BLIND CHANNEL ESTIMATION IN DS-CDMA SYSTEMS WITH UNKNOWN WIDE-SENSE STATIONARY NOISE USING GENERALIZED CORRELATION DECOMPOSITION

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ABSTRACT

A novel blind subspace-based channel estimation technique is developed for direct-sequence code division multiple-access (DS-CDMA) systems operating in unknown wide-sense stationary noise environments. Unlike the existing blind algorithms designed for unknown noise environments, the proposed technique is applicable to any symbol constellation and does not require any auxiliary antennas at the receiver side. The proposed technique is based on the generalized correlation decomposition (GCD) that is used to obtain more accurate estimates of the noise subspace and the user-of-interest channel vector. Simulation results show that when the optimal GCD weighting matrices are used, the estimation performance is substantially improved as compared to the conventional singular value decomposition (SVD)-based blind channel estimation techniques.

Index Terms— Blind channel estimation, canonical correlation decomposition, DS-CDMA, generalized correlation decomposition.

1. INTRODUCTION

Conventional blind subspace-based channel estimation techniques are based on the assumption that the noise is white, and, as a result, the signal and noise subspaces can be identified from the eigendecomposition of the data covariance matrix [1], [2]. However, this assumption can be easily violated in practice [3]-[5] and, therefore, some other approaches should be exploited to identify the signal and noise subspaces. In [3], a blind subspace-based channel estimation technique has been developed for unknown correlated noise environments. This technique uses quite a common assumption that the noise is wide-sense stationary [4]-[6], while, unlike other estimation techniques proposed in this context [4]-[5], it is applicable to the case of an arbitrary signal constellation and does not require any auxiliary receive antenna or any knowledge of the spreading sequence of any user other than that of the user-of-interest.

The method proposed in [3] uses the centro-Hermitian property [6] of the noise covariance matrix to form a low-rank matrix that depends only on the second-order statistics of the received signals while is independent from the correlated noise. Then, the signal and noise subspaces are identified using the SVD of the so-obtained low-rank matrix. In practice, the exact data covariance matrix is not available at the receiver, but it can be estimated from the data samples. This may result in substantial errors in the noise subspace estimate and in the subsequent channel estimate.

In this paper, a technique based on GCD [7] is developed to accurately estimate the noise subspace from given data samples. It is shown that the user channel can be more precisely estimated if, instead of the conventional SVD-based noise subspace estimate of [3], the aforementioned GCD-based estimate is used. The GCD technique uses two weighting matrices that can be properly adjusted to improve the noise subspace estimation performance. We obtain the optimal GCD weighting matrices for which the mean-square error (MSE) of the orthogonal projection of any arbitrary vector in the actual signal subspace onto the estimated noise subspace is minimized in the high SNR regime. It is shown that if such optimal weighting matrices are used, the corresponding GCD can be viewed as an extension of the classical canonical correlation decomposition (CCD) [8].

The rest of the paper is organized as follows. The signal model and a brief overview on the background of the problem are given in Section 2. The proposed algorithm is presented in Section 3 and its performance is analyzed in Section 4. Simulation results are presented in Section 5 and concluding remarks are drawn in Section 6.

2. BACKGROUND

The received baseband signal of the synchronous DS-CDMA system can be represented as $x(t) = \sum_{m=-\infty}^{\infty}\sum_{k=1}^{K} A_k b_k(m) w_k(t-mT_s) + v(t)$ [3], [5] where K is the number of users, T_s is the symbol duration, $v(t)$ is the unknown correlated noise that is assumed to be wide-sense stationary [3]-[6], and $b_k(m)$, A_k, and $w_k(t)$ denote the mth i.i.d. zero-mean unit-variance data symbol, the signal amplitude, and the signature waveform of the kth user, respectively.

