QUANTIFYING DIVERSITY FOR WIRELESS SYSTEMS WITH FINITE-BIT REPRESENTATION

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ABSTRACT

Diversity order is widely adopted as a critical metric on the reliability of wireless transmissions over fading channels. Numerous designs have been proposed to collect the diversity from different types of channels. However, since the simulators and practical systems can only afford finite bits to represent the real/complex numbers, the definition of diversity needs to be revisited. In this paper, we thoroughly investigate the effects of finite-bit representation on the statistical properties of the channel and the diversity with different transmitters/receivers. We show that finite-bit representation may cause diversity loss relative to the infinite precision representation. For different transmission systems, the diversity loss follows different patterns as the number of bits decreases. Furthermore, we find that the number of non-vanishing eigenvalues plays an important role in quantizing the diversity for infinite lattice/constellation. Numerical examples verify our theoretical findings.

***Index Terms*—** Diversity, finite-bit representation, maximum likelihood equalizer, lattice reduction

1. INTRODUCTION

To deal with the deleterious channel fading effects on the system performance, diversity-enriched transmitters and receivers have well-appreciated merits. Many transmission designs have been proposed to exploit the diversity from different channels, e.g., golden code [1, 10] for multi-antenna channels and linear complex field coded (LCFC-) OFDM systems [5] for frequency-selective channels. To our best knowledge, all these existing results on diversity are based on the mathematical derivation in real or complex field, which assumes the numbers are represented in infinite accuracy. However, in practical systems and even simulation tools (including MATLAB), only finite bits are afforded to represent real/complex numbers. Surely, this will affect the performance of the communication systems [4]. Therefore, the diversity has to be revisited and quantified for systems with finite-bit representation.

In this paper, we consider finite-bit representation for the complex channel, signal and noise. This means that every complex number is mapped to finite bits according to a certain mapper (or quantizer) [4]. We first analyze the effects of finite-bit representation on the Gaussian channel. We show that the Gaussian complex channel represented by finite bits loses diversity when signal-to-noise ratio (SNR) is high enough. We then study the diversity of different systems with finite-bit representation and maximum-likelihood equalizer (MLE). We also compare the sensitivity of different receivers (MLE and lattice reduction aided equalizers (LRAEs)) with finite-bit representation. It is shown that although theoretically LRAE may collect the same diversity as MLE in the real/complex field, it may show different diversity when one considers finite-bit representation. Note that the key player of this analysis is not how good the quantizer is, but lower and upper bounds of the finite-bit representation cause the diversity loss.

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2. SYSTEM MODEL AND PROBLEM STATEMENT

Consider block transmissions

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{w}, \tag{1}$$

where $\boldsymbol{H}$ is the $M \times N$ complex Gaussian channel matrix with zero mean, the $N \times 1$ vector $\boldsymbol{s}$ consists of the information symbols, $\boldsymbol{y}$ is the $M \times 1$ received vector, and $\boldsymbol{w}$ is independent and identically distributed (i.i.d.) complex additive white Gaussian noise with variance σ_w^2. We assume that the channel matrix $\boldsymbol{H}$ is known at the receiver, but unknown at the transmitter. Note that the channel matrix $\boldsymbol{H}$ is general enough to represent a number of cases, e.g., multi-antenna, precoded OFDM, and multiuser channels.

The MLE for the model in (1) is given as

$$\hat{\boldsymbol{s}}_{ml} = \arg \min_{\tilde{\boldsymbol{s}} \in \mathcal{S}^N} \|\boldsymbol{y} - \boldsymbol{H}\tilde{\boldsymbol{s}}\|^2, \tag{2}$$

where $\mathcal{S}$ is the constellation of the symbols. MLE in (2) provides optimal error performance with the price paid on high decoding complexity ($\mathcal{O}(|\mathcal{S}|^N)$). Suppose that signal $\boldsymbol{s}$ is transmitted while vector $\tilde{\boldsymbol{s}} \neq \boldsymbol{s}$ is detected at the receiver. The pairwise error probability (PEP) of MLE in (2) is quantified as [1, 5, 10]

