BAYESIAN MLSD FOR MULTIPATH RAYLEIGH FADING CHANNELS

Hossein Roufarshbaf, Jill K. Nelson

George Mason University
Department of Electrical and Computer Engineering
Fairfax, VA 22030

ABSTRACT

We propose a tree-search based Bayesian approach to blind maximum likelihood sequence detection (MLSD) of convolutionally encoded data transmitted over a multipath Rayleigh fading channel. In deriving the path metric for searching the channel-code tree, the proposed algorithm incorporates a forgetting factor matched to the time variation of the channel to generate accurate estimates of the correlation across the transmitted and received data. In addition, an augmented metric is presented to address the challenge of unknown channel order in time-varying systems. Simulation results show that the proposed algorithm can achieve significant improvement in bit error rate over competing schemes, even when channel order information is unavailable at the receiver.

***Index Terms*—** Maximum likelihood detection, equalizers, time-varying channels, multipath channels

1. INTRODUCTION

As the need for higher bandwidth and greater mobility increases, detecting data transmitted over time-varying channels is a significant challenge in modern telecommunication systems. The intersymbol interference (ISI) caused by multipath propagation, filtering, and bandwidth limitation degrades the performance of such systems. Traditionally, ISI is combatted through channel estimation using training sequences, but training wastes valuable bandwidth. Additionally, the channel may vary during the transmission of a single packet, in which case training-based techniques are not applicable.

A variety of approaches for joint channel and data estimation have been explored in previous research, e.g. [1, 2]. In contrast, we propose a technique for sequential detection of transmitted data without explicit channel estimation. Our approach builds on the tree-based Bayesian maximum likelihood sequence detector (BMLSD), which was presented in [3] for time-invariant channels. We consider its development for time-varying complex multipath Rayleigh fading channels and employ a forgetting factor whose value is optimized to match the rate of change of the channel. We also address the challenge of detection under channel order uncertainty.

This work was supported in part by a Ralph E. Powe Junior Faculty Enhancement Award from Oak Ridge Associated Universities.

2. SYSTEM MODEL

The system model under consideration is shown in Figure 1. Information bits are encoded using a rate $R = \frac{1}{r}$ convolutional encoder prior to transmission over the channel. The information bits are transmitted in blocks of length N, denoted by $\mathbf{b}_1^N$, yielding blocks of coded bits of length rN, denoted by $\mathbf{x}_1^{rN}$, at the output of the encoder. The encoded bits are transmitted over a linear time-varying (LTV) complex channel $\mathbf{h}_t = \mathbf{h}_t^c + j\mathbf{h}_t^s$ with additive white Gaussian noise (AWGN) $w_t = w_t^c + jw_t^s$, where w_t^c and w_t^s are uncorrelated white Gaussian noise processes with variance $\sigma^2 = N_0/2$. For simplicity, we consider transmission of binary phase shift keying (BPSK) encoded data. The effect of the channel can be modeled as

$$\begin{aligned} z_n = z_n^c + jz_n^s = \left[\sum_{k=0}^{L-1} h_{t,k} x_{t-kT_s} + w_t\right]_{t=nT_s} \quad (1) \\ = \left[\sum_{k=0}^{L-1} h_{t,k}^c x_{t-kT_s} + w_t^c\right]_{t=nT_s} \\ +j \left[\sum_{k=0}^{L-1} h_{t,k}^s x_{t-kT_s} + w_t^s\right]_{t=nT_s}, \end{aligned}$$

where z_n denotes the nth sample of the channel output, T_s is the sampling time period at the receiver, and $h_{t,k}$ is the kth tap of the length-L channel vector $\mathbf{h}_t = [h_{t,0} \cdots h_{t,L-1}]$ at time t.

Following Jakes' model for a Rayleigh fading channel, each channel tap is comprised of Q propagation paths as

$$\begin{aligned} h_{t,l} &= h_{t,l}^c + jh_{t,l}^s \quad (2) \\ &= E_0 \sum_{q=1}^{Q} C_{q,l} \exp(j(\omega_d t \cos\alpha_q + \phi_q)), \end{aligned}$$

where E_0 is a scaling constant, $C_{q,l}$, α_q, and ϕ_q are the random path gain, angle of incoming wave, and initial phase associated with the qth propagation path, respectively, and ω_d

