OPTIMAL ADAPTIVE TRANSMISSION FOR A COGNITIVE RADIO WITH SENSING

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ABSTRACT

We propose a randomized transmission scheme for minimizing a time-averaged cost metric in a cognitive radio. We assume that a single cognitive radio (i.e., a transmitter and receiver) hops over N orthogonal channels, each occupied by a primary user whose ON-OFF activity is modeled by a two-state Markov chain. We assume that the cognitive radio senses the activity in each channel at the beginning of every symbol period, and that a usage cost is assigned to each channel that depends on the channel's physical-layer characteristics and the sensing outcome. We fully characterize the transmission scheme that minimizes the time-averaged cost, subject to interference constraints imposed by the primaries. Finally, we evaluate the performance for two special cases of the cost: the bit error rate and (lower and upper bounds on) the channel capacity.

***Index Terms*—** cognitive radio, dynamic spectrum access, frequency hopping, probabilistic transmission, interference constraints

1. INTRODUCTION

The dramatic growth in the number of devices using wireless services has led to an overcrowding of the Federal Communications Commission (FCC)-licensed spectrum [6]. Dynamic spectrum access (DSA) techniques seek to increase the number of devices that can use the licensed spectrum by allowing cognitive (or low-priority) users to seek out and exploit holes in the transmissions of primary (or high-priority) users, subject to interference constraints imposed by the primaries [12]. In a basic model of (decentralized) DSA, the cognitive radio operates in two phases [13], [1], [4], [11]. First, it senses the activity of primary users at regular time epochs. Second, based on these sensing results, its statistical knowledge of how the primaries behave, and its knowledge of the interference power constraints, it adapts its transmission strategy in order to maximize the expected number of successfully delivered packets (or bits) while satisfying the interference constraints.

In this paper, we propose a randomized transmission scheme for optimizing an arbitrary time-averaged cost metric in DSA. We assume that a single cognitive radio (consisting of a transmitter and receiver) hops over N orthogonal channels, each occupied by a primary user whose ON-OFF activity is modeled by a two-state continuous-time Markov chain (CTMC). We assume that the cognitive radio senses the activity in each channel at the beginning of every symbol period, and that a usage cost is assigned to each channel that depends on the channel's physical-layer characteristics and the sensing outcome. We fully characterize the (optimal) transmission scheme that minimizes the time-averaged cost subject to interference constraints imposed by the primaries, under that assumption that these interference constraints must be satisfied over sufficiently small time intervals. Finally, we evaluate the performance for two special cases of the cost metric: the bit error rate (BER) and (lower and upper bounds on) the channel capacity.

1.1. Related Work

There is a growing body of literature on DSA for wireless cognitive systems. We focus on decentralized and hierarchical approaches to DSA, in which the cognitive system constrains the interference that it inflicts on the primary receiver.[1] For collocated systems (in which spatial separation is small), overlay and underlay systems are natural design approaches [12]. In overlay systems, cognitive users detect and transmit when primary users are silent, thus making use of idle periods between packet transmissions of the primary system. In underlay systems, cognitive users may transmit at the same time as the primary, but must use spread-spectrum or related techniques to keep their transmissions below the noise floor. Common analysis tools for DSA include information theory, game theory [14], and Markov decision processes.

Overlay approaches include [13], [1], [4], and [11]. These works focus on time-slotted cognitive systems that sense primary activity in a subset of all channels at the beginning of every slot. Based on such sensing results, a Markov decision process framework is used to maximize the cognitive throughput, i.e. the number of successful packet transmissions per unit time, subject to collision constraints at the primary receivers. In [13], it is assumed that the primary users are also slotted, and that the cognitive and primaries use the same symbol slots. Optimal and suboptimal channel sensing and access strategies are derived. In [1], it is shown that the design of the access policy in this scenario can be decoupled from the sensing policy without loss of optimality. Both [13] and [1] provide an analysis for the case of sensing errors. In [4] and [11], the primary users are modeled by CTMCs. In [4], a closed-form optimal transmission policy is derived when the interference constraints are in terms of the percentage of permissible packet collisions, and when all channels are sensed in each symbol period. In [11], a closed-form optimal transmission policy is given when a single channel is sensed in each symbol period. Underlay approaches have been approached using spread spectrum and ultra-wideband techniques [5], as well information theoretic concepts [3]. For example, in [3], information-theoretic concepts are used to derive bounds on the largest rate that the cognitive radio can achieve while not affecting the maximum rate of the primary user, when the primary and secondary transmit simultaneously.

