# RESOURCE ALLOCATION IN DMT TRANSMITTERS WITH PER-TONE PULSE SHAPING

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## ABSTRACT

Per-tone pulse shaping has been proposed as an alternative to time domain spectral shaping for DMT transmitters, e.g. VDSL modems. It shapes the spectrum of individual tones such that the stop band energy of each tone can be minimized. This in particular enables transmitter to use more tones without violating the PSD mask constraint for data transmission. In per-tone pulse shaping based DMT transmitters a fixed length pulse shaping filter is typically used for every tone. The tones in the middle of the pass band however, contribute less to the overall stop band energy, so that using a high order pulse shaping filter for these tones does not result in a significant reduction of the stop band energy. As a result a significant number of pulse shaping filter taps are wasted on the tones in the middle of the pass band and do not bring any performance gain. Using a variable length pulse shaping filter which is designed such that the PSD mask constraint is not violated can then significantly reduce the total number of pulse shaping filter taps without compromising performance. In this paper, a resource allocation technique is presented for variable length pulse shaping filter design using a dual problem formulation. This optimally solves the problem of pulse shaping filter tap distribution over tones for given PSD mask constraints, with a relatively low complexity.

***Index Terms***— Multicarrier Communication, VDSL, Resource management

## 1. INTRODUCTION

Very high speed Digital Subscriber Line (VDSL) modems use Discrete Multi-Tone (DMT) modulation [1]. DMT divides the available spectrum into smaller parallel sub-bands or tones. Each tone corresponds to an orthogonal carrier. The input bit-stream is divided into several independent parallel streams which then QAM-modulate the different carriers. These QAM modulations are implemented based on an IFFT [2]. A cyclic prefix is added to each resulting time domain symbol before transmission. If the cyclic prefix is too short namely shorter than the channel impulse response, then this results in inter-symbol interference (ISI) and intercarrier interference (ICI). Highly dispersive channels such as the VDSL channel have a very long channel impulse response hence to mitigate ISI/ICI a very long cyclic prefix is needed.

The DFT (Discrete Fourier Transform) filters used to implement the DMT have a poor frequency response. Firstly, the first side lobe is only 13 dB below the main lobe and secondly the rate of decay of the sidelobe energy is only inversely proportional to frequency. In some applications like VDSL the PSD of the transmitting signal is not allowed to exceed a predefined PSD mask defining one or more pass bands and stop bands. In order to satisfy this constraint, many tones near the band edges can not be used, which then significantly reduces the available bandwidth for data transmission. There are many techniques used to mitigate this problem such as the time domain windowing method used in the VDSL standard [3]. Recently, Phoong et al. proposed a per tone spectrum shaping method to reduce the energy in the stop band [4]. In this method a pulse shaping filter is designed for each tone such that its stop band energy contribution is minimized. This method allows for asymmetric filters which helps in reducing the stop band energy of the tones at the band edges. This in turn helps to increase the number of tones that can be used for data transmission without violating the PSD mask constraint compared to time domain spectrum shaping. The resulting transmitter structure was shown to be a dual of the per tone equalization structure [5] proposed by Van acker et al, where the implementation complexity of the transmitter is reduced significantly through the usage of sliding FFT operations with difference terms. The length of cyclic prefix for this setup increases by the order of the pulse shaping filter used.

In the DMT transmitter using per-tone pulse shaping [4], every tone is shaped using a shaping filter of fixed length. However tones in the middle of the pass band contribute less

This research work was carried out at the ESAT laboratory of the Katholieke Universiteit Leuven, in the frame of K.U.Leuven Research Council CoE EF/05/006 Optimization in Engineering and the Marie-Curie Fellowship EST-SIGNAL program (http://est-signal.i3s.unice.fr) under contract No. MEST-CT-2005-021175. The scientific responsibility is assumed by its authors.

to the overall stop band energy and therefore the higher order pulse shaping filter does not improve the performance significantly. Therefore using a constant length pulse shaping filter for all tones may correspond to a waste of system resources. A better alternative is then to consider a variable length pulse shaping filter for every tone. Furthermore as it is also pointed out in [4], the variable length pulse shaping filter will further reduce the complexity of the transmitter. Therefore we need an efficient method to distribute the filter taps among the given number of tones, such that a minimum number of taps is used without violating the PSD mask constraint.

For a given number of tones, the optimum filter tap distribution problem can be turned into a dual problem using Lagrange multipliers. In this paper, a resource allocation algorithm to determine the optimum lengths of the pulse shaping filters is proposed using this dual problem formulation.

In section 2, a system model of the DMT transmitter employing the per-tone pulse shaping is presented. In section 3, a resource allocation problem is formulated,which can be used to compute the optimal number of filter taps for each tone under PSD mask constraint. An algorithm to solve this optimization problem will then be presented. Section 4 contains some simulation results. Finally conclusion are presented in section 5.