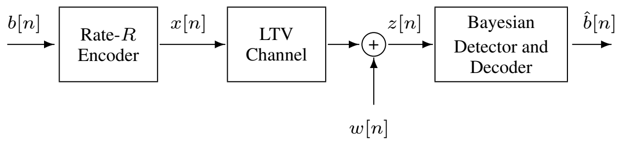

Fig. 1. System model for the proposed blind detector. Information bits are passed through a convolutional encoder, transmitted over a linear time-varying channel with unknown taps, and processed by a stack-based detection and decoding block.

is the maximum radian Doppler frequency [4]. The channel autocorrelation is given by

$$\begin{aligned} E[h^c_{t,l}h^c_{t+\tau,l}] &= E[h^s_{t,l}h^s_{t+\tau,l}] = J_0(\omega_d\tau) \\ E[h^c_{t,l}h^s_{t+\tau,l}] &= E[h^s_{t,l}h^c_{t+\tau,l}] = 0, \end{aligned} \tag{3}$$

where $J_0(.)$ denotes the zero-order Bessel function of the first kind [4].

Each block of received samples $\mathbf{z}_1^{rN} = \mathbf{z}_1^{c,rN} + j\mathbf{z}_1^{s,rN}$ serves as input to the detection and decoding block, which uses a stack-based algorithm to estimate the data sequence $\mathbf{b}_1^N$ by navigating the tree generated by the combined code and channel. We assume that the receiver has knowledge of the error-control code used at the transmitter, the variance of the AWGN, and the initial state of the encoder.

3. METRIC DERIVATION FOR TIME-VARYING CHANNELS

In the presence of channel information at the receiver, ML sequence detection can be performed efficiently via the Viterbi Algorithm. When the channel response is unknown, the likelihood no longer factors onto a trellis, and hence the Viterbi Algorithm cannot be directly applied. Instead, we employ a Bayesian detector that traverses a tree to find the most likely transmitted sequence. At each stage of the tree search algorithm, the BMLSD extends a path within the stack and computes the likelihood metric for the bit sequence associated with each extended path. The proposed Bayesian approach assumes a prior over the channel taps, and the conditional likelihood of a bit sequence $\mathbf{b}_1^n$ is averaged over the unknown quantities:

$$\begin{aligned} & p(\mathbf{b}_1^n|\mathbf{z}_1^{rN}, C) \\ &= \int_{\mathbf{h}_t} p(\mathbf{b}_1^n, \mathbf{h}_t^c, \mathbf{h}_t^s | \mathbf{z}_1^{c,rN}\mathbf{z}_1^{s,rN}, C) d\mathbf{h}_t \\ &= \int_{\mathbf{h}_t^c} p(\mathbf{b}_1^n, \mathbf{h}_t^c | \mathbf{z}_1^{c,rN}, C) d\mathbf{h}_t^c \int_{\mathbf{h}_t^s} p(\mathbf{b}_1^n, \mathbf{h}_t^s | \mathbf{z}_1^{s,rN}, C) d\mathbf{h}_t^s, \end{aligned} \tag{4}$$

where C denotes the explored tree at the current iteration. Because the real and imaginary parts of the complex channel are uncorrelated under Jakes' model, we can write the overall likelihood as the product of the metrics for each component.

If the channel taps are assumed to be drawn from a zero-mean Gaussian prior with an identity covariance matrix, the metric for a real time-invariant channel can be written as

$$\begin{aligned} & \int_{\mathbf{h}^c} p(\mathbf{b}_1^n, \mathbf{h}^c | \mathbf{z}_1^{rN}, C) d\mathbf{h}^c \\ \propto \quad & P(\mathbf{b}_1^n) \int_{\mathbf{h}^c} p(\mathbf{z}_1^{rN}|\mathbf{b}_1^n, \mathbf{h}^c, C) p(\mathbf{h}^c) d\mathbf{h}^c \\ \approx \quad & \frac{\sigma^L_{h^c}}{2^n} \left(\frac{(\sigma^2+1)^{\frac{r(n-N)}{2}}}{\sigma^{rn}} \right) \left| \frac{R^{rn}_{xx}}{\sigma^2} + \frac{I}{\sigma^2_h} \right|^{-1/2} \times \\ & \exp\left\{ -\frac{R^{rn}_{z^c z^c}[0]}{2\sigma^2} + \frac{1}{2\sigma^4} \mathbf{r}^{rnT}_{z^c x} \left(\frac{R^{rn}_{xx}}{\sigma^2} + \frac{I}{\sigma^2_h} \right)^{-1} \mathbf{r}^{rn}_{z^c x} \right\} \times \\ & \exp\left\{ -\frac{1}{2(\sigma^2+1)} \sum_{i=rn+1}^{rN} z_i^{c2} \right\}, \end{aligned} \tag{5}$$

where $R^k_{z^c z^c}[0] = \sum_{i=1}^k z_i^{c2}$, $\mathbf{r}^k_{z^c x} = \sum_{i=1}^k z_i^c \mathbf{x}^i_{i-L+1}$, and $R^k_{xx} = \sum_{i=1}^k (\mathbf{x}^i_{i-L+1})(\mathbf{x}^i_{i-L+1})^T$ [3]. The metric for the imaginary component can be computed similarly.

