BLIND MAXIMUM-LIKELIHOOD DATA RECOVERY IN OFDM

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ABSTRACT

OFDM modulation combines the advantages of high achievable rates and relatively easy implementation. In this paper, we show how to perform blind maximum-likelihood data recovery in OFDM transmission from the output symbol and cyclic prefix. Our approach relies on decomposing the OFDM channel into two subchannels (cyclic and linear) that share the same input and are characterized by the same channel parameters. This fact enables us to estimate the channel parameters from one subchannel and substitute the estimate into the other, thus obtaining a nonlinear relationship involving the input and output data only. The relationship does not show any channel dependence whatsoever and can be exhaustively searched for the maximum likelihood estimate of the input. This shows that OFDM systems are completely identifiable using output data only, irrespective of the channel zeros, as long as the channel delay spread is less than the length of the cyclic prefix.

Index Terms— OFDM, Blind channel estimation, Maximum likelihood detection.

1. INTRODUCTION

There has been increasing interest in OFDM as it combines the advantages of high achievable rates and easy implementation. This is reflected by the many standards that considered and adopted OFDM [1]. For proper operation of an OFDM system, the receiver needs to estimate the channel and eliminate its effect. Many techniques have been proposed in the literature for this purpose (see, e.g., [1], [2], [3], [4], [5], and the references therein).

In this paper, we perform channel identification and equalization from output data only (i.e. OFDM output symbol and associated cyclic prefix (CP)), and without the need for a training sequence or a priori channel information. The advantage of our approach is three fold:

1. It provides a blind estimate of the data from one output symbol without the need for training or averaging (contrary to the common practice where averaging over several symbols is required). Thus, the method lends itself to block fading channels.
2. Data detection is done without any restriction on the channel (as long as the delay spread is shorter than the (CP)). In fact, data detection can be performed even in the presence of zeros on the FFT grid.[1]
3. The fact that we use two observations (the OFDM symbol and CP) to recover the input symbol enhances the diversity of the system as can be seen from simulations.

Our approach is based on the transformation of the linear OFDM channel into two parallel subachannels due to the presence of a cyclic prefix at the input. One is a cyclic channel that relates the input and output OFDM symbols and thus is free of any intersymbol interference (ISI) effects and is best described in the frequency domain. The other one is a linear channel that carries the burden of ISI and that relates the input and output prefixes through linear convolution. This channel is best studied in the time domain.

It can be shown that the two subchannels are characterized by the same set of parameters (or impulse response(IR)) and are driven by the same stream of data. They only differ in the way in which they operate on the data (i.e. linear vs circular convolution). This fact enables us to estimate the IR from one subchannel and eliminate its effect from the other, thus obtaining a nonlinear relationship that involves the input and output data only. This relationship can in turn be optimized for the ML data estimate; something that can be achieved through exhaustive search (in the worst case scenario).

2. NOTATION

We denote scalars with small-case letters, vectors with small-case boldface letters, and matrices with uppercase boldface letters. Calligraphic notation (e.g. $\boldsymbol{\mathcal{X}}$) is reserved for vectors in the frequency domain. The individual entries of a variable like $\boldsymbol{h}$ are denoted by $h(l)$. A hat over a variable indicates an estimate of the variable (e.g., $\hat{\boldsymbol{h}}$ is an estimate of $\boldsymbol{h}$). When any of these variables become a function of time, the time index i appears as a subscript.

Now consider a length-N vector $\boldsymbol{x}_i$. We deal with three derivatives associated with this vector. The first two are obtained by partitioning $\boldsymbol{x}_i$ into a lower (trailing) part $\underline{\boldsymbol{x}}_i$ (known

[1] This comes contrary to the common belief that OFDM using CP cannot be equalized for channels with zeros on the FFT grid [1] and [6]

as the cyclic prefix) and an upper vector $\tilde{\boldsymbol{x}}_i$. The third derivative, $\overline{\boldsymbol{x}}_i$, is created by concatenating $\boldsymbol{x}_i$ with a copy of CP i.e. $\underline{\boldsymbol{x}}_i$. Thus, we have

$$\overline{\boldsymbol{x}}_i = \begin{bmatrix} \underline{\boldsymbol{x}}_i \\ \boldsymbol{x}_i \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{x}}_i \\ \tilde{\boldsymbol{x}}_i \\ \underline{\boldsymbol{x}}_i \end{bmatrix} \quad (1)$$

This notational convention of underlined and overlined variables will be extended to matrices as well and when it is not clear, the number of rows will appear as a subscript.