Let $\mathbf{c}_k \triangleq \left[c_k[0], c_k[1], \ldots, c_k[L_c-1]\right]^T$ be the spreading sequence of the kth user where L_c is the spreading factor and $(\cdot)^T$ stands for the transpose. The signature waveform of this user can be written as $w_k(t) = \sum_{l=0}^{L_c-1} c_k[l] h_k\big(t - lT_c\big)$ where $h_k(t)$ is the unknown multipath channel impulse response of the kth user that is assumed to be fixed during the observation period [3]-[5], and $T_c = T_s/L_c$ is the chip period. Assume that $h_k(t)$ is nonzero in $[0, \alpha_k T_c]$ where $L-1 \le \max\{\alpha_1, \ldots, \alpha_K\} < L$ and L is a positive integer. We consider the case $L \ll L_c$, so that the effect of intersymbol-interference (ISI) is negligible [3]-[5]. Sampling the received signal in the nth transmitted user symbols interval, the ISI-free part of the sampled data vector is given by $\mathbf{x}(n) = \sum_{k=1}^{K} A_k b_k(n)\mathbf{w}_k + \mathbf{v}(n)$ [3]-[5] where $\mathbf{x}(n) = [x(nT_s + (L-1)T_c), x(nT_s + LT_c), \ldots, x(nT_s + (L_c-1)T_c)]^T$, $\mathbf{v}(n) = [v(nT_s + (L-1)T_c), v(nT_s + LT_c), \ldots, v((nT_s + L_c - 1)T_c)$, and $\mathbf{w}_k = [w_k((L-1)T_c), w_k(LT_c), \cdots, w_k((L_c-1)T_c)]^T$. Note that the signature vector $\mathbf{w}_k$ is given by [3]-[5]

$$\mathbf{w}_k = \begin{bmatrix} c_k[L-1] & \ldots & c_k[0] \\ c_k[L] & \ldots & c_k[1] \\ \vdots & \ddots & \vdots \\ c_k[L_c-1] & \ldots & c_k[L_c-L] \end{bmatrix} \mathbf{h}_k \triangleq \mathbf{C}_k \mathbf{h}_k \qquad (1)$$

where $\mathbf{h}_k \triangleq [h_k(0), h_k(T_c), \ldots, h_k((L-1)T_c)]^T$. We assume without any loss of generality that $\mathbf{h}_k$ is a unit-norm vector and the normalization factor of the channel vector is absorbed in A_k [3]-[5]. Denoting $\mathbf{W} \triangleq [A_1\mathbf{w}_1, A_2\mathbf{w}_2, \ldots, A_K\mathbf{w}_K]$, we can rewrite the sampled data vector model as

$$\mathbf{x}(n) = \mathbf{W}\mathbf{b}(n) + \mathbf{v}(n) \qquad (2)$$

where $\mathbf{b}(n) \triangleq [b_1(n), b_2(n), \ldots, b_K(n)]^T$. From (2), we have $\mathbf{R} \triangleq \mathrm{E}\{\mathbf{x}(n)\mathbf{x}(n)^H\} = \mathbf{W}\mathbf{W}^H + \mathbf{\Sigma}$ where $\mathbf{\Sigma} \triangleq \mathrm{E}\{\mathbf{v}(n)\mathbf{v}(n)^H\}$, $\mathrm{E}\{\cdot\}$ is the statistical expectation, and $(\cdot)^H$ stands for the Hermitian transpose. As $\mathbf{v}(n)$ is wide-sense stationary, the noise covariance matrix $\mathbf{\Sigma}$ is centro-Hermitian [3], [6], that is, $\mathbf{J}\mathbf{\Sigma}^*\mathbf{J} = \mathbf{\Sigma}$ where $(\cdot)^*$ denotes the complex conjugate and $\mathbf{J}$ is the exchange matrix with ones on its antidiagonal and zeros elsewhere.