When the channel response varies with time, equation (5) is no longer valid, since the ensemble estimates of $R_{z^c z^c}$, $\mathbf{r}_{z^c x}$ and R_{xx} are not equal to time average estimates due to non-stationarity. In order to allow for a time-varying channel, we incorporate a forgetting factor λ that allows the estimates of $R_{z^c z^c}$, $\mathbf{r}_{z^c x}$ and R_{xx} to vary with the channel:

$$\begin{aligned} \hat{R}^k_{z^c z^c}[0] &= \frac{1-\lambda}{1-\lambda^{k+1}} \sum_{i=1}^k \lambda^{k-i} z_i^{c2} \\ \hat{\mathbf{r}}^k_{z^c x} &= \frac{1-\lambda}{1-\lambda^{k+1}} \sum_{i=1}^k \lambda^{k-i} z_i^c \mathbf{x}^i_{i-L+1} \\ \hat{R}^k_{xx} &= \frac{1-\lambda}{1-\lambda^{k+1}} \sum_{i=1}^k \lambda^{k-i} \left(\mathbf{x}^i_{i-L+1}\right)\left(\mathbf{x}^i_{i-L+1}\right)^T, \end{aligned} \tag{6}$$

where $(1-\lambda)/(1-\lambda^{k+1})$ is a scaling factor over k iterations.

3.1. Choosing the Forgetting Factor

To generate reliable estimates of the auto-correlations and cross-correlations that appear in the path metric, it is natural to choose a value for λ that reflects the rate at which the channel is changing with time. Note that the first exponential term (fourth line) in (5) is equivalent to the squared difference between the true channel output and the estimated output when a least squares estimate of the channel is employed. Hence, viewing the Bayesian detector as performing implicit channel tracking, we can define the mean square error (MSE) of the estimated channel output as

$$\epsilon = \hat{R}^{rn}_{z^c z^c}[0] - \frac{1}{2\sigma^2} \hat{\mathbf{r}}^{rnT}_{z^c x} \left(\frac{\hat{R}^{rn}_{xx}}{\sigma^2} + \frac{I}{\sigma^2_h} \right)^{-1} \hat{\mathbf{r}}^{rn}_{z^c x}. \tag{7}$$

When the channel is perfectly known, the MSE is equal to the noise variance, σ^2. When the channel is both unknown and time varying, however, additional MSE is introduced via estimation and lag error through the auto- and cross- correlation expressions. Our goal is to choose λ such that the total MSE is minimized for the time-varying channel model of interest.

It has been shown in [5] that, for the recursive least squares (RLS) algorithm tracking a time-varying system, the MSE components due to estimation error and lag can be approximated as

$$\epsilon_{est} \approx \frac{1-\lambda}{1+\lambda}\sigma^2, \quad \text{and} \tag{8}$$

$$\epsilon_{lag} = \frac{\sigma_v^2}{2(1-\lambda)} L\sigma_x^2.$$

Leveraging the RLS channel estimation implicit in the BMLSD metric, we employ the approach presented in [5] to select a value for λ. MSE due to lag is computed based on the assumption that the time-varying channel follows a first-order Markov model, e.g.

$$\mathbf{h}_{nT_s} = \mathbf{h}_{(n-1)T_s} + \mathbf{v}_n, \tag{9}$$

where $\mathbf{v}_n$ is a zero mean Gaussian random vector with diagonal covariance matrix $\sigma_v^2 I$. The total excess MSE is then given by

$$\epsilon_{tot} = \frac{1-\lambda}{1+\lambda} L\sigma^2 + \frac{1}{2(1-\lambda)} L\sigma_x^2\sigma_v^2. \tag{10}$$