3. ESSENTIAL ELEMENTS OF OFDM TRANSMISSION

In an OFDM system, data is transmitted in symbols $\boldsymbol{\mathcal{X}}_i$ of length N each. The symbol undergoes an IFFT operation to produce the time domain symbol $\boldsymbol{x}_i$, i.e. $\boldsymbol{x}_i = \sqrt{N}\boldsymbol{Q}\boldsymbol{\mathcal{X}}_i$, where $\boldsymbol{Q}$ is the $N \times N$ IFFT matrix.

When juxtaposed, these symbols result in the sequence $\{x_k\}$. [2] We assume a non-ideal channel $\underline{\boldsymbol{h}}$ of maximum length $L+1$. To avoid ISI caused by passing through the channel, a cyclic prefix (CP) $\underline{x}_i$ (of length L) is appended to x_i, resulting finally in super-symbol $\overline{\boldsymbol{x}}_i$ as defined in (1). The concatenation of these symbols produces the underlying sequence $\{\overline{x}_k\}$.

When passed through the channel $\underline{\boldsymbol{h}}$, the sequence $\{\overline{x}_k\}$ produces the output sequence $\{\overline{y}_k\}$ i.e.

$$\overline{y}_k = \underline{h}_k * \overline{x}_k + \overline{n}_k \quad (2)$$

where $\overline{n}_k$ is the additive white Gaussian noise and $*$ stands for linear convolution.

Motivated by the symbol structure of the input, it is convenient to partition the output into symbols of length $M = N + L$, i.e.

$$\overline{\boldsymbol{y}}_i = \begin{bmatrix} \underline{\boldsymbol{y}}_i \\ \boldsymbol{y}_i \end{bmatrix} \quad (3)$$

This is a natural way to partition the output because the prefix $\underline{\boldsymbol{y}}_i$ actually absorbs all ISI that takes place between the adjacent symbols $\overline{\boldsymbol{x}}_{i-1}$ and $\overline{\boldsymbol{x}}_i$. Moreover, the remaining part $\boldsymbol{y}_i$ of the symbol depends on the ith input OFDM symbol $\boldsymbol{x}_i$ only. These facts allow us to partition the total OFDM channel described by (2) into two subchannels that we describe next.

3.1. Circular Convolution (Subchannel)

Due to the presence of the cyclic prefix, the input and output OFDM symbols $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ are related by circular convolution (denoted by $\circledast$), i.e.

$$\boxed{\boldsymbol{y}_i = \boldsymbol{h}_i \circledast \boldsymbol{x}_i + \boldsymbol{n}_i} \quad (4)$$

where $\boldsymbol{h}_i$ is a length-N zero-padded version of $\underline{\boldsymbol{h}}_i$. In the frequency domain, the cyclic convolution (4) reduces to the element-by-element operation

$$\boxed{\boldsymbol{\mathcal{Y}}_i = \boldsymbol{\mathcal{H}} \odot \boldsymbol{\mathcal{X}}_i + \boldsymbol{\mathcal{N}}_i} \quad (5)$$

where $\boldsymbol{\mathcal{H}}$, $\boldsymbol{\mathcal{X}}_i$, $\boldsymbol{\mathcal{N}}_i$, and $\boldsymbol{\mathcal{Y}}_i$, are the DFT's of $\boldsymbol{h}$, $\boldsymbol{x}_i$, $\boldsymbol{n}_i$, and $\boldsymbol{y}_i$ respectively

$$\begin{gathered} \boldsymbol{\mathcal{H}} = \boldsymbol{Q}^*\boldsymbol{h}, \quad \boldsymbol{\mathcal{X}}_i = \frac{1}{\sqrt{N}}\boldsymbol{Q}^*\boldsymbol{x}_i, \\ \boldsymbol{\mathcal{N}}_i = \frac{1}{\sqrt{N}}\boldsymbol{Q}^*\boldsymbol{n}_i, \text{ and } \boldsymbol{\mathcal{Y}}_i = \frac{1}{\sqrt{N}}\boldsymbol{Q}^*\boldsymbol{y}_i \end{gathered} \quad (6)$$

