$$

In practical applications the exact frequency band is known only with a certain degree of accuracy. In addition, especially in mobile applications, the channel is evolving in time. Therefore, only some relatively wide frequency band defined by the velocity of the mobile and the carrier frequency is expected to be known. In such situations, a one-band-fits-all approach may not produce a sparse and accurate approximation of the channel. To resolve this problem, it was previously suggested to use a band of bases with different widths to account for different speeds of the mobile [6]. However, such a representation once again ignores the fact that the actual channel bandwidth $2W$ could be much less than $2\nu_D$ dictated by the maximum normalized Doppler frequency $\nu_D = f_D T$.

To improve the estimator robustness, we suggest the use of multiple bases, better known as frames [8], precomputed in such a way as to reflect various scattering scenarios. In order to construct such multiple bases, we assume that a certain estimate (or rather its upper bound) of the maximum Doppler frequency ν_D is available. The first few bases in the frame are obtained using traditional DPSS with bandwidth $2\nu_D$. Additional bases can be constructed by partitioning the band $[-\nu_D;\nu_D]$ into K subbands with the boundaries of each subband given by $[\nu_k;\nu_{k+1}]$, where $0 \le k \le K-1$, $\nu_{k+1} > \nu_k$, and $\nu_0 = -\nu_D$, $\nu_{K-1} = \nu_D$. Hence, each set of MDPSS has a bandwidth equal to $\nu_{k+1} - \nu_k$ and a modulation frequency equal to $\nu_m = 0.5(\nu_k + \nu_{k+1})$. Obviously, a set of such functions again forms a basis of functions limited to the bandwidth $[-\nu_D;\nu_D]$. It is a convention in the signal processing community to call each basis function *an atom*. While particular partition is arbitrary for every level $K \ge 1$, we can chose to partition the bandwidth in equal blocks to reduce the amount of stored precomputed DPSS, or to partition according to the angular resolution of the receive antenna, *etc*, as shown in Fig. 2.

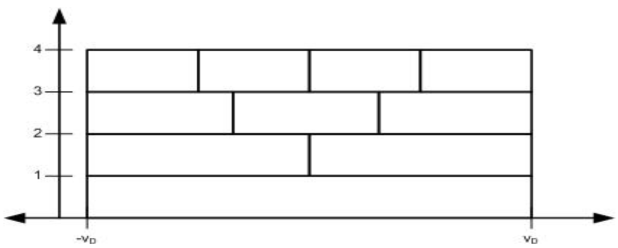

Fig. 2. Sample partition of the bandwidth for $K = 4$.

Representation in the overcomplete basis can be made sparse due to the richness of such a basis. Since the expansion into simple bases is not unique, a fast, convenient and unique projection algorithm cannot be used. Fortunately, efficient algorithms, known generically as pursuits [7], [9], can be used and they are briefly described in the next section.

3.3. Matching Pursuit with MDPSS frames

From the few approaches which can be applied for expansion in overcomplete bases, we choose the so-called matching pursuit [9]. The main feature of the algorithm is that when stopped after a few steps, it yields an approximation using only a few atoms [9]. The matching pursuit was originally introduced in the signal processing community as an algorithm that decomposes any signal into a linear expansion of waveforms that are selected from a redundant dictionary of functions [9]. It is a general, greedy, sparse function approximation scheme based on minimizing the squared error, which iteratively adds new functions (i.e. basis functions) to the linear expansion. In comparison to a basis pursuit, it significantly reduces the computational complexity, since the basis pursuit minimizes a global cost function over all bases present in the dictionary [9]. If the dictionary is orthogonal, the method works perfectly. Also, to achieve compact representation of the signal, it is necessary that the atoms are representative of the signal behaviour and that the appropriate atoms from the dictionary are chosen.

The algorithm for the matching pursuit starts with an initial approximation for the signal, $\widehat{x}$, and the residual, R:

$$\widehat{x}^{(0)} = 0 \tag{14}$$

$$R^{(0)} = x \tag{15}$$

and it builds up a sequence of sparse approximation stepwise by trying to reduce the norm of the residue, $R = \widehat{x} - x$. At stage k, it identifies the dictionary atom that best correlates with the residual and then adds to the current approximation a scalar multiple of that atom, such that

$$\widehat{x}^{(k)} = \widehat{x}^{(k-1)} + \alpha_k \phi_k \tag{16}$$

$$R^{(k)} = x - \widehat{x}^{(k)}, \tag{17}$$

where $\alpha_k = \langle R^{(k-1)}, \phi_k \rangle / \|\phi_k\|^2$. The process continues until the norm of the residual $R^{(k)}$ does not exceed required margin of error $\epsilon > 0$: $||R^{(k)}|| \le \epsilon$ [9]. In our approach, a stopping rule mandates that the number of bases, χ_B, needed for signal approximation should satisfy $\chi_B \le \lceil 2N\nu_D \rceil + 1$. Hence, a matching pursuit approximates the signal using χ_B bases as

$$x = \sum_{n=1}^{\chi_B} \langle x, \phi_n \rangle \phi_n + R^{(\chi_B)}, \tag{18}$$

where ϕ_n are χ_B bases from the dictionary with the strongest contributions.