$$P_e(\boldsymbol{s} \to \tilde{\boldsymbol{s}}|\boldsymbol{H}) = Q\left(\frac{1}{\sigma_w^2}\|\boldsymbol{H}(\tilde{\boldsymbol{s}} - \boldsymbol{s})\|^2\right) := Q\left(\frac{1}{\sigma_w^2}\|\boldsymbol{H}\boldsymbol{e}\|^2\right), \tag{3}$$

where $\boldsymbol{e} = \tilde{\boldsymbol{s}} - \boldsymbol{s}$ is the error vector, and $Q(\cdot)$ is the Gaussian tail function. By defining an $MN \times 1$ vector $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_N^T]^T$, where $\boldsymbol{h}_n$ is the nth column of $\boldsymbol{H}$, we have

$$\|\boldsymbol{H}\boldsymbol{e}\|^2 = \boldsymbol{h}^{\mathcal{H}}\boldsymbol{E}\boldsymbol{h} \sim \tilde{\boldsymbol{h}}^{\mathcal{H}}\boldsymbol{A}_e\tilde{\boldsymbol{h}}, \tag{4}$$

where $\boldsymbol{E} = \boldsymbol{I}_M \otimes \left((\boldsymbol{e}^T)^{\mathcal{H}}\boldsymbol{e}^T\right)$, the correlation matrix of $\boldsymbol{h}$ is $\boldsymbol{R}_h = E[\boldsymbol{h}^{\mathcal{H}}\boldsymbol{h}]$ with rank ρ_h, for which the SVD is $\boldsymbol{U}_h^{\mathcal{H}}\boldsymbol{\Lambda}_h\boldsymbol{U}_h$, an $\rho_h \times 1$ vector $\tilde{\boldsymbol{h}}$ has i.i.d. complex Gaussian entries, $\boldsymbol{A}_e = \boldsymbol{\Lambda}_h^{\frac{1}{2}}\boldsymbol{U}_h^{\mathcal{H}}\boldsymbol{E}\boldsymbol{U}_h\boldsymbol{\Lambda}_h^{\frac{1}{2}}$ is determined by the error vector $\boldsymbol{e}$ and $\boldsymbol{R}_h$, and "$\sim$" denotes the identical distributions. Since each entry of the signal $\boldsymbol{s}$ is drawn from the constellation $\mathcal{S}$, the error signal $\boldsymbol{e}$ belongs to a set $\mathcal{S}_e = \{\boldsymbol{e} := \tilde{\boldsymbol{s}} - \boldsymbol{s} | \tilde{\boldsymbol{s}} \neq \boldsymbol{s}, \tilde{\boldsymbol{s}}, \boldsymbol{s} \in \mathcal{S}^N\}$. With infinite bits representation, the diversity order $G_d^{(i)}$ collected by the MLE is defined as (see e.g., [1, 5, 10])

$$G_d^{(i)} = \min_{\boldsymbol{e} \in \mathcal{S}_e} \text{rank}(\boldsymbol{A}_e). \tag{5}$$

In this paper, instead of assuming each number (the real or complex numbers) is represented by infinite bits, we consider the situation where finite-bit representation is adopted, e.g., fixed-point or floating-point number representation. In this case, a small real number may be quantified to zero. Thus, the performance of MLE in (2) is affected because the rank of $\boldsymbol{A}_e$ in (4) may be different due to the following two cases:

S1) The statistical property of $\boldsymbol{H}$ is changed when the number of bits adopted is too small;

S2) $\boldsymbol{H}$ is still well approximated as a complex Gaussian random matrix, but the constellation $\mathcal{S}$ spans a wide range and thus the rank of finite-bit represented $\boldsymbol{A}_e$ is smaller than the rank of original $\boldsymbol{A}_e$.

In the following, we analyze the diversity order for these two cases with finite-bit representation.