It is well known that, as the data covariance matrix $\mathbf{R}$ depends on the unknown noise covariance matrix $\mathbf{\Sigma}$, the signal and noise subspaces cannot be obtained from the eigendecomposition of $\mathbf{R}$. To get around this problem, it has been proposed in [3] (see also [6]) to identify the signal and noise subspaces from the SVD of the so-called covariance difference matrix

$$\mathbf{R}_d \triangleq \mathbf{R} - \mathbf{J}\mathbf{R}^*\mathbf{J} = \mathbf{W}\mathbf{W}^H - \mathbf{J}\mathbf{W}^*\mathbf{W}^T\mathbf{J}. \qquad (3)$$

rom equation (3), it can be observed that $\mathbf{R}_d$ depends only on the user signatures and is independent from the unknown noise covariance matrix $\mathbf{\Sigma}$. Therefore, the signal and noise subspaces can be directly obtained from the SVD of $\mathbf{R}_d$. It has been shown in [3] that under certain mild identifiability conditions, the orthogonality of the so-obtained noise subspace onto $\mathbf{w}_k = \mathbf{C}_k\mathbf{h}_k$ can be used to uniquely identify user channel vectors.

In practice, $\mathbf{R}$ is not known exactly and is usually estimated from a finite number of data samples. This may induce substantial errors in the noise subspace estimate, and, consequently, cause a significant discrepancy between the estimated channel vector and its actual value.

3. THE PROPOSED TECHNIQUE

Consider two arbitrary positive-definite matrices $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ and form

$$\mathbf{\Pi}_1^{-\frac{1}{2}}\mathbf{R}_d\mathbf{\Pi}_2^{-\frac{1}{2}} = \mathbf{\Pi}_1^{-\frac{1}{2}}[\mathbf{W}\ \mathbf{J}\mathbf{W}^*]\begin{bmatrix} \mathbf{I}_K & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_K \end{bmatrix}[\mathbf{W}\ \mathbf{J}\mathbf{W}^*]^H\mathbf{\Pi}_2^{-\frac{1}{2}} \qquad (4)$$

where $\mathbf{\Pi}_1^{1/2}$ and $\mathbf{\Pi}_2^{1/2}$ are the unique Hermitian square roots of $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$, respectively. As $[\mathbf{W}\ \ \mathbf{J}\mathbf{W}^*]$ is an $(L_c - L + 1) \times 2K$ matrix, we assume that $L_c > 2K + L - 1$ so that $\mathbf{\Pi}_1^{-1/2}\mathbf{R}_d\mathbf{\Pi}_2^{-1/2}$ becomes rank-deficient.[1] The GCD of $\mathbf{R}_d$ is defined as the SVD of $\mathbf{\Pi}_1^{-1/2}\mathbf{R}_d\mathbf{\Pi}_2^{-1/2}$ [7], that is,

$$\mathbf{\Pi}_1^{-\frac{1}{2}}\mathbf{R}_d\mathbf{\Pi}_2^{-\frac{1}{2}} = \mathbf{U}_1\mathbf{\Lambda}\mathbf{U}_2^H = [\mathbf{U}_{s1}\ \mathbf{U}_{n1}]\begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}[\mathbf{U}_{s2}\ \mathbf{U}_{n2}]^H \qquad (5)$$

where $\mathbf{\Lambda}_s$ is the $2K \times 2K$ diagonal matrix whose diagonal elements are the non-zero singular values of $\mathbf{\Pi}_1^{-1/2}\mathbf{R}_d\mathbf{\Pi}_2^{-1/2}$ and $\mathbf{U}_{s1}$ and $\mathbf{U}_{s2}$ consist of the left and right singular vectors associated those singular values, respectively. From (4) and (5), we have that

$$\mathtt{span}\big(\mathbf{\Pi}_1^{-\frac{1}{2}}[\mathbf{W}\ \ \mathbf{J}\mathbf{W}^*]\big) = \mathtt{span}(\mathbf{U}_{s1}) = \overline{\mathtt{span}}(\mathbf{U}_{n1}) \qquad (6)$$

where $\mathtt{span}(\cdot)$ and $\overline{\mathtt{span}}(\cdot)$ stand for the column span of a matrix and its orthogonal complement, respectively. Denoting $\mathbf{L}_i = [\mathbf{L}_{si}\ \mathbf{L}_{ni}] \triangleq \mathbf{\Pi}_i^{-\frac{1}{2}}[\mathbf{U}_{si}\ \mathbf{U}_{ni}] = \mathbf{\Pi}_i^{-\frac{1}{2}}\mathbf{U}_i$ for $i = 1, 2$, it can be directly obtained from (6) that [7]

$$\mathtt{span}\,([\mathbf{W}\ \ \mathbf{J}\mathbf{W}^*]) = \overline{\mathtt{span}}(\mathbf{L}_{n1}). \qquad (7)$$