Since $\underline{\boldsymbol{h}}$ corresponds to the first $L+1$ elements of $\boldsymbol{h}$, we can show that

$$\boldsymbol{\mathcal{H}} = \boldsymbol{Q}^*_{L+1}\underline{\boldsymbol{h}} \quad \text{and} \quad \underline{\boldsymbol{h}} = \boldsymbol{Q}_{L+1}\boldsymbol{\mathcal{H}} \quad (7)$$

where $\boldsymbol{Q}^*_{L+1}$ consists of the first $L+1$ columns of $\boldsymbol{Q}^*$ and $\boldsymbol{Q}_{L+1}$ consists of first $L+1$ rows of $\boldsymbol{Q}$. This allows us to rewrite (5) as

$$\boxed{\boldsymbol{\mathcal{Y}}_i = \text{diag}(\boldsymbol{\mathcal{X}}_i)\boldsymbol{Q}^*_{L+1}\underline{\boldsymbol{h}} + \boldsymbol{\mathcal{N}}_i} \quad (8)$$

3.2. Linear Convolution (Subchannel)

From (2), we can also deduce that the cyclic prefixes at the input and output are related by linear convolution. Specifically, if we concatenate all cyclic prefixes at the input into a sequence $\{\underline{x}_k\}$ and the cyclic prefixes at the output into the corresponding sequence $\{\underline{y}_k\}$, then we can show that the two sequences are related by linear convolution [7]

$$\boxed{\underline{y}_k = \underline{h}_k * \underline{x}_k + \underline{n}_i} \quad (9)$$

From this we deduce that the cyclic prefix of OFDM symbol $\boldsymbol{y}_i$ is related to the input cyclic prefixes $\underline{x}_{i-1}$ and $\underline{x}_i$ by

$$\boxed{\underline{\boldsymbol{y}}_i = \underline{\boldsymbol{X}}_i\underline{\boldsymbol{h}} + \underline{\boldsymbol{n}}_i} \quad (10)$$

where $\underline{\boldsymbol{X}}_i$ is constructed from $\underline{x}_{i-1}$ and $\underline{x}_i$ according to

$$\underline{\boldsymbol{X}}_i = \begin{bmatrix} \underline{x}_i(0) & \underline{x}_{i-1}(L-1) & \cdots & \underline{x}_{i-1}(0) \\ \underline{x}_i(1) & \underline{x}_i(0) & \cdots & \underline{x}_{i-1}(1) \\ \vdots & \ddots & \ddots & \vdots \\ \underline{x}_i(L-1) & \cdots & \underline{x}_i(0) & \underline{x}_{i-1}(L-1) \end{bmatrix} \quad (11)$$

This fact together with the FFT relationship (7) yields the desired time-frequency form

$$\underline{\boldsymbol{y}}_i = \underline{\boldsymbol{X}}_i\boldsymbol{Q}_{L+1}\boldsymbol{\mathcal{H}} + \underline{\boldsymbol{n}}_i \quad (12)$$

[2] The time indices in the sequence $\boldsymbol{x}_i$ and the underlying sequence $\{x_k\}$ are dummy variables. Nevertheless, we chose to index the two sequences differently to avoid any confusion that might arise from choosing identical indicies.

4. MAXIMUM-LIKELIHOOD ESTIMATION

Consider the frequency domain description of the cyclic subchannel (5). To obtain the ML estimate of $\boldsymbol{\mathcal{H}}$, we assume that the sequence $\boldsymbol{\mathcal{X}}_i$ is deterministic and perform an element-by-element division of (5) by $\boldsymbol{\mathcal{X}}_i$ to get

$$\boldsymbol{D}_{\mathcal{X}}^{-1}\boldsymbol{\mathcal{Y}}_i = \boldsymbol{\mathcal{H}} + \boldsymbol{\mathcal{N}}_i^{'} \tag{13}$$

where

$$\boldsymbol{D}_{\mathcal{X}} = \text{diag}(\boldsymbol{\mathcal{X}}_i) \tag{14}$$

and $\boldsymbol{\mathcal{N}}_i^{'}$ is Gaussian distributed with zero mean and autocorrelation matrix

$$\boldsymbol{R}_{n'} = \sigma_n^2 \boldsymbol{D}_{\mathcal{X}}^{-1}\boldsymbol{D}_{\mathcal{X}}^{-*} = \sigma_n^2 |\boldsymbol{D}_{\mathcal{X}}|^{-2} \tag{15}$$