3. FINITE BIT REPRESENTED CHANNELS

A Gaussian random variable h is represented by finite bits as (using fixed-point with two's complement arithmetic)

$$\mathcal{F}(h, G, F) = -b_{G-1}2^{G-1} + \sum_{i=0}^{G-2} b_i 2^i + \sum_{i=1}^{F} a_i 2^{-i}, \tag{6}$$

where G and F are the number of integer and fractional bits, and a_i and b_i are binary bits. When the number of bits is not enough, the Gaussian variable can not be well approximated, and thus the diversity may be lost. Here, we use one example to show how the number of bits adopted will affect the diversity.

Example 1 (Finite-bit represented Gaussian channel): We assume 4-QAM constellation and $M = N = 1$ in (1). We plot the bit-error-rate (BER) curves in Fig. 1 with fixed-point arithmetic, and the numbers of integer and fractional bits (G, F) are $(10, 10)$, $(8, 8)$, $(6, 6)$, and $(4, 4)$, respectively. We also plot one curve for the same system with 15 digits scaled fixed-point format in MATLAB as a benchmark. From the figure we can see, the diversity is either one or zero (error floor), since H now is a complex Gaussian random variable.

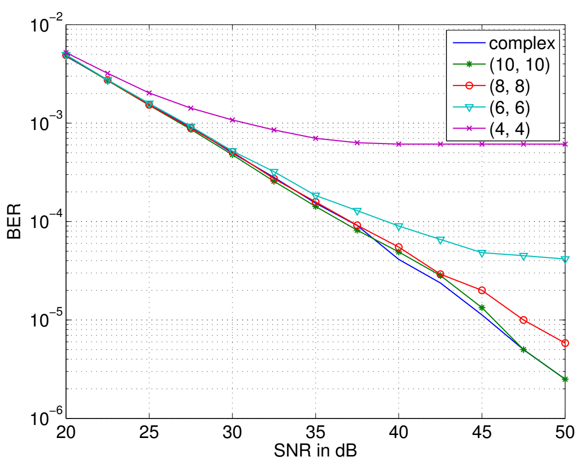

Fig. 1. *Effects of finite-bit representation on diversity*

Given finite-bit representation, when SNR is high enough, error floor of the BER curves shows up because finite bits cannot differentiate large numbers above a certain bound. How many bits are adopted determines when the error floor (zero diversity) appears. Approximately, based on (6), the error floor appears at:

$$\mathrm{SNR} = (G + F)10\log_{10} 2 \approx 3(G + F)\mathrm{dB}.$$

Thus, in practical systems, the number of bits is chosen large enough ($>$ (8, 8)) so that within a reasonable SNR range ($<$ 50 dB), the channel will not lose its diversity.

4. DIVERSITY WITH DIFFERENT TRANSMITTERS

To analyze the diversity for **ANY** QAM constellation with finite-bit representation, we need to focus on the structure of $\boldsymbol{A}_e$. Suppose that the SVD of $\boldsymbol{A}_e$ in (4) is $\boldsymbol{U}_e^{\mathcal{H}}\boldsymbol{\Lambda}_e\boldsymbol{U}_e$, where $\boldsymbol{\Lambda}_e$ is a diagonal matrix with the diagonal entries $\alpha_1, \alpha_2, \ldots, \alpha_{\rho_e}$ as the eigenvalues of $\boldsymbol{A}_e$ and ρ_e is the rank of $\boldsymbol{A}_e$. Thus, we have $\tilde{\boldsymbol{h}}^{\mathcal{H}}\boldsymbol{A}_e\tilde{\boldsymbol{h}} = \sum_{n=1}^{\rho_e}\alpha_n|\bar{h}_n|^2$, where $\bar{h}_n$ is the nth entry of $\bar{\boldsymbol{h}} = \boldsymbol{U}_e\tilde{\boldsymbol{h}}$. Since $\boldsymbol{U}_e$ is a unitary matrix, we know $\bar{\boldsymbol{h}}$ is a vector with i.i.d. complex Gaussian distributed entries. For systems with finite-bit representation, if α_n is less than ϵ_{th} which is the lower bound of the finite-bit representation, it will be quantified to zero. Thus, for systems with finite-bit representation, the rank of $\boldsymbol{A}_e$ needs to be revisited. The result is summarized as follows.