Assuming without any loss of generality that the first user is the user-of-interest, from (7) it follows that $\mathbf{L}_{n1}^H\mathbf{w}_1 = \mathbf{0}$. Using the latter equation along with the fact that $\mathbf{w}_1 = \mathbf{C}_1\mathbf{h}_1$, we have that $\mathbf{h}_1$ is a solution to

$$\mathbf{T}\mathbf{h} = \mathbf{0} \qquad (8)$$

where $\mathbf{T} \triangleq \mathbf{L}_{n1}^H\mathbf{C}_1$. Using the similar argument as in [3], it can be readily shown that up to a scaling factor, $\mathbf{h}_1$ is the unique solution to (8) if and only if $\mathbf{C}_1$ is a full column-rank matrix and $\mathtt{dim}\{\mathtt{span}(\mathbf{C}_1) \cap \mathtt{span}\,([\mathbf{W}\ \ \mathbf{J}\mathbf{W}^*])\} = 1$.

Note that if $\mathbf{R}$ is exactly known and the identifiability conditions hold, the choice of $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ becomes immaterial as $\mathbf{h}_1$ can be uniquely identified from (8) for any *arbitrary* positive-definite matrices $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$. However, $\mathbf{R}$ is usually estimated as $\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n)$. In such a case, the choice of $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ has a significant effect on the accuracy of the resulting noise subspace estimate and, correspondingly, to the channel vector estimate. It will be shown in Section 4 that an accurate noise subspace estimate can be obtained if $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ are judiciously chosen based on the second-order statistics of the received data. This means that in practice, such proper values of $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ should be estimated from the received data samples. In such cases, the proposed channel estimation algorithm can be summarized as the following number od steps:

1. Compute $\hat{\mathbf{R}}_d = \hat{\mathbf{R}} - \mathbf{J}\hat{\mathbf{R}}^*\mathbf{J}$.
2. Choose suitable weighting matrices $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ and obtain their sample estimates[2] $\hat{\mathbf{\Pi}}_1$ and $\hat{\mathbf{\Pi}}_2$.
3. Use SVD of $\hat{\mathbf{\Pi}}_1^{-1/2}\hat{\mathbf{R}}_d\hat{\mathbf{\Pi}}_2^{-1/2}$ to obtain

$$\hat{\mathbf{\Pi}}_1^{-\frac{1}{2}}\hat{\mathbf{R}}_d\hat{\mathbf{\Pi}}_2^{-\frac{1}{2}} = \Big[\hat{\mathbf{U}}_{s1}\ \hat{\mathbf{U}}_{n1}\Big]\begin{bmatrix} \hat{\mathbf{\Lambda}}_s & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{\Lambda}}_n \end{bmatrix}\Big[\hat{\mathbf{U}}_{s2}\ \hat{\mathbf{U}}_{n2}\Big]^H \qquad (9)$$

where (9) is the finite-sample counterpart of (5).

[1] If $L_c > 2K + L - 1$ does not hold, we can use temporal oversampling to increase the dimension of the observation space so that $\mathbf{R}_d$ becomes low-rank [3].

[2] A proper approach to select the weighting matrices and obtain their sample estimates is discussed in Section 4.