The maximum-likelihood estimate of $\boldsymbol{\mathcal{H}}$ can now be obtained by solving the system of equations (13) in the least-squares (LS) sense subject to the constraint

$$\tilde{\boldsymbol{Q}}_{N-L-1}\boldsymbol{\mathcal{H}} \triangleq \tilde{\boldsymbol{Q}}\boldsymbol{\mathcal{H}} = 0 \tag{16}$$

We can show that the ML estimate is given by [8]

$$\begin{aligned}\hat{\boldsymbol{\mathcal{H}}}^{ML} &= \left[\boldsymbol{I} - \boldsymbol{R}_{n'}\tilde{\boldsymbol{Q}}^*\left(\tilde{\boldsymbol{Q}}\boldsymbol{R}_{n'}\tilde{\boldsymbol{Q}}^*\right)^{-1}\tilde{\boldsymbol{Q}}\right]\boldsymbol{D}_{\mathcal{X}}^{-1}\boldsymbol{\mathcal{Y}}_i \\ &= \left[\boldsymbol{I} - |\boldsymbol{D}_{\mathcal{X}}|^{-2}\tilde{\boldsymbol{Q}}^*\left(\tilde{\boldsymbol{Q}}|\boldsymbol{D}_{\mathcal{X}}|^{-2}\tilde{\boldsymbol{Q}}^*\right)^{-1}\tilde{\boldsymbol{Q}}\right]\boldsymbol{D}_{\mathcal{X}}^{-1}\boldsymbol{\mathcal{Y}}_i\end{aligned} \tag{17}$$

The ML estimate (17) was obtained solely from the circular convolution subchannel. Upon replacing $\boldsymbol{\mathcal{H}}$ that appears in the time-frequency form (12) (corresponding to the linear subchannel) with its ML estimate (17), we obtain

$$\begin{aligned}\underline{\boldsymbol{y}}_i = \underline{\boldsymbol{X}}_i\boldsymbol{Q}_{L+1}&\left[\boldsymbol{I} - |\boldsymbol{D}_{\mathcal{X}}|^{-2}\tilde{\boldsymbol{Q}}^*\left(\tilde{\boldsymbol{Q}}|\boldsymbol{D}_{\mathcal{X}}|^{-2}\tilde{\boldsymbol{Q}}^*\right)^{-1}\tilde{\boldsymbol{Q}}\right] \\ &\boldsymbol{D}_{\mathcal{X}}^{-1}\boldsymbol{\mathcal{Y}}_i + \underline{\boldsymbol{n}}_i\end{aligned} \tag{18}$$

This is an input/output relationship that does not depend on any channel information whatsoever. Since the data is assumed deterministic, maximum-likelihood estimation is the optimum way to detect it, i.e. we minimize

$$\begin{aligned}\hat{\boldsymbol{\mathcal{X}}}_i^{ML} = \arg\min_{\mathcal{X}_i}\Bigg\|\underline{\boldsymbol{y}}_i - \underline{\boldsymbol{X}}_i\boldsymbol{Q}_{L+1}&\Big[\boldsymbol{I} - |\boldsymbol{D}_{\mathcal{X}}|^{-2} \\ \tilde{\boldsymbol{Q}}^*\left(\tilde{\boldsymbol{Q}}|\boldsymbol{D}_{\mathcal{X}}|^{-2}\tilde{\boldsymbol{Q}}^*\right)^{-1}&\tilde{\boldsymbol{Q}}\Big]\boldsymbol{D}_{\mathcal{X}}^{-1}\boldsymbol{\mathcal{Y}}_i\Bigg\|^2\end{aligned} \tag{19}$$

This is a nonlinear least-squares problem in the data. In the worst case scenario, it can be solved by an exhaustive search over all possible sequences $\boldsymbol{\mathcal{X}}_i$. To gain more insight into this problem, we now treat the case of constant modulus data, in which we have

$$|\boldsymbol{D}_{\mathcal{X}}|^{-2} = \frac{1}{\mathcal{E}_X}\boldsymbol{I} \quad \text{and} \quad \boldsymbol{D}_{\mathcal{X}}^{-1} = \frac{1}{\mathcal{E}_X}\boldsymbol{D}_{\mathcal{X}}^* \tag{20}$$