Our model is neither a pure underlay nor a pure overlay model. Unlike the overlay models described above, we seek to minimize an arbitrary cost metric (that includes packet collisions as a special case) that depends on the physical layer parameters, the power that the cognitive imparts to each primary receiver, and the power that each primary transmitter imparts onto the cognitive receiver. Unlike underlay systems, we sense the channel and attempt to avoid colliding with the primaries according to a function of the probability, expected duration, and power of a collision. Our scheme most resembles an underlay system with sensing-based adaptation.

[1] For a discussion on centralized schemes, see, e.g., [7].

1.2. Notation

We use the following notation: (a) $\mathbb{E}[.]$ denotes expectation, (b) $X \sim \mathcal{N}(0, \sigma^2)$ indicates that X a zero mean Gaussian random variable with variance σ^2, and (c) if $\{a_k\}_{k=1}^N$ is a set of numbers, then $[a_k]$ $= [a_1, \ldots, a_N]$ denotes the associated row vector.

2. SYSTEM MODEL

We consider the operation of a single cognitive user in the presence of N primary users, as described below.

2.1. Primary Users

Assume that there exist N primary users and that each user operates independently and occupies a separate orthogonal channel.[2] At a time t, the kth primary transmitter is either ON (state 1) or OFF (state 0). Let $\delta_k(t) \in \{0, 1\}$ denote the random process describing the ON-OFF activity of the kth primary transmitter. We assume that the statistics $\{\delta_k(t)\}_t$ are described by a two-state CTMC with holding parameter λ_k in state 1 and μ_k in state 0. The transition matrix of $\delta_k(t)$ is [8, p.391]

$$P_k(t) = \frac{1}{\lambda_k + \mu_k} \begin{bmatrix} \lambda_k + \mu_k e^{-(\lambda_k+\mu_k)t} & \mu_k - \mu_k e^{-(\lambda_k+\mu_k)t} \\ \lambda_k - \lambda_k e^{-(\lambda_k+\mu_k)t} & \mu_k + \lambda_k e^{-(\lambda_k+\mu_k)t} \end{bmatrix}, \quad (1)$$

where the (i, j)th entry above is the probability that $\delta_k(t)$ is in state $(j-1)$ at time t, given that it was it was in state $(i-1)$ at time 0.

2.2. Cognitive User

The cognitive user operates using a slotted structure with symbol period T_s according to the protocol depicted in Figure 1. Consider an arbitrary symbol slot. At the beginning of the slot, the cognitive senses the activity in all N channels. Let $\underline{\omega} \triangleq [\omega_1, \ldots, \omega_N], \omega_k \in \{0, 1\}$, denote the sensing outcome. The cognitive then determines the optimal probability transmission vector based on $\underline{\omega}$. Denote this

$$\underline{p}^*(\underline{\omega}) \triangleq [p_1(\underline{\omega}), \ldots, p_N(\underline{\omega}), p_{N+1}(\underline{\omega})], \quad (2)$$

where $p_{N+1}(\underline{\omega})$ denotes the probability of not transmitting in the current slot. As described below, an optimal probability transmission vector is one that minimizes the time-averaged cost subject to interference constraints. We will sometimes omit the index $\underline{\omega}$ for terms appearing in (2) for brevity.

2.3. Interference Constraint and Cost Metric

Interference constraints limit the fraction of time that each channel can be used. Assume that the kth primary receiver advertises an average interference power constraint γ_k (Watts) to the cognitive, and let v_k be the average power that the cognitive transmitter imparts onto the kth primary receiver when it transmits. For each fixed $\underline{\omega}$, we require that

$$v_k \ p_k(\underline{\omega}) \leq \gamma_k, \ k \in \{1, \ldots, N\}. \quad (3)$$

Under this constraint, the interference power to the kth primary is no greater than γ_k, conditioned on the times that the primary is transmitting.[3] For each $\underline{\omega}$, constraint set on $\underline{p}^*$ is

$$\mathcal{C} = \left\{ \begin{array}{c} p_k \in \left[0, \min\left(\frac{\gamma_k}{v_k}, 1\right)\right] \\ k \in \{1, \ldots, N\} \end{array}, p_{N+1} \in [0, 1], \sum_{k=1}^{N+1} p_k = 1 \right\}.$$