4. NUMERICAL SIMULATION

In this section, the performance of the MDPSS estimator is compared with the Slepian basis expansion DPPS approach [2] for a certain radio environment. The channel model used in the simulations is presented in Section 2 and it is simulated using the AR approach suggested in [16]. The parameters of the simulated system are the same as in [2]: the carrier frequency is 2 GHz, the symbol rate used is 48600 $1/s$, the speed of the user is 102.5 km/h, 10 pilots per data block are used, and the data block length is $M = 256$. The number of DPSSs used in estimation is given by $\lceil 2M\nu_D \rceil + 1$. The same number of bases is used for MDPSS, while $K = 15$ subbands is used in generation of MDPSS.

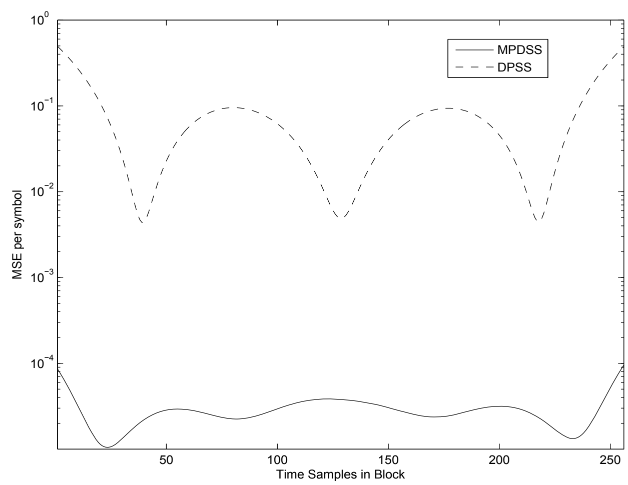

Fig. 3. Mean square error per symbol for MDPSS (solid) and DPSS (dashed) mobile channel estimators for the noise-free case.

As an introductory example, consider the estimation accuracy for the WSSUS channel with a uniform power angle profile (PAS) with central AoA $\phi_0 = 5$ degrees and spread $\Delta = 20$ degrees. We used 1000 channel realizations and Fig. 3 depicts the results for the considered channel model. The mean square errors (MSE) for both MDPSS and DPSS estimators have the highest values at the edges of the data block. However, the MSE for MDPSS estimator is several orders of magnitude lower than the value for the Slepian basis expansion estimator based on DPSS.

Next, let's examine the estimation accuracy for the WSSUS channels with uniform PAS, central AoAs $\phi_1 = 45$ and $\phi_1 = 75$, and spread $0 < \Delta \leq 2\pi/3$. Furthermore, it is assumed that the channel is noisy. Figs. 4 and 5 depict the results for $SNR = 10$ dB and $SNR = 20$ dB, respectively.

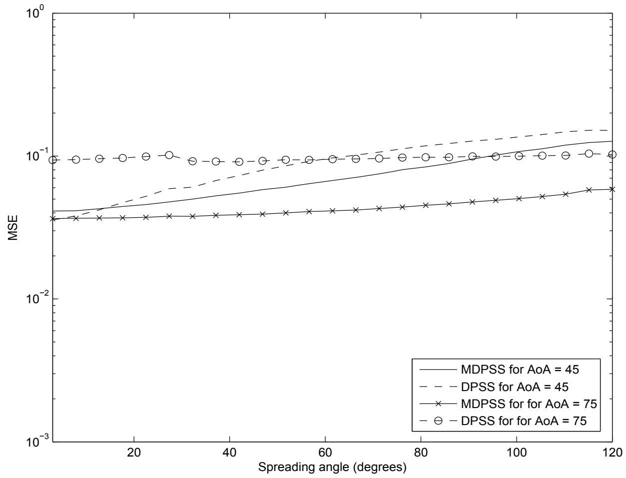

Fig. 4. Dependence of the MSE on the angular spread Δ and the mean angle of arrival for $SNR = 10$ dB.

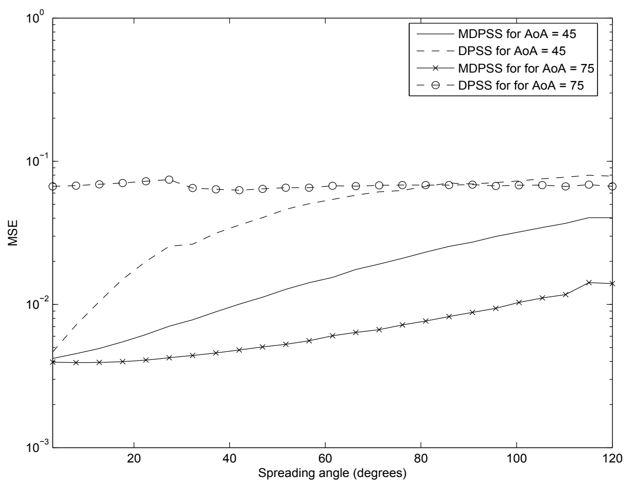

Fig. 5. Dependence of the MSE on the angular spread Δ and the mean angle of arrival for $SNR = 20$ dB.