Proposition 1 *Suppose the transmission system is based on finite-bit representation. Given the specific constellation, the diversity order collected by the MLE in* (2) *is*

$$G_d^{(f)} = \min_{\boldsymbol{e}\in\mathcal{S}_e} \mathrm{rank}(\mathcal{F}(\boldsymbol{A}_e, G, F)), \tag{7}$$

where $\mathcal{F}(a, G, F)$ is defined in (6) *and $\boldsymbol{A}_e$ is expressed as in* (4).

Obviously, the system diversity with finite-bit representation in (7) depends on two terms: i) the range of the values in $\boldsymbol{A}_e$; ii) the number of bits adopted to represent the numbers. For fixed number of bits, the diversity is determined by the rank of quantized $\boldsymbol{A}_e$, i.e., the non-zero eigenvalues of $\boldsymbol{A}_e$. In general, the eigenvalues of $\boldsymbol{A}_e$ depend on the constellation size and the transmitter structure. To find the non-zero eigenvalues for any constellation is equivalent to finding non-vanishing eigenvalues which are defined as follows.

Definition 1 *Suppose $\alpha_1 \le \alpha_2 \le \cdots \le \alpha_{\rho_e}$ are the ordered nonzero eigenvalues of $\boldsymbol{A}_e$ in* (4)*. If there exists a constant $\epsilon > 0$ such that*

$$\inf_{|\mathcal{S}|\to\infty} |\alpha_n| > \epsilon, \tag{8}$$

then α_n is called a non-vanishing eigenvalue as the constellation size increases to infinity.

If the eigenvalue α_n is vanishing, then given a finite number of bits (no matter how many bits are adopted), α_n will be smaller than ϵ_{th} and quantified to zero when the constellation size is large enough. Furthermore, with some constraints of the transmitter, e.g., the fixed norm of $\boldsymbol{e}$ or the determinant is nonzero, not all eigenvalues of $\boldsymbol{A}_e$ vanish simultaneously. Thus, we can see, the asymptotic diversity as the constellation size increases to infinity is the smallest number of non-vanishing eigenvalues of $\boldsymbol{A}_e$. Now we summarize our results as follows.

Proposition 2 *The asymptotic diversity collected by MLE as the constellation size increases to infinity is defined as*

$$G_d^{(a)} = \lim_{|\mathcal{S}|\to\infty,\forall\mathcal{F}} \min_{\boldsymbol{e}\in\mathcal{S}_e} \operatorname{rank}(\mathcal{F}(\boldsymbol{A}_e, G, F)). \qquad (9)$$

In other words, the asymptotic diversity equals the number of non-vanishing eigenvalues of $\boldsymbol{A}_e$.

Proposition 2 shows that given finite-bit representation, when the constellation size keeps increasing, even MLE may lose diversity. A natural question now is how fast the schemes lose diversity. In the following, we use three examples [1, 3, 5, 10] to illustrate this phenomenon.

Example 2 (V-BLAST systems): Suppose the channel coefficients are i.i.d. complex Gaussian distributed [3]. Eq. (4) is rewritten as

$$\|\boldsymbol{He}\|^2 = \boldsymbol{h}^{\mathcal{H}} \left(\boldsymbol{I}_M \otimes \left((\boldsymbol{e}^T)^{\mathcal{H}}\boldsymbol{e}^T\right)\right)\boldsymbol{h}, \qquad (10)$$

where $\boldsymbol{h}$ is an $MN \times 1$ column vector by stacking all columns of $\boldsymbol{H}$ into one column. Thus, according to [3], the maximum diversity is M. Specifically, in this case, the M nonzero eigenvalues of $\boldsymbol{I}_M \otimes ((\boldsymbol{e}^T)^{\mathcal{H}}\boldsymbol{e}^T)$ are the same as $\|\boldsymbol{e}\|^2$, which is lower bounded by the minimum Euclidean distance ($d_{\min}$) of the constellation. Thus, if the minimum value ϵ_{th} that the finite bits can represent is smaller than $d_{\min}$, then the diversity is M. Otherwise, the diversity is zero. All eigenvalues vanish simultaneously when we reduce the number of bits.