## 2. SYSTEM MODEL

### 2.1. Per tone pulse shaping

The following notation is adopted in the description of the DMT system. $\{.\}^T$ denotes the transpose, $N$ is the size of the IDFT and $i$ denotes the tone index. $P$ is the order of the pulse shaping filter, $L$ is the cyclic prefix length. $\mathbf{x}_l$ is the output of the transmitter i.e. a vector of length $K = N + P + L$, corresponding to one transmitted symbol.

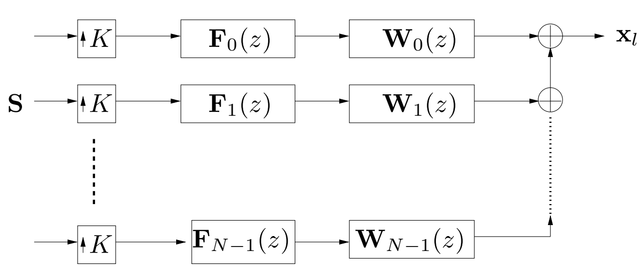


**Fig. 1**. DMT transmitter with per tone shaping filter

**Figure 1** shows the transmitter structure of the DMT with the per-tone shaping as derived in [4]. The per-tone shaping filters are designed using a minimum stop band energy criterion. We will adopt the following design criterion for every filter $W_i(e^{j\omega})$:

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & (1-\beta)\int_{\Omega 1} |F_i(e^{j\omega})W_i(e^{j\omega})|^2 d\omega \\ & +\beta \int_{\Omega 2} |F_i(e^{j\omega})W_i(e^{j\omega})|^2 d\omega \\ \text{subject to} \quad & W_i(e^{j\omega i/N}) = 1 \end{aligned}$$

where
$\Omega 1$=stop band frequencies as defined by PSD mask
$\Omega 2$=pass band frequencies as defined by PSD mask
$W_i(ej\omega) = \mathbf{e}^H \mathbf{w}_i$
$\mathbf{e}^H = \left[1 \; e^{j\omega} \cdots e^{jP\omega}\right]$
and where $\beta$ is a small constant $0 < \beta = \epsilon < 1$. This is included to prevent the stopband from amplifying which can result in a signficant increase of transmitting power [4]. The design criterion

$$\min_{\mathbf{w}_i} \quad \mathbf{w}_i^H \mathbf{Q}_i \mathbf{w}_i \quad \text{subject to} \quad W_i(e^{j2\pi i/N}) = 1 \qquad (1)$$

where,
$\mathbf{Q}_i = (1-\beta)\int_{\Omega 1} |F_i(e^{j\omega})|^2 \mathbf{e}\mathbf{e}^H \, d\omega + \beta \int_{\Omega 2} |F_i(e^{j\omega})|^2 \mathbf{e}\mathbf{e}^H \, d\omega$
This is a linearly constrained least squares problem, which is easily solved for every $i$ and for every filter order $P_i$.

### 2.2. Modified transmitter

From **Figure 1** we can see that the transmitter output is

$$\mathbf{x}_l = \sum_i s_i \mathbf{f}_i \star \mathbf{w}_i \qquad (2)$$

where $s_i$ is the input signal at tone $i$, $\mathbf{f}_i$ is the DFT filter and $\mathbf{w}_i$ is the shaping filter for tone $i$. In [4] it has been shown that this transmitter output can be generated cheaply based on as

$$\mathbf{x_l} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_L \\ \multicolumn{2}{c}{\mathbf{I}_N} \\ \multicolumn{2}{c}{\mathbf{0}_{P\times N}} \end{pmatrix} \mathbf{\Phi}^H \mathbf{D}\mathbf{s} + \sum_{i=0}^{N-1} \begin{pmatrix} -\alpha^{i(L+1)}\bar{\mathbf{v}}_i \\ \mathbf{0}_{(K-P)\times 1} \\ \alpha^i \bar{\mathbf{v}}_i \end{pmatrix} s_i. \quad (3)$$

Here $\mathbf{I}_k$ is the $k$ by $k$ identity matrix, $\mathbf{\Phi}^H$ is the IDFT matrix, **s** is the input vector, $\alpha = e^{-j2\pi/N}$, $\mathbf{v}_i = \mathbf{U}_i\mathbf{w}_i = [v_{i,0} \; \bar{\mathbf{v}}_i^T]^T$, $\mathbf{U}_i$ is an $(P+1)$ by $(P+1)$ upper triangular Toeplitz matrix whose first row is $[1 \; \alpha^i \cdots \alpha^{iP}]$ $\mathbf{D}$ is a diagonal matrix with diagonal elements $[v_{0,0} \; v_{1,0} \cdots v_{N-1,0}]$.