4. Compute $\hat{\mathbf{L}}_{n1} = \hat{\mathbf{\Pi}}_1^{-1/2}\hat{\mathbf{U}}_{n1}$ and $\hat{\mathbf{T}} = \hat{\mathbf{L}}_{n1}^H\mathbf{C}_1$. Then, obtain the least-square (LS) estimate of the channel $\mathbf{h}_1$ as $\hat{\mathbf{h}}_1 = \mho\{\hat{\mathbf{T}}^H\hat{\mathbf{T}}\}$ where $\mho\{\cdot\}$ denotes the eigenvector associated with the smallest eigenvalue of a matrix.

Note that if $\mathbf{\Pi}_1 = \mathbf{\Pi}_2 = \mathbf{I}$, then the above algorithm reduces to the algorithm proposed in [3]. However, as shown in Section 4, the estimation performance of [3] can be substantially improved if $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ are properly selected based on the received data.

4. PERFORMANCE ANALYSIS

The channel estimation error in the proposed technique is caused by the fact that $\hat{\mathbf{L}}_{n1}$ does not inherit the orthogonality property of $\mathbf{L}_{n1}$ to $\mathtt{span}([\mathbf{W}\ \ \mathbf{J}\mathbf{W}^*])$, and, therefore, any non-zero vector $\mathbf{w}$ in the signal subspace may have a non-zero orthogonal projection onto the estimated noise subspace. Therefore, to improve the channel estimation performance, it is reasonable to find $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ for which the MSE of the orthogonal projection of any arbitrary $\mathbf{w} \in \mathtt{span}([\mathbf{W}\ \ \mathbf{J}\mathbf{W}^*])$ onto the estimated noise subspace is minimized. We refer to such a subspace as the *optimal* GCD-based noise subspace estimate as, among all GCD-based noise subspace estimates, it preserves the orthogonality property to the signal subspace in the best way. The matrices $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ that result in such an optimal GCD-based noise subspace estimate are also denoted as $\tilde{\mathbf{\Pi}}_1$ and $\tilde{\mathbf{\Pi}}_2$, respectively.

Let $\mathbf{P}_{n1} \triangleq \mathbf{L}_{n1}\left(\mathbf{L}_{n1}^H\mathbf{L}_{n1}\right)^{-1}\mathbf{L}_{n1}^H$ be the orthogonal projection matrix onto the noise subspace. As $\mathbf{P}_{n1}\mathbf{w} = \mathbf{0}$ for any $\mathbf{w} \in \mathtt{span}([\mathbf{W}\ \mathbf{J}\mathbf{W}^*])$, our objective is to find the optimal matrices $\tilde{\mathbf{\Pi}}_1$ and $\tilde{\mathbf{\Pi}}_2$ that minimize $e_o \triangleq \mathrm{E}\{\|\hat{\mathbf{P}}_{n1}\mathbf{w}\|^2\}$ where $\hat{\mathbf{P}}_{n1} \triangleq \hat{\mathbf{L}}_{n1}(\hat{\mathbf{L}}_{n1}^H\hat{\mathbf{L}}_{n1})^{-1}\hat{\mathbf{L}}_{n1}^H$ is the sample estimate of $\mathbf{P}_{n1}$. In the sequel, the first-order perturbation analysis is used to obtain approximate expressions for $\|\hat{\mathbf{P}}_{n1}\mathbf{w}\|^2$ and e_o. As the derived expression for e_o appears to depend on up to the fourth-order unknown noise statistics, we limit our analysis to the high SNR scenario to make the problem tractable. Then, we determine the matrices $\tilde{\mathbf{\Pi}}_1$ and $\tilde{\mathbf{\Pi}}_2$ that result into the minimal value of e_o in such a scenario. Proof of the following theorem is given in [9].