Taking the derivative with respect to λ and setting it to zero yields the MSE-minimizing forgetting factor:

$$\lambda^* = \frac{1 - \left(\frac{{\sigma_x}^2{\sigma_v}^2}{4\sigma^2}\right)^{\frac{1}{2}}}{1 + \left(\frac{{\sigma_x}^2{\sigma_v}^2}{4\sigma^2}\right)^{\frac{1}{2}}}. \tag{11}$$

For the channel model described in Section 2, σ_v^2 is easily calculated from (3) and (9) as

$$\sigma_v^2 = 2J_0(0) - 2J_0(\omega_d). \tag{12}$$

3.2. Addressing Unknown Channel Order

When the communication channel is unknown and time varying, the effective length will likely also be unknown to the receiver. In order to overcome uncertainty in channel order, we evaluate the path metric (4) across a range of channel lengths. Typically, some statistical information about the nature of the channel is available, and hence we have an estimate of the maximum channel duration L_{max}. At each stage of the BMLSD tree search and for each path in the stack, we employ a metric given by

$$m_B(\mathbf{b}_1^n) = \max_{l=1,\ldots,L_{max}} P(\mathbf{b}_1^n | \mathbf{z}_1^{rN}, C, l). \tag{13}$$

Empirical results indicate that the proposed max approach achieves faster and better performance than marginalizing the channel order, e.g. $P(\mathbf{b}_1^n|\mathbf{z}_1^{rN}, C) = \sum_{l=1}^{L_{max}} P(\mathbf{b}_1^n, l|\mathbf{z}_1^{rN}, C)$.

Because the path metric can be separated into real and imaginary components, channel order may be estimated differently for each part, which is particularly beneficial when only one of the components enters a deep fade.

4. SIMULATION RESULTS

To evaluate the performance of the proposed detector, we have simulated a time-varying channel using Jakes' reference model with three independent channel taps, $Q = 100$ propagation paths, a maximum Doppler shift of $f_D = 40$Hz, a sampling frequency of $T_s = 10\mu$sec, and a carrier frequency of 900MHz. For each information block, an independent time-varying channel is generated. The channel taps are weighted according to $[0.407 \;\; 0.815 \;\; 0.407]$, and the average channel energy is normalized to unity for each block. A rate $R = \frac{1}{2}$ convolutional encoder with generator matrix $G(x) = [x+1 \;\; x^2+x+1]$ is employed prior to transmission over the channel.

Figure 2 shows the performance of the proposed algorithm as a function of the forgetting factor λ when SNR is held constant at 7 dB. The performance of the BMLSD approach is compared to that of MLSD (Viterbi) with a known channel response. The MSE-minimizing forgetting factor based on the channel parameters considered can be calculated using (11) as $\lambda^* = 0.992$. The simulation results show that, when the time-varying channel taps do not enter a deep fade, the forgetting factor that minimizes bit error rate (BER) matches well with that predicted to minimize MSE. Empirical performance in this case is robust to some change in the forgetting factor; choosing λ as low as approximately 0.97 has little effect on BER. When the channel taps may enter a deep fade, the simulation results reveal that the value of λ for which minimum BER is achieved is somewhat smaller than that analytically computed for minimizing MSE, perhaps indicating faster channel variation than the MSE-minimizing analysis predicts. Since a forgetting factor of $\lambda = 0.97$ achieves strong performance in both good and deep fade conditions, we have fixed λ at this value for the remaining simulations.

Figure 3 shows the simulated performance of the proposed BMLSD approach with known channel order for block sizes of $N = 20$ and $N = 50$ bits. In addition, the performance of the BMLSD approach when channel order is unknown (and the metric is calculated according to (13)) is plotted for $N = 50$ bits. The maximum channel order is assumed to be $L_{max} = 5$. The performance of the BMLSD for $N = 20$ is nearly identical to that for $N = 50$, indicating the attractiveness of the algorithm for applications with very small packet sizes. Note that increasing the block size beyond 50 bits does not improve the performance of the algorithm due to the memory limit (approximated by $1/(1-\lambda)$ [5]) imposed by the forgetting factor.

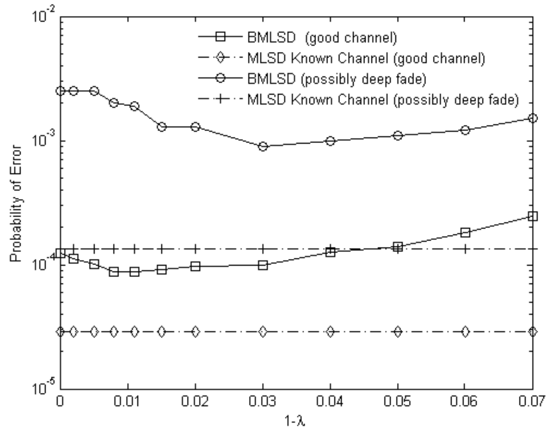

Fig. 2. BMLSD performance as a function of the forgetting factor λ. SNR is fixed at 7 dB, and blocks of length $N = 50$ are transmitted.

