LOW-COMPLEXITY DYNAMIC SPECTRUM MANAGEMENT ALGORITHMS FOR DIGITAL SUBSCRIBER LINES

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ABSTRACT