Theorem 1: The first-order perturbation analysis based approximation of $\|\hat{\mathbf{P}}_{n1}\mathbf{w}\|^2$ is given by

$$\|\hat{\mathbf{P}}_{n1}\mathbf{w}\|^2 = \mathbf{w}^H\mathbf{L}_{s1}\mathbf{\Lambda}_s^{-1}\mathbf{L}_{s2}^H\Delta\mathbf{R}_d\mathbf{P}_{n1}\Delta\mathbf{R}_d\mathbf{L}_{s2}\mathbf{\Lambda}_s^{-1}\mathbf{L}_{s1}^H\mathbf{w} \tag{10}$$

where $\Delta\mathbf{R}_d \triangleq \mathbf{R}_d - \hat{\mathbf{R}}_d$. Applying the statistical expectation to both sides of (10), we have

$$e_o = \frac{1}{N}\mathbf{z}^H\big(\mathrm{tr}\left(\mathbf{\Sigma}\mathbf{P}_{n1}\right)(\mathbf{W}\mathbf{W}^H + \mathbf{J}\mathbf{W}^*\mathbf{W}^T\mathbf{J}) + \mathbf{\Theta}_{\mathbf{P}_{n1}}\big)\mathbf{z} \tag{11}$$

where $\mathrm{tr}(\cdot)$ stands for the trace of a matrix, $\mathbf{z} \triangleq \mathbf{L}_{s2}\mathbf{\Lambda}_s^{-1}\mathbf{L}_{s1}^H\mathbf{w}$, and $\mathbf{\Theta}_{\mathbf{P}_{n1}} \triangleq \mathrm{E}\left\{(\mathbf{v}\mathbf{v}^H - \mathbf{J}\mathbf{v}^*\mathbf{v}^T\mathbf{J})\mathbf{P}_{n1}(\mathbf{v}\mathbf{v}^H - \mathbf{J}\mathbf{v}^*\mathbf{v}^T\mathbf{J})\right\}$. In the high SNR regime, the optimal matrices $\tilde{\mathbf{\Pi}}_1$ and $\tilde{\mathbf{\Pi}}_2$ that minimize (11) are given by

$$\tilde{\mathbf{\Pi}}_1 = \tilde{\mathbf{\Pi}}_2 = \mathbf{R} + \mathbf{J}\mathbf{R}^*\mathbf{J}. \tag{12}$$

The following remarks are in order.

Remark 1: In practice, $\tilde{\mathbf{\Pi}}_1$ and $\tilde{\mathbf{\Pi}}_2$ are unknown and have to be estimated using

$$\hat{\tilde{\mathbf{\Pi}}}_1 = \hat{\tilde{\mathbf{\Pi}}}_2 = \hat{\mathbf{R}} + \mathbf{J}\hat{\mathbf{R}}^*\mathbf{J} \tag{13}$$

Note from (10) that $\|\hat{\mathbf{P}}_{n1}\mathbf{w}\|^2$ is only a function of $\Delta\mathbf{R}_d$ and is independent from the estimation errors in $\hat{\mathbf{\Pi}}_1$ and $\hat{\mathbf{\Pi}}_2$. From the latter fact it can be inferred that the distortion of the resulting noise subspace estimate caused by the use of the estimated weighting matrices instead of the true ones is negligible as compared to the distortion induced by the use of $\hat{\mathbf{R}}_d$ in lieu of $\mathbf{R}_d$.

Remark 2: If $\tilde{\mathbf{\Pi}}_1$ and $\tilde{\mathbf{\Pi}}_2$ are used as the weighting matrices, then the GCD in (5) is transformed to an extended form of CCD. To show this, let us introduce $\mathbf{X}_1 = [\mathbf{x}\ \mathbf{J}\mathbf{x}^*]$ and $\mathbf{X}_2 = [\mathbf{x}\ -\mathbf{J}\mathbf{x}^*]$ where, for the sake of notational brevity, the time index n has been omitted from $\mathbf{x}(n)$. Then, it can be observed that $\mathbf{R}_d = \mathbf{R} - \mathbf{J}\mathbf{R}^*\mathbf{J} = \mathrm{E}\left\{\mathbf{X}_1\mathbf{X}_2^H\right\}$, $\tilde{\mathbf{\Pi}}_1 = \mathbf{R} + \mathbf{J}\mathbf{R}^*\mathbf{J} = \mathrm{E}\left\{\mathbf{X}_1\mathbf{X}_1^H\right\}$ and $\tilde{\mathbf{\Pi}}_2 = \mathbf{R} + \mathbf{J}\mathbf{R}^*\mathbf{J} = \mathrm{E}\left\{\mathbf{X}_2\mathbf{X}_2^H\right\}$. Therefore, when $\tilde{\mathbf{\Pi}}_1$ and $\tilde{\mathbf{\Pi}}_2$ are used, the GCD in (5) is in fact the CCD of $\mathbf{R}_d = \mathrm{E}\left\{\mathbf{X}_1\mathbf{X}_2^H\right\}$. Hence, Theorem 1 proves that the optimal GCD-based noise subspace estimate in the high SNR regime is obtained through the CCD of $\mathbf{R}_d = \mathrm{E}\left\{\mathbf{X}_1\mathbf{X}_2^H\right\}$, or, equivalently, the SVD of