Let $M(k|\omega_k) \geq 0$ denote the cost of transmitting over channel k given that ω_k was last sensed. It is assumed that the cognitive computes or else acquires access to the $2N$ scalar values, $\{M(k|\omega_k)\}_{k \in \{1,\ldots,N\}, \omega_k \in \{0,1\}}$ prior to operation. The cognitive seeks the transmission vector which minimizes the time-averaged cost subject to the interference constraints *and* subject to using the OFF state only when probability cannot be allocated to any other state because of the interference constraint. It can be shown that the cognitive determines $\underline{p}^*(\underline{\omega})$ according to

$$\underline{p}^* = \arg \min_{\underline{p} \in \mathcal{C}} \left\{ \sum_{k=1}^{N} p_k \ M(k|\omega_k) + p_{N+1} \left(\max_k M(k|\omega_k) + \epsilon \right) \right\}, \quad (4)$$

where $\epsilon > 0$ is an arbitrary constant which ensures that the "cost" assigned to p_{N+1} (the OFF state) is strictly greater than the cost assigned to any of the N channels.[4]

3. OPTIMAL TRANSMISSION VECTOR

The optimization problem above has a linear objective function with non-negative weights and linear constraints. We provide the solution in closed form. Define $\bar{p}_\ell \triangleq \gamma_\ell / v_\ell$. It will be instructive to give the optimal solution separately depending on the value of $\sum_{\ell=1}^N \bar{p}_\ell$. The proofs follow by contradiction and are omitted.

3.1. Non-Adaptive Transmission

Consider first the case that $\sum_{\ell=1}^N \bar{p}_\ell \leq 1$. The solution is

$$p_k = \bar{p}_k, \forall k \in \{1, \ldots, N\} \quad \text{and} \quad p_{N+1} = 1 - \sum_{\ell=1}^{N} \bar{p}_\ell.$$

This solution has some important properties. First, all N channels are utilized. However, if $\sum_{\ell=1}^N \bar{p}_\ell < 1$, then not all times-slots are utilized since $p_{N+1} > 0$. Second, the solution is non-adaptive, since it (clearly) does not depend on $\underline{\omega}$.

3.2. Adaptive Transmission

Consider next the case that $\sum_{\ell=1}^N \bar{p}_\ell > 1$. The solution is given by the following steps

1. Order channels. Let $k_1, \ldots, k_N \in \{1, \ldots, N\}$ be chosen distinctly so that $M(k_1|\omega_{k_1}) \leq \cdots \leq M(k_N|\omega_{k_N})$.
2. Find the number channels to use. Let $\Gamma \in \{1, \ldots, N-1\}$ be the largest integer such that $\sum_{\ell=1}^\Gamma \bar{p}_{k_\ell} \leq 1$.
3. Set probabilities. Set $p_{k_\ell} = \bar{p}_{k_\ell}, \forall \ell \in \{1, \ldots, \Gamma\}$, $p_{k_{\Gamma+1}} = 1 - \sum_{\ell=1}^\Gamma \bar{p}_{k_\ell}$, and $p_k = 0$ otherwise.

This solution also has some important properties. First, we only utilize all N channels if $\Gamma = N-1$ and $\sum_{\ell=1}^{N-1} \bar{p}_{k_\ell} \neq 1$. Second, the OFF state is not used, since it can be verified that $p_{N+1} = 0$. Third, this strategy is adaptive since it depends on $\{\omega_k\}$. An illustrative example will be given in Section 5.

[2] Channels could be made orthogonal in time (e.g., TDMA), frequency (e.g., OFDM), or via spreading codes (e.g., CDMA).

[3] An alternative to (3) is the constraint $v_k \ \mathbb{E}[p_k(\underline{\omega})] \leq \gamma_k$. However, under this formulation, the interference constraint could be violated over long time intervals, when the primaries (therefore, $\underline{\omega}$) evolve slowly relative to T_s.

[4] The rightmost term in (4) is necessary because the optimization has been cast as a minimization. This term does not appear in an equivalent maximization formulation.