Example 3 (Precoded OFDM systems): The LCFC-OFDM system [5] is designed to collect multipath diversity of frequency-selective channels. The equivalent channel matrix for LCFC-OFDM systems is $\boldsymbol{H} = \boldsymbol{D}_H\boldsymbol{\Theta}$, where $\boldsymbol{D}_H = \operatorname{diag}[H(0), H(1), \ldots, H(N-1)]$ with $H(n)$ as the channel response at subcarrier n, and $\boldsymbol{\Theta}$ is an $N \times N$ full-rank square unitary precoder. By stacking $H(n)$ into one $N \times 1$ column $\boldsymbol{h}$, we can rewrite (4) as

$$\|\boldsymbol{He}\|^2 = \boldsymbol{h}^{\mathcal{H}} \left(\operatorname{diag}(\boldsymbol{u})^{\mathcal{H}}\operatorname{diag}(\boldsymbol{u})\right)\boldsymbol{h}, \qquad (11)$$

where $\boldsymbol{u} = \boldsymbol{\Theta}(\tilde{\boldsymbol{s}} - \boldsymbol{s})$. The eigenvalues of $\boldsymbol{A}_e$ are the norm of the entries of $\boldsymbol{u}$. As shown in [5], the minimum entry of $|\boldsymbol{u}|$ only depends on the minimum distance $d_{\min}$ of the constellation adopted. Thus, the minimum eigenvalue will not approach zero as the constellation size increases to infinity. Neither will other eigenvalues. As the number of bits decreases, not like V-BLAST, LCFC-OFDM loses diversity gradually since the eigenvalues are not all equal.

Example 4 (Golden code for 2×2 systems): The golden code is a full-diversity-full-rate space-time coding scheme for 2×2 i.i.d. channels [1, 10]. Since golden code is implemented in two time slots, the error pattern $\boldsymbol{e}$ now is a 2×2 matrix. Thus, (4) can be expressed as

$$\|\boldsymbol{He}\|^2 = \boldsymbol{h}^{\mathcal{H}} \left(\boldsymbol{I}_2 \otimes \boldsymbol{A}^{\mathcal{H}}\boldsymbol{A}\right)\boldsymbol{h}, \qquad (12)$$

where $\boldsymbol{h}$ is a 4×1 vector by stacking all four channel coefficients into one column, and

$$\boldsymbol{A} = \begin{bmatrix} e_{11}\cos\theta_1 - e_{22}\sin\theta_1 & e_{12}\cos\theta_2 - e_{21}\sin\theta_2 \\ e_{12}\sin\theta_2 + e_{21}\cos\theta_2 & e_{11}\sin\theta_1 + e_{22}\cos\theta_1 \end{bmatrix}, \qquad (13)$$

where e_{mn} is the $(2(m-1)+n)$th entry of $\boldsymbol{e}$. The two eigenvalues of $\boldsymbol{A}^{\mathcal{H}}\boldsymbol{A}$ are

$$\alpha_{1,2} = \|\boldsymbol{e}\|^2 \left(1 \pm \sqrt{1 - \frac{4\det(\boldsymbol{A}^{\mathcal{H}}\boldsymbol{A})}{\|\boldsymbol{e}\|^4}}\right). \qquad (14)$$