Thus, the ML estimate of $\boldsymbol{\mathcal{X}}_i$ (19) simplifies to

$$\hat{\boldsymbol{\mathcal{X}}}_i^{ML} = \arg\min_{\mathcal{X}_i}\left\|\underline{\boldsymbol{y}}_i - \frac{1}{\mathcal{E}_X}\underline{\boldsymbol{X}}_i\boldsymbol{Q}_{L+1}\left[\boldsymbol{I} - \tilde{\boldsymbol{Q}}^*\tilde{\boldsymbol{Q}}\right]\boldsymbol{\mathcal{Y}}_i \odot \boldsymbol{\mathcal{X}}_i^*\right\|^2 \tag{21}$$

where in (21), we used the fact that $\tilde{\boldsymbol{Q}}$ is a left-inverse of $\tilde{\boldsymbol{Q}}^*$ - a consequence of the unitary nature of $\boldsymbol{Q}$

$$\boldsymbol{I} = \boldsymbol{Q}\boldsymbol{Q}^* = \begin{bmatrix}\boldsymbol{Q}_{L+1} \\ \tilde{\boldsymbol{Q}}_{N-L-1}\end{bmatrix}\begin{bmatrix}\boldsymbol{Q}_{L+1}^* & \tilde{\boldsymbol{Q}}_{N-L-1}^*\end{bmatrix} \tag{22}$$

From (22), we can also deduce that

$$\boldsymbol{Q}_{L+1}\tilde{\boldsymbol{Q}} = \boldsymbol{Q}_{L+1}\tilde{\boldsymbol{Q}}_{N-L-1} = 0$$

So, the ML estimate of $\boldsymbol{\mathcal{X}}_i$, for the constant modulus case, is now obtained by performing the minimization

$$\hat{\boldsymbol{\mathcal{X}}}_i^{ML} = \arg\min_{\mathcal{X}_i}\left\|\underline{\boldsymbol{y}}_i - \frac{1}{\mathcal{E}_X}\underline{\boldsymbol{X}}_i\boldsymbol{Q}_{L+1}\boldsymbol{\mathcal{Y}}_i \odot \boldsymbol{\mathcal{X}}_i^*\right\|^2 \tag{23}$$

Notice that the only unknowns in this minimization are $\underline{\boldsymbol{X}}_i$ and $\boldsymbol{\mathcal{X}}_i$, i.e. the input data sequence. This minimization is nothing but a *nonlinear least-squares* problem in the data. In the worst case scenario, we can obtain the ML estimate through an exhaustive search.

4.1. Reducing computational complexity

(19) and (23) are computationally very complex but the matrices involved in them are very sparse. So the computational complexity can be reduced by relying on the sparsity of these matrices. It is not discussed here as it is out of scope of this paper and is a subject of future research.

5. SIMULATIONS AND RESULTS

We consider an OFDM system with $N = 16$ and cyclic prefix of length $L = 4$. The OFDM symbol consists of BPSK or 4-QAM symbols. The channel IR consists of 5 iid Rayleigh fading taps. We compare the BER performance of three methods: (i) Perfectly known channel, (ii) Channel estimated using $L + 1$ pilots and (iii) Blind based estimation.

In Figure 1, we compare the three mentioned approaches of signal estimation for BPSK modulated data over a Rayleigh fading channel. As expected, the best performance is achieved by the perfectly known channel, followed by that obtained by training based estimated channel.

The same conclusion can be made for the 4-QAM input (see Figure 2). Note however that in the high SNR region, the BER curve of blind based estimation exhibits steeper slope (higher diversity) which can be explained from the fact that two channels (linear and cyclic) are used to detect the data.