$$\boldsymbol{\mathcal{R}} = \mathrm{E}\{\mathbf{X}_1\mathbf{X}_1^H\}^{-\frac{1}{2}}\mathrm{E}\{\mathbf{X}_1\mathbf{X}_2^H\}\mathrm{E}\{\mathbf{X}_2\mathbf{X}_2^H\}^{-\frac{1}{2}}. \tag{14}$$

Note that the above definition of CCD can be considered as an extension to its conventional definition [8] as the matrix $\boldsymbol{\mathcal{R}}$ in (14) is formed based on two random *matrices* $\mathbf{X}_1$ and $\mathbf{X}_2$ rather than two random *vectors*.

A related optimality property of conventional CCD has been earlier explored in [7] in the direction-of-arrival estimation context. However, apart from extended definition of CCD used in our technique, there is a fundamental difference between our algorithm and that of [7]: The algorithm proposed in [7] is based on CCD of the cross-correlation matrix of two data vectors received at a pair of well-separated antennas. In contrast to [7], our technique uses the data vector $\mathbf{x}$ received at a single antenna to build up two *virtual* data matrices $\mathbf{X}_1$ and $\mathbf{X}_2$. Then, the extended CCD technique is applied to the cross-correlation of $\mathbf{X}_1$ and $\mathbf{X}_2$ to obtain the optimal GCD-based noise subspace estimate.

Without any performance analysis or any proof of the optimality of CCD, some other applications of the conventional CCD principle can be found in [4], [5]. Interestingly, in contrast to (12), the choice of the GCD weighting matrices in the approaches of [4], [5] and [7] appears to be equal to the data covariance matrix at the receive antenna(s).

5. SIMULATIONS

In our simulation examples, $K = 7$, $L_c = 40$, and $L = 5$ are chosen. The user symbols are drawn from the QPSK constellation and the entries of the channel vector associated with each user are randomly drawn from a zero-mean complex Gaussian process. The results are averaged over 1000 independent simulation runs.

Fig. 1 shows the MSE curves of the channel vector estimate versus N for SNR = 10 dB. In Fig. 1, the noise is modelled as an unknown Gaussian random process [3] whose (l,k)-th covariance matrix entry is equal to $[\mathbf{\Sigma}]_{lk} = 0.95^{|l-k|}$. The MSE values are displayed in both the cases when $\mathbf{\Pi}_i = \mathbf{I}$ and $\mathbf{\Pi}_i = \tilde{\mathbf{\Pi}}_i$ for $i = 1, 2$. Note that in the latter case, the exact optimal weighting matrices are unavailable and their sample estimates (13) are used in the simulation. As can be

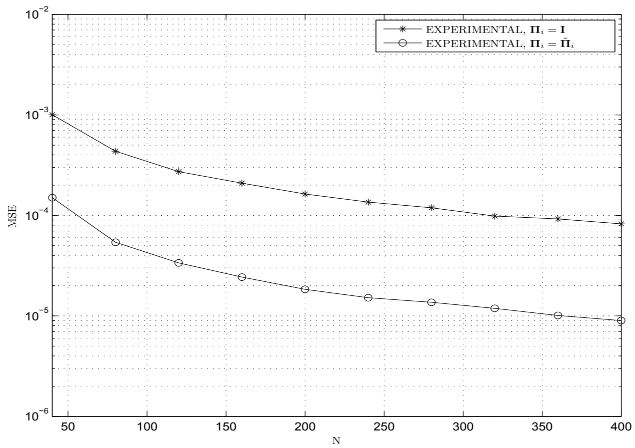

Fig. 1. MSEs of the estimated channel versus the number of data samples N for the correlated Gaussian noise.