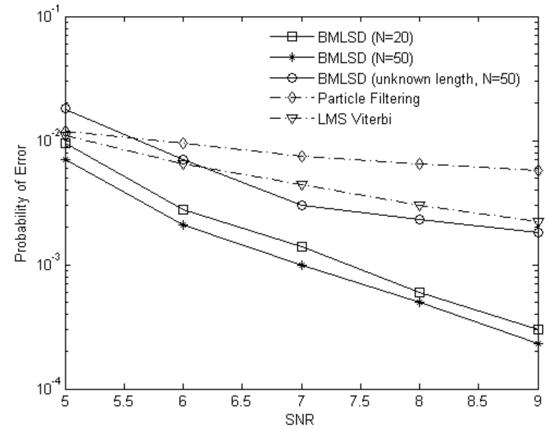

Fig. 3. Performance of the BMLSD, particle filtering, and LMS-Viterbi Algorithm as a function of SNR. BMLSD performance is considered for varying block size, as well as with and without knowledge of the channel order.

For comparison, Figure 3 also shows the performance of stochastic particle filtering (SPF) based joint blind equalization and decoding [6] with 500 particles, as well as the LMS-based Viterbi Algorithm [1] (performed across both the code and channel) retaining 16 paths to each state. Blocks of length $N = 50$ were transmitted, and the channel length was assumed known for both algorithms. Simulation results reveal that the proposed BMLSD approach achieves significantly lower BER than both the LMS-Viterbi and SPF algorithms.

The complexity of the algorithms considered is a function of both the number of paths explored and the complexity of the path metric update. The LMS-Viterbi metric update is the least complex as it requires no matrix inversion. The metric updates of the BMLSD and SPF algorithms are similar to each other in complexity. The SPF algorithm must update the metric for all particles (paths) at each iteration, however, while the BMLSD algorithm updates only one path. Note that the number of paths explored in the BMLSD tree-search is a random quantity. By setting the BMLSD stack size to the number of particles in the SPF approach, we guarantee that the worst-case complexity of the BMLSD algorithm is equal to that of SPF. For SNR of at least 7 dB, our simulations show that the BMLSD approach requires significantly fewer path extensions than the worst-case scenario and thus boasts lower overall complexity than SPF.

5. CONCLUSIONS

We have developed a Bayesian approach to blind detection of data transmitted over a complex time-varying multipath channel. Using a forgetting factor matched to the rate of variation of the channel and a tree-search metric augmented to eliminate the need for channel length knowledge, we address the challenges of implicitly estimating a dispersive fading channel whose effective order may also vary with time. Simulation results show that the proposed detector can achieve lower BER than competing blind detection schemes, even when only the BMLSD approach faces uncertainty in the channel order.

6. REFERENCES

[1] N. Seshadri, "Joint data and channel estimation using blind trellis search techniques," *IEEE Trans. on Comm.*, vol. 42, pp. 1000–1011, February-April 1994.

[2] F. Alberge, M. Nikolova, and P. Duhamel, "Blind identification/equalization using deterministic maximum likelihood and a partial prior on the input," *IEEE Trans. on Signal Processing*, vol. 54, pp. 724–737, February 2006.

[3] J. K. Nelson and A. C. Singer, "Bayesian ML sequence detection for ISI channels," in *Proc. of the IEEE Conf. on Info. Sciences and Systems*, March 2006, pp. 693–698.

[4] Y. R. Zheng and C. Xiao, "Simulation models with correct statistical properties for Rayleigh fading channels," *IEEE Trans. On Signal Processing*, vol. 51, pp. 920–928, June 2003.

[5] E. Eleftheriou and D. Falconer, "Tracking properties and steady-state performance of RLS adaptive filter algorithms," *IEEE Trans. on Acoustics, Speech and Signal Processing*, vol. 34, pp. 1097–1110, October 1986.

[6] J. Miguez and P. M. Djuric, "Blind equalization of frequency-selective channels by sequential importance sampling," *IEEE Trans. on Signal Processing*, vol. 52, pp. 2738–2748, October 2004.