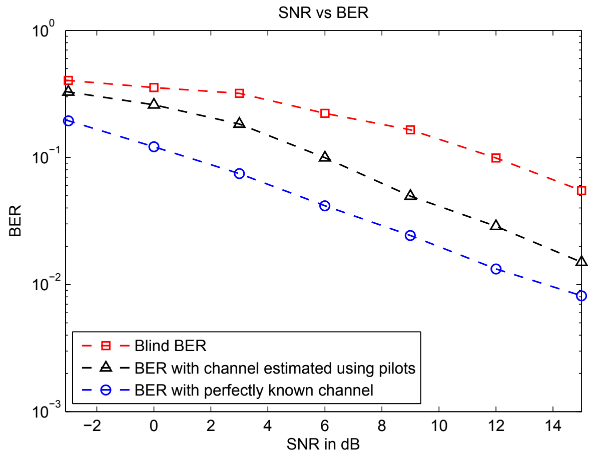

Fig. 1. BER vs SNR for BPSK-OFDM over a Rayleigh channel

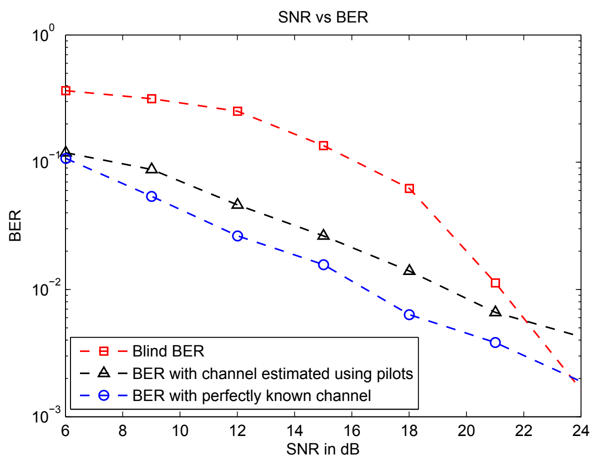

Fig. 2. BER vs SNR for 4QAM-OFDM over a Rayleigh channel

In Figure 3, the three approaches are compared for BPSK modulated data when the channel IR has zeros on the FFT grid. We note that at high SNR, the BER for perfectly known channel and that of the estimated channel reach an error floor. Our blind method does not suffer from this problem and thus blind case outperforms the other two cases.

6. CONCLUSION

In this paper, we demonstrated how to perform blind ML data recovery in OFDM transmission. This is done using a single output OFDM symbol and associated CP. In particular, it was shown that the ML data estimate is the solution of an integer nonlinear-least squares problem. This proves that the data recovery is possible from output data only, irrespective of the channel zero locations and irrespective of the quality

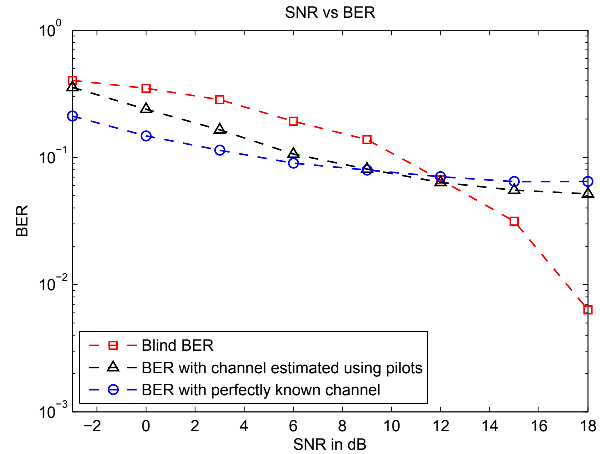

Fig. 3. BER vs SNR for BPSK-OFDM over channel with zeros on FFT grid

of the channel estimates or of its exact order. The algorithm developed in this paper entails exponential complexity whose reduction is the subject of future research.

7. REFERENCES

[1] Z. Wang and G. B. Giannakis, "Wireless Multicarrier Communications: Where Fourier Meets Shannon," IEEE Signal Processing Mag., vol. 17, May 2000.

[2] T. Y. Al-Naffouri, "An EM-based forward-backward Kalman for the estimation of time-variant channels in OFDM," IEEE Trans. Signal Process., vol. 55, Jul. 2007.

[3] M. Chang and Yu T. Su, "Blind and semiblind detections of OFDM signals in fading channels," IEEE Transactions on Communications, vol 52, Issue 5, May 2004.

[4] R. W. Heath and G. B. Giannakis, "Exploiting input cyclostationarity for blind channel identification in OFDM systems," IEEE Trans. Signal Process., vol 47, Mar. 1999.

[5] B. Muquet, M. de Courville, P. Duhamel, and V. Buzenae, "A subspace based blind and semi-blind channel identification method for OFDM systems," Proc. IEEE Workshop for Signal Process., May 1999.

[6] E. Chen, R. Tao, and X. Zhao, "Channel Equalization for OFDM System Based on the BP Neural Network," The 8th International Conference on Signal Processing, 2006.

[7] X. Wang and R. Liu, "Adaptive channel estimation using cyclic prefix in multicarrier modulation system," IEEE Comm. Lett., vol. 3, no. 10, Oct. 1999.

[8] Ali H. Sayed, "Fundamentals of Adaptive Filtering," John Wiley and Sons, Inc., 2003.