4. EXAMPLE 1 - BIT ERROR RATE

We assign to $M(k|\omega_k)$ the average BER when we use the kth channel when ω_k is sensed. To derive values of this metric we model the cognitive channel as an additive white Gaussian noise (AWGN) channel with additional interference due to primary transmissions, and for simplicity, assume that the cognitive uses Binary Phase Shift Keying (BPSK) modulation. We assume the following symbol-level input-output relationship in the kth channel[5]

$$Z_\ell = \pm\sqrt{E_s} + V_\ell + I_\ell, \tag{5}$$

where ℓ denotes discrete time, E_s is the received energy per symbol, and $V_\ell \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_V^2)$ models receiver noise of average power σ_V^2. Let σ_k^2 denote the average power that the kth primary imparts onto the cognitive receiver when it transmits, and define the random variable (r.v.) $Y_{k,\ell} \triangleq \sigma_k^2 \, \frac{1}{T_s} \int_{t=(\ell-1)T_s}^{\ell T_s} \delta_k(t) dt$. Conditioned on $Y_{k,\ell}$, I_ℓ is a zero-mean Gaussian r.v. with variance $Y_{k,\ell}$, i.e., $I_\ell | Y_{k,\ell} \sim \mathcal{N}(0,\ Y_{k,\ell})$ (the marginal distribution of I_ℓ is difficult to obtain and will not be used in the remainder of the analysis). Note that, for any *fixed* value of $Y_{k,\ell}$, (5) is a AWGN channel. Finally, define $Y_k \triangleq Y_{k,1}$ for use when the subscript ℓ is irrelevant.

The BER-optimal symbol-level detector compares Z_ℓ to the zero threshold for each ℓ [9]. The ergodic BER is easily found by conditioning on $Y_{k,\ell}$ and averaging over $Y_{k,\ell}$. We get

$$M(k \mid \omega_k) = \mathbb{E}\left[Q\left(\sqrt{\frac{E_s}{\sigma_V^2 + Y_k}} \right) \middle| \omega_k \right], \tag{6}$$

where $Q(.)$ is the Gaussian Q-function [9, p.84]. Consider the special case that $\sigma_k^2 \ll 1, \forall k$, as this might model the case where the cognitive operates in the periphery of the primary network. We can use an asymptotic expansion on $Q(.)$ to show that

$$\lim_{\sigma_k^2 \to 0} M(k|\omega_k) = Q\left(\sqrt{\frac{E_s}{\sigma_V^2}}\right) + \sqrt{\frac{E_s}{2\pi\sigma_V^2}}\; e^{-\frac{E_s}{2\sigma_V^2}}\; \mathbb{E}\left[Y_k \mid \omega_k\right].$$

Only the right-most term in the above equation depends on k. Thus, using the metric (6) when $\sigma_k^2 \ll 1, \forall k$, is asymptotically equivalent to minimizing the expected interference power, i.e., $\mathbb{E}[Y_k|w_k]$.

We now compare the BER-performance of (6), which selects channels specifically to minimize the BER, to the metric which selects channels to minimize the expected number of collisions. A collision is defined as occurring if the primary transmits at any point during which the cognitive transmits. The metric for this scenario is [4]

$$M(k|\omega_k) = \begin{cases} 1, & \text{if } \omega_k = 1, \\ 1 - e^{-\lambda_k T_s}, & \text{if } \omega_k = 0, \end{cases} \tag{7}$$

where the second line is the probability that the cognitive starts to transmit in the next T_s seconds given that $\omega_k = 0$.

We plot the time-averaged BER versus the signal to noise ratio (SNR) $\triangleq E_s/\sigma_V^2$ for these two metrics in Figure 2 (parameters are given in the caption). Specifically, we assume that $\underline{\omega}$ starts in a state governed by the stationary distribution of the Markov chain, and then simulate the evolution of $\underline{\omega}$ in time according to (1). At high SNR, when receiver noise is negligible and the dominating factor is interference noise from the primary, (6) results in a BER several orders of magnitude smaller than (7). This is because (6) takes into the account the probability, expected duration, and interference power of a collision, whereas (7) takes into account only the probability of a collision. However, the derivation of (6) requires detailed assumptions on the cognitive channel that are not necessary in (7).

[5]We have derived this model from continuous-time waveforms by assuming that the interferer is a zero-mean Gaussian random process, perfect frequency and phase synchronization, a square matched filter at the receiver, and using the standard methodology [9]. We omit these details for brevity.

5. EXAMPLE 2 - CHANNEL CAPACITY

Let $M(k|\omega_k)$ denote the capacity of the kth channel when ω_k is sensed, and using the channel model (5).[6] Unfortunately, a technique similar to the one used to derive $M(k|\omega_k)$ in the last section cannot be used here. This is because the ergodic capacity requires that the receiver know $Y_{k,\ell}$ [10], which is not the case. Thus, we rely on upper and lower bounds on capacity.