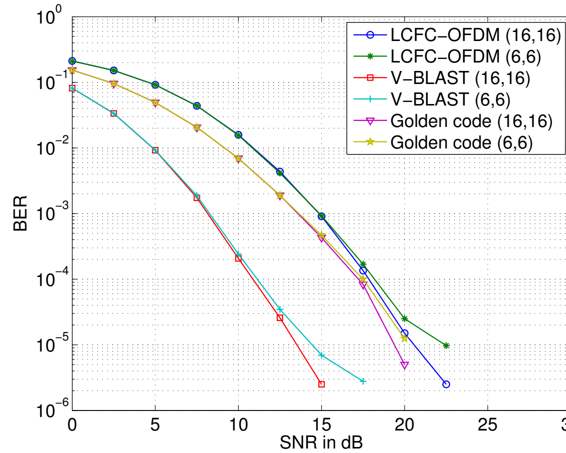

Fig. 2. *Performance with finite-bit representation*

Although it has been shown that $|\det(\boldsymbol{A})| \geq \frac{1}{2\sqrt{5}}$ with the optimal $(\theta_1,\theta_2) = (\frac{1}{2}\arctan\frac{1}{2}, \frac{1}{2}\arctan 2)$, one of the eigenvalues may go to zero.

Given an error pattern $e_{21} = e_{22} = 0$, and e_{11}, e_{12} real integers such that

$$|e_{11} - \sqrt{2}e_{12}| < \epsilon\sqrt{e_{12}}\,, \quad \forall\, \epsilon > 0, \qquad (15)$$

then it can be verified that $\det(\boldsymbol{A}^{\mathcal{H}}\boldsymbol{A}) = \frac{1}{20}(e_{11}^2 - 2e_{12}^2)^2 < \frac{1}{20}(\epsilon^2 e_{12} + 2\sqrt{2}\epsilon e_{12}^{\frac{3}{2}})^2$. Thus, it is ready to show that when e_{11} and e_{12} go to infinity, one of the eigenvalues in (14) approaches zero.

Because there are two groups of identical eigenvalues in $\boldsymbol{A}_e$, the number of non-vanishing eigenvalues is 2. It is not difficult to find such an error pattern that makes one of the eigenvalues be quantized to zero. For example, using MATLAB with 15 digits scaled fixed point format, when $\boldsymbol{e} = [65780, 85786; 59796, 69848]$, one eigenvalue of $\boldsymbol{A}^{\mathcal{H}}\boldsymbol{A}$ is quantified to zero, while the determinant of $\boldsymbol{A}^{\mathcal{H}}\boldsymbol{A}$ is 2.2×10^4 and $\operatorname{rank}(\boldsymbol{A}^{\mathcal{H}}\boldsymbol{A}) = 1$. Although an extremely large constellation is needed to reach this error pattern, the diversity for this particular constellation is 2, according to the code design in [10].

Example 5 (Diversity of systems with finite-bit representation): We plot the performance of LCFC-OFDM systems [5] for frequency-selective channels with channel order 3 (multipath diversity is 4), V-BLAST systems [3] for 4×4 i.i.d. channels, and golden code [10] for 2×2 i.i.d. channels. We adopt 4-QAM and fix the number of integer bits G and number of fractional bits F as $(16, 16)$ and $(6, 6)$, respectively. From Fig. 2 we observe that diversity 4 is collected by all these three systems when the number of bits is high enough. However, when the number of bits is low, diversity is lost when SNR is high.

5. DIVERSITY COLLECTED BY DIFFERENT RECEIVERS

In this section, we illustrate that finite-bit representation also affects the diversity that a receiver can collect. Instead of MLE, we consider LRAEs, which are proposed in [2, 6, 8, 9] to improve the performance of LEs without increasing complexity much. We adopt the complex LLL (CLLL) algorithm [2, 6] to perform LRAE on the channel matrix $\boldsymbol{H}$. A reduced lattice basis $\tilde{\boldsymbol{H}} = \boldsymbol{HT}$ is obtained by the CLLL algorithm, where $\boldsymbol{T}$ is a unimodular matrix with all the entries being Gaussian integers and the determinant of $\boldsymbol{T}$ being ± 1 or $\pm j$. Then we perform the ZF equalizer $\tilde{\boldsymbol{H}}^\dagger$ instead of $\boldsymbol{H}^\dagger$ to the