## 3. PROBLEM FORMULATION

Tones in the middle of the pass band do not contribute much to the over all stop band energy, hence can be assigned a pulse shaping filter with fewer taps without significantly affecting the overall performance. This motivates the use of variable length pulse shaping filter for the different tones. This then requires procedure to distribute the filter taps over the tones optimally, and such that the PSD mask constraint is still fulfilled. One possibility is to perform an exhaustive search over all possible combinations of shaping filter lengths which satisfy the PSD mask constraint. Even though it can be implemented with a simple algorithm, this method has a prohibitively high complexity.

Here we present an alternative approach that is based on restating the optimization problem as a dual problem. Our optimization problem can be written as

$$\max_{P_1 \cdots P_N} \quad -\sum_i P_i \quad \text{subject to} \quad \mathbf{T}_{mask} \geq \mathbf{T}_{spec} \qquad (4)$$

where $P_i$ is the shaping filter order for the $i$-th tone, $\mathbf{T}_{mask}$ is the vector containing $M$ sample points of the PSD mask at different frequencies and $\mathbf{T}_{spec}$ is the vector containing this similarly sampled PSD of the transmitter output. Now we can use Lagrange multipliers to integrate the constraint in the objective function, readily to a dual problem formulation

$$\max_{P_1 \cdots P_N} \quad -\sum_i P_i + \sum_j \lambda_j (T_{mask,j} - T_{spec,j})$$

where $\lambda_j > 0$ and $j$ is the index for the frequency samples. With,

$$\begin{aligned} T_{spec,j} &= \sum_i \sigma_i^2 |F_i(\omega_j) W_i(\omega_j)|^2 \\ &= \sum_i \sigma_i^2 |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2 \end{aligned} \qquad (5)$$

,where $\sigma_i^2 = E\{|s_i|^2\}$, we can write equation (5) as,

$$\max_{P_1 \cdots P_N} \quad -\sum_i P_i - \sum_j \lambda_j \sum_i \sigma_i^2 |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{w}_i|^2 + \sum_j \lambda_j T_{mask,j}$$

which is equivalent to

$$\max_{P_1 \cdots P_N} \quad -\sum_i \{P_i + \sum_j \lambda_j \sigma_i^2 |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2\} + \underbrace{\sum_j \lambda_j T_{mask,j}}_{\text{constant for a given } \lambda_j} \qquad (6)$$

For a given $\lambda_j$, $\sum_j \lambda_j T_{mask,j}$ is a constant and therefore maximizing equation ( 6) is equivalent to

$$\max_{P_1 \cdots P_N} \quad -\sum_i \{P_i + \sum_j \lambda_j \sigma_i^2 |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2\} \qquad (7)$$

Hence, the problem is decoupled over tones, and can be written for each tone as,

$$\min_{P_i} \quad P_i + \sum_j \lambda_j \sigma_i^2 |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2 \qquad (8)$$

Here, $\lambda_j$ depends on the difference between the PSD mask and the output PSD, hence the update formula can be written as

$$\lambda_j^{t+1} = \lambda_j^t - \mu_j (T_{mask,j} - T_{spec,j}) \qquad (9)$$

where $\mu_j$ is the step size for the update which is always positive. $\mu_j$ can be varied in order to drop the energy at a frequency point quickly if the output spectrum at that frequency exceeds the PSD mask and to cautiously increase it if the output spectrum is under the PSD mask. A typical initialization value for $\lambda = \{\lambda_j\}$ is 0 and then $\mu$ can be initialized with a value generally much larger than 1 as we need to establish a $\lambda$ very high value for $\lambda$ in order to make the $\sum_j \lambda_j \sigma_i^2 |F_i(\omega_j) \mathbf{e}^H(\omega_j) \mathbf{W}_i|^2$ comparable to the value $P_i$. $P_{max}$ is the maximum order of the pulse shaping filter allowed per tone as dictated by the prefix length and the chosen impulse response length [4]. The algorithm stops only when the total number of taps stops updating. The complete algorithm is given below.