The results clearly indicate that the MDPSS frames are a more accurate estimation tool for the assumed channel model. For the considered angles of arrival and spreading angles, the MDPSS estimator consistently provided lower MSE in comparison to the Slepian basis expansion estimator based on DPSS. The advantage of the MDPSS stems from the fact that these bases are able to describe different scattering scenarios.

5. CONCLUSIONS

In this paper, MDPSS frames are proposed for estimation of fast fading channels in order to preserve sparsity of the representation and enhance the estimation accuracy. The members of the frame were obtained by modulation and bandwidth variation of DPSSs in order to reflect various scattering scenarios. The matching pursuit approach was used to achieve a sparse representation of the channel. The proposed scheme was tested for various mobile channels, and its performance was compared with the Slepian basis expansion estimator based on DPSS. The results showed that the MDPSS method provides more accurate estimation than the DPSS scheme.

6. REFERENCES

[1] H. L. V. Trees, *Detection, Estimation, and Moduclation Theory: Part I*. New York: Wiley-Interscience, 2001.

[2] T. Zemen and C. F. Mecklenbräuker, "Time-variant channel estimation using discrete prolate spheroidal sequences," *IEEE Transactions on Signal Processing*, vol. 53, no. 9, pp. 3597–3607, Sep. 2005.

[3] D. B. Percival and A. T. Walden, *Spectral Analysis for Physical Applications: Multitaper and Conventional Univeriate Techniques*. New York: Cambridge University Press, 1993.

[4] T. Zemen, C. F. Mecklenbräuker, and B. Fleury, "Time-variant channel prediction using time-concentrated and band-limited sequences," in *Proc. of 2006 IEEE International Conference on Communications (ICC '06)*, vol. 12, Istanbul, Turkey, Jun. 11–15, 2006, pp. 5660–5665.

[5] A. Alcocer-Ochoa, V. Y. Kontorovitch, and R. Parra-Michel, "A MIMO channel simulator applying the universal basis," in *Proc. of 64th IEEE Vehicular Technology Conference (VTC-2006)*, Montreal, QC, Canada, Sep. 25–28, 2006.

[6] T. Zemen, H. Hofstetter, and G. Steinbck, "Slepian subspace projection in time and frequency for time-variant channel estimation," in *14th IST Mobile and Wireless Summit*, Dresden, Germany, Jun. 19–23, 2005.

[7] S. G. Mallat, *A Wavelet Tour of Signal Processing*, 2nd ed. San Diego: Academic Press, 1999.

[8] J. Kovačević and A. Chabira, "Life beyond bases: The advent of the frames (part I)," *IEEE Signal Processing Magazine*, vol. 24, no. 4, pp. 86–104, Jul. 2007.

[9] S. G. Mallat and Z. Zhang, "Matching pursuits with time-frequency dictionaries," *IEEE Transactions on Signal Processing*, vol. 41, no. 12, pp. 3397–3415, Dec. 1993.

[10] W. C. Jakes, Ed., *Microwave Mobile Communications*. New York: Wiley, 1974.

[11] K. E. Baddour, C. C. Squires, and T. J. Willink, "Mobile channel prediction with application to transmitter antenna selection for Alamouti systems," in *Proc. of 64th IEEE Vehicular Technology Conference (VTC-2006)*, Montreal, QC, Canada, Sep. 25–28, 2006.

[12] J. Salz and J. H. Winters, "Effect of fading correlation on adaptive arrays in digital mobile radio," *IEEE Transactions on Vehicular Technology*, vol. 43, no. 4, pp. 1049–1057, Nov. 1994.

[13] A. F. Molisch, H. Asplund, R. Heddergott, M. Steinbauer, and T. Zwick, "The COST 259 directional channel model-part i: Overview and methodology," *IEEE Transactions on Wireless Communications*, vol. 5, no. 12, pp. 3421–3433, Dec. 2006.

[14] D. Slepian, "Prolate spheroidal wave functions, Fourier analysis, and uncertainty - V: The discrete case," *The Bell System Technical Journal*, vol. 57, no. 5, pp. 1371–1430, May/Jun. 1978.

[15] J. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2001.

[16] K. E. Baddour and N. C. Beaulieu, "Autoregressive modeling for fading channel simulation," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1650 – 1662, Jul. 2005.