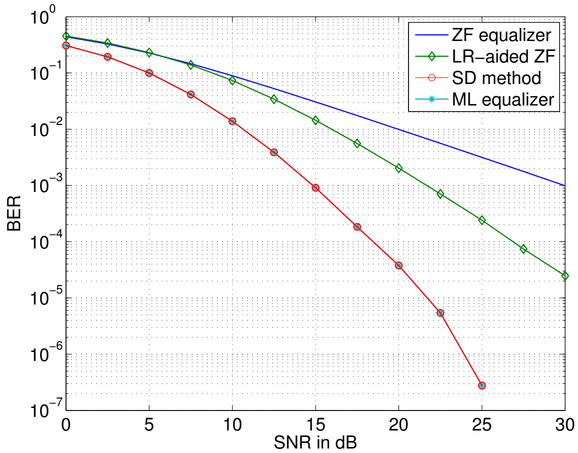

Fig. 3. *Performance of golden code*

observation vector as

$$x = \tilde{H}^{\dagger} y = T^{-1} s + \tilde{H}^{\dagger} w := z + n. \quad (16)$$

Since all the entries of T^{-1} and the signal constellation belong to Gaussian integer ring, the entries of z are also Gaussian integers. Thus, we perform the first hard-decoding step by rounding x to the nearest Gaussian integers to get $\hat{z}$. The second hard-decoding step is to quantize $T\hat{z}$ to the signal constellation $\mathcal{S}$ to obtain the estimated symbols $\hat{s}$. The detailed algorithm can be found in [2, 6].

Note that the constellation size of z is infinite because the entries of T can be arbitrarily large. Furthermore, as stated in [2, 6], the CLLL algorithm upper bounds the orthogonality deficiency of $\tilde{H}$. Thus, according to the results in [11], LEs based on $\tilde{H}$ have the same diversity as that of MLE based on $\tilde{H}$. For the MLE based on $\tilde{H}$, we can express the PEP as in (3)

$$P_e(z \to \tilde{z} | \tilde{H}, \{\tilde{z}, z\} \in \mathbb{Z}[j]^{N\times 1}) = Q\left(\frac{1}{\sigma_w^2} \|\tilde{H}(\tilde{z} - z)\|^2\right),$$

where $\mathbb{Z}[j]$ denotes the complex integer set whose elements have the form $\mathbb{Z} + j\mathbb{Z}$, with $j = \sqrt{-1}$. Then, in the first quantization step, LRAEs achieve the same diversity as MLE based on infinite constellation under finite-bit representation. Now, we summarize the result as follows.

Proposition 3 *Given finite-bit representation, the diversity collected by LRAEs is the same as the asymptotic diversity enabled by the transmitter with infinite constellation.*

With Proposition 3, we can then quantify the diversity collected by LRAEs for general systems. For example, for V-BLAST and LCFC-OFDM systems, LRAEs collect the same diversity as MLEs, because the minimum eigenvalue is non-vanishing as constellation size increases. This is consistent with the theoretical results in [6, 7]. In general, we claim that to design a coding scheme which could achieve full diversity with LRAEs at fairly low complexity, the matrix A_e in (4) needs to be designed so that the minimum eigenvalue is non-vanishing when constellation size increases to infinity.

Example 6 (Diversity of golden code): In this example, we implement the golden code in [10] for a 2×2 system in MATLAB. The channel coefficients are assumed to be i.i.d. complex Gaussian random variables. Four detectors are adopted to recover the signal: ZF, LR-aided ZF, sphere decoding (SD) and ML detectors. It can be observed from Fig. 3 that both SD and ML detectors exploit full diversity 4. ZF equalizer collects diversity 1 [7] while LR-aided ZF equalizer only has diversity 2. LRAE loses diversity 2 because of the finite-bit representation of MATLAB, which verifies our analysis in Proposition 3.

6. CONCLUSIONS

In this paper, we analyze the effects of finite-bit representation on determining system diversity. We show that for a specific constellation, the diversity enabled by the transmitter depends on the number of bits adopted. Furthermore, the asymptotic diversity as the constellation increases to infinity equals the number of non-vanishing eigenvalues. We also apply these results to quantify the diversity of LRAEs for systems with finite-bit representation.[1]

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