**Algorithm: Optimization of pulse shaping filter length**

```
Initialize λ,μ
repeat
    For i ∈ {used tones}
        For P_i = 0 ··· P_max
            compute w_i
            L_i ⇐(argmin) {P_i + Σ_j λ_j σ_i^2 |F_i(ω_j) W_i(ω_j)|^2}
        End For
    End For
    compute T_spec
    For j = 1 ··· M
        If ((T_mask,j − T_spec,j) < 0)
            μ_j = 2μ_j
        Else
            μ_j = μ_j/2
        End If
        λ_j^{t+1} = λ_j^t − μ_j (T_mask,j − T_spec,j)
        If (λ_j^{t+1} < 0)
            λ_j^{t+1} = 0
        End If
    End For
While (change in total filter taps)
```

## 4. SIMULATION RESULTS

In our simulation, we consider VDSL downstream transmission with a PSD mask corresponding to the FTTCab M1 deployment scenario [3]. The size of the FFT is 1024, the cyclic prefix plus the overhead due to the pulse shaping filter is 80 samples. We use $P_{max} = 16$ to be comparable to [4]. The initial values are $\lambda_j = 0$ and $\mu_j = 10000$. All the tones carry an equal amount of power.

In **Figure 2** we can see that the algorithm terminates after the 13-th iteration as there is no further change in the total number of taps anymore. This is the minimum total number of taps needed in to satisfy the PSD mask with the given tones. The value at the first iteration is 375, which is equal to the total number tones, as for $\lambda_j = 0$ all the tones have $P_i = 1$. It can be seen that the final total number of taps is reduced from 6375(=17*375) to 3787 (i.e. if a fixed filter of length 17 is used for all the 375 tone as in [4]). This corresponds to a significant reduction of almost 40%. As the tone with less power can have fewer number of filter taps without affecting the overall performance we can further reduce the number of taps if we also apply power loading. The distribution of the filter taps across the tones is given in **Figure 3**. It can be clearly seen that the tones at the band edges receive more filter taps than the tones in the middle of the band. **Figure 4** shows the resulting output PSD and the PSD mask.

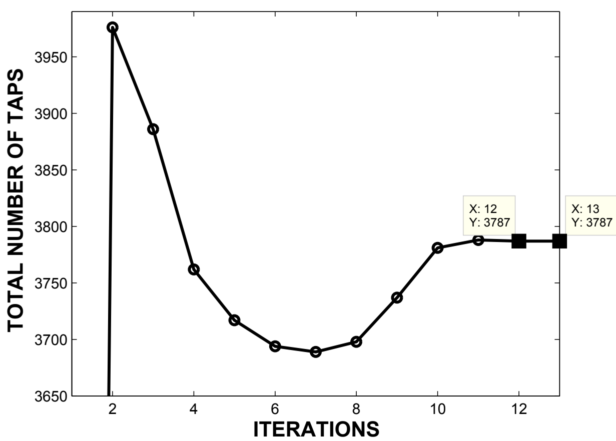


**Fig. 2**. Convergence of total number of taps

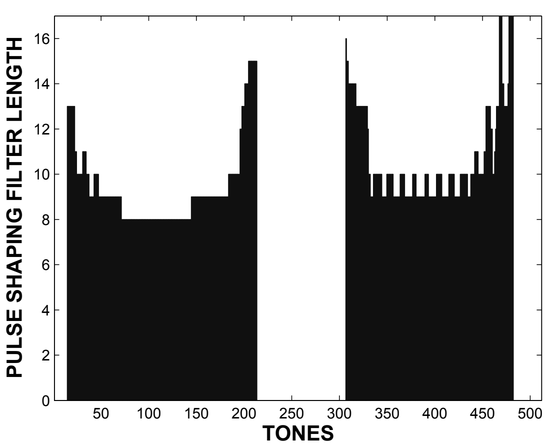


**Fig. 3**. Optimum distribution of taps amongst the tones

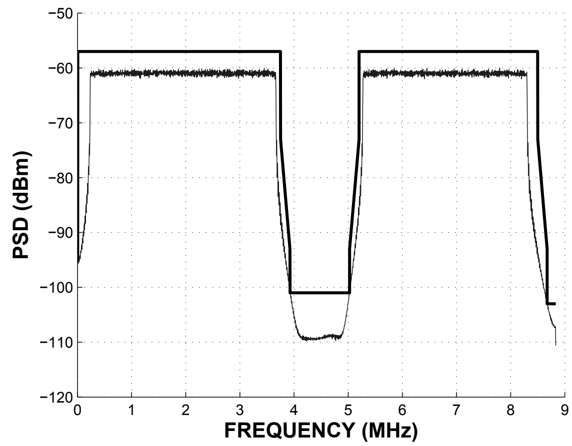


**Fig. 4**. Output PSD after the optimum distribution of number of taps amongst the tones

## 5. CONCLUSION

In this paper, we have proposed an efficient resource allocation algorithm to distribute the filter taps amongst the used tones in a DMT transmitter with per-tone pulse shaping. We have shown that a dual problem formulation leads to an optimization that is instead decoupled over tones and furthermore the number of taps can be significantly reduced without violating the spectrum mask. A VDSL simulation demonstrates that the number of taps can be reduced to approximately 40% without sacrificing performance.

## 6. REFERENCES


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