A lower bound on $M(k|\omega_k)$ is found by replacing the r.v. $I_\ell|\omega_k$ with a Gaussian r.v. of the same variance [2, p.263]. The variance is

$$\begin{aligned} \text{VAR}[I_\ell|\omega_k] &= \mathbb{E}[I_\ell^2|\omega_k] = \mathbb{E}\left[\mathbb{E}[I_\ell^2|\omega_k, Y_{k,\ell}] \mid \omega_k\right] \\ &= \mathbb{E}\left[\mathbb{E}[I_\ell^2|Y_{k,\ell}] \mid \omega_k\right] = \mathbb{E}\left[Y_{k,\ell}|\omega_k\right], \end{aligned}$$

which can easily computed in closed form using (1). Using the expression for the capacity of the AWGN, we get the following tractable lower bound

$$M(k|\omega_k) \geq \frac{1}{2}\log\left(1 + \frac{E_s}{\sigma_V^2 + \mathbb{E}\left[Y_k \mid \omega_k\right]}\right). \tag{8}$$

An upper bound on capacity is found as follows. Assume that by sensing, the transmitter and receiver receive *full* knowledge of the interference pattern in the ℓth symbol interval, i.e., $\delta_k(t), t \in [(\ell-1)T_s, \ell T_s]$, rather than just $w_k = \delta_k(\ell T_s)$. The interference is then Gaussian distributed with *known* variance. We get the following tractable upper bound on capacity

$$M(k|\omega_k) \leq \mathbb{E}\left[\frac{1}{2}\log\left(1 + \frac{E_s}{\sigma_V^2 + Y_k}\right) \middle| \omega_k\right]. \tag{9}$$

The lower and upper bounds converge to the same value, and therefore the exact capacity, as $\text{VAR}[Y_{k,\ell}|\omega_k] \to 0$. The proof follows from the fact that (8) can be related to (9) using Jensen's inequality. Using (1), it can be verified that for $\omega_k \in \{0, 1\}$

$$\lim_{T_s \to 0} \text{VAR}[Y_{k,\ell}|\omega_k] = \lim_{T_s \to \infty} \text{VAR}[Y_{k,\ell}|\omega_k] = 0. \tag{10}$$

This behavior is verified in Figure 3, where we plot time-averaged values of the lower (8) and upper (9) bounds on capacity versus the symbol period when $N = 5$ for SNR $\in \{0, 10\}$ dB (other parameters are given in the caption). As expected, the lower and upper bounds are tight for values of the values of T_s indicated by (10).

In Figure 4, we verify the observations on $\underline{p}^*(\underline{\omega})$ made in Section 3. We plot $p_k(\underline{\omega}), k \in \{2, 3, 6\}$, over 200,000 realizations of $\underline{\omega}$, assuming that $\underline{\omega}$ is initially in a state governed by the stationary distribution of the Markov chain and that it evolves according to (1). We use the same parameters as in Figure 3 but fixing SNR $= 10$ dB and $T_s = 0.01$. For clarity, we have plotted a histogram of the values taken on by each function (i.e., we have sorted the x-axis). Note that $\sum \bar{p}_\ell > 1$. From Section 3.2, we expect $p_6(\underline{\omega}) = 0$ and for $p_2(\underline{\omega})$ and $p_3(\underline{\omega})$ to adapt with $\underline{\omega}$. Indeed, it is seen that both $p_2(\underline{\omega})$ and $p_3(\underline{\omega})$ take on four different values depending on $\underline{\omega}$, including zero. We have repeated this figure for $[\bar{p}_k] = [0.12, 0.18, 0.22, 0.25, 0.20]$ (not shown). Here, $\sum \bar{p}_\ell = 0.97 < 1$. As predicted in Section 3.1, it is seen that there is a non-zero probability of not transmitting, $p_6(\underline{\omega}) = 0.03 > 0$, and that the scheme does not adapt with $\underline{\omega}$, as $p_2(\underline{\omega}) = 0.18$ and $p_3(\underline{\omega}) = 0.22$ independently of $\underline{\omega}$.

[6]We seek to *maximize* the metric in this case. This requires only trivial modification of optimal solution given in Section 3.