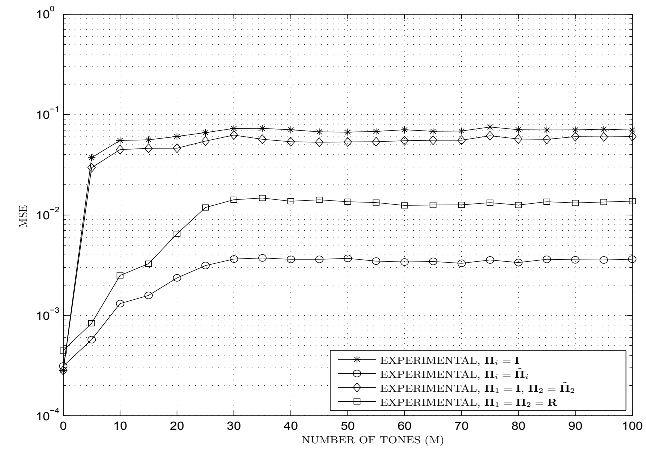

Fig. 2. MSEs of the estimated channel versus the number of noise harmonics M for the multiple-harmonics noise model.

observed from this figure, the MSE of the channel vector estimate is much lower in the case when $\mathbf{\Pi}_i = \tilde{\mathbf{\Pi}}_i$ than in the case when $\mathbf{\Pi}_i = \mathbf{I}$.

In Fig. 2 the receiver noise is modelled as multiple harmonics with the background white noise [3], [6]. In this figure, $[\mathbf{v}(n)]_k = \sum_{m=1}^{M} \sqrt{P_{vm}} e^{j(\Omega_m k + \theta(n,m))} + \xi_k(n)$ where the frequency offsets Ω_m $(m = 1, \ldots, M)$ are randomly and independently drawn from the interval $[0, \pi/2]$ and $\theta(n, m)$ are the random phases uniformly distributed in the interval $[0, 2\pi]$. Fig. 2 shows the experimental MSE curves versus the number of the noise harmonics M for $N = 100$. User powers are equal and $\eta \triangleq A_k^2/\sigma^2 = 10$ dB is assumed. The powers of all the noise harmonics are also equal and the ratio of the total power of these harmonics to the power of the white noise is given by $\sum_{m=1}^{M} P_{vm}/\sigma^2 = 20$ dB. As can be observed from Fig. 2, the best estimation performance is achieved when $\mathbf{\Pi}_i = \tilde{\mathbf{\Pi}}_i$ for $i = 1, 2$. This figure also shows that the estimation performance is quite robust against the increase of the number of noise harmonics when $\mathbf{\Pi}_i = \tilde{\mathbf{\Pi}}_i$ for $i = 1, 2$.

6. CONCLUSIONS

A novel blind subspace-based channel estimation algorithm has been proposed for DS-CDMA systems operating in unknown wide-sense stationary noise environments. The proposed approach employs the centro-Hermitian property of the noise covariance matrix along with the generalized correlation decomposition (GCD) to estimate the channel vector of the user-of-interest. The optimal values of the GCD weighting matrices which maximally preserve the orthogonality of the estimated noise subspace to the actual signal subspace are obtained in the high SNR regime. It has been demonstrated that such an optimal choice of the weighting matrices transforms GCD to an extended form of the canonical correlation decomposition. Simulation results show that the proposed GCD-based approach can achieve a substantially improved estimation performance as compared to its conventional SVD-based counterpart.

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