6. SUMMARY AND FUTURE WORK

We have proposed an optimal randomized transmission scheme for minimizing an arbitrary time-averaged cost metric in DSA. For a summary of our model and key results, see the second paragraph of Section 1. It was seen that by designing directly for the metric of interest (e.g., BER or capacity), we can improve performance and provide additional insights relative to schemes which minimize only the expected number of packet collisions. We made some simplifications to facilitate the analysis. It was assumed that sensing consumes no resources (time or energy). As future work, we would like to study sensing as part of resource budget. An optimization would potentially answer how many channels to sense, and which ones. We assumed that both the transmitter and receiver sense channels perfectly. When sensor errors are introduced, the transmitter and receiver will occasionally lose synchronization in their mutual hopping pattern. It would be interesting to see if there exist analytic approaches to this problem. Finally, we are interested in studying energy allocation and adaptive modulation based on $\underline{\omega}$.

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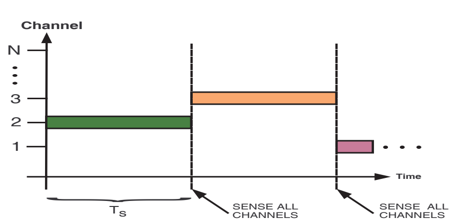

Fig. 1. At the start of each symbol slot, the cognitive senses the (instantaneous) primary activity in all N channels. Based on the sensing outcome, it constructs a distribution on the next channel on which to transmit. It then transmits according to this distribution.

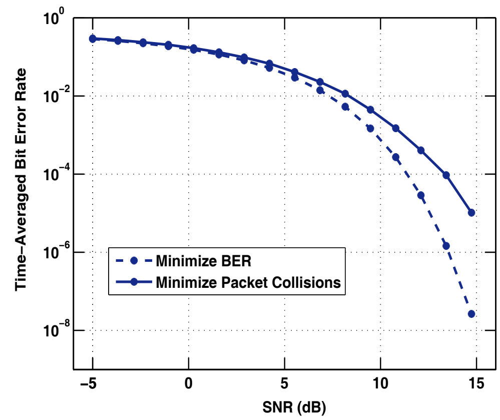

Fig. 2. The BER versus SNR for the cost metrics (6) and (7). Parameters are $N = 4$, $[\lambda_k] = [\mu_k] = [0.1, 0.1, 10, 10]$, $T_s = 0.01$, $[\sigma_k^2] = [1, 1, 0.01, 0.01]$, and $[\bar{p}_k] = [0.5, 0.5, 0.5, 0.5]$.

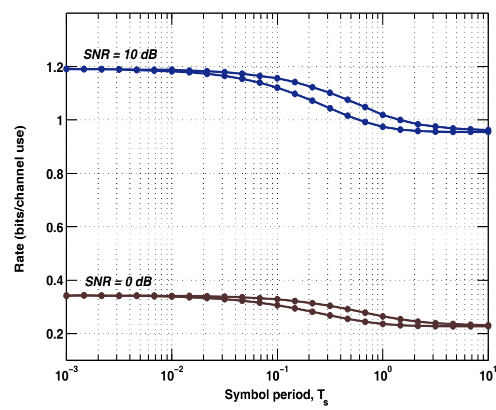

Fig. 3. The lower and upper bounds on the channel capacity versus the symbol period T_s for SNR $\in \{0, 10\}$ dB. Other parameters: $N = 5, [\lambda_k] = [1, 3, 2, 5, 2.5]$, $[\mu_k] = [1, 1, 1, 1, 1]$, $[\sigma_k^2] = [10, 5, 2, 5, 8]$, and $[\bar{p}_k] = [0.36, 0.60, 0.56, 0.60, 0.44]$.

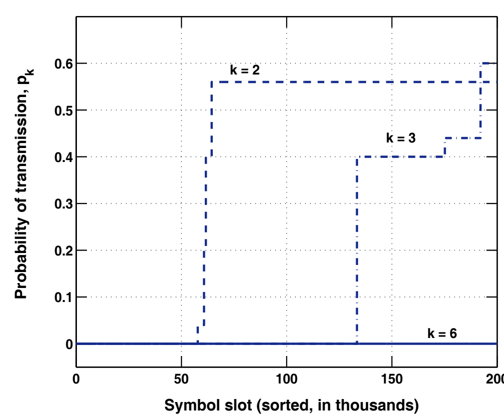

Fig. 4. The optimal transmission probabilities $p_2(\underline{\omega}), p_3(\underline{\omega}), p_6(\underline{\omega})$ versus symbol slot number for the upper bound on capacity. The parameters are the same as in Figure 3 except we have fixed SNR = 10 dB and $T_s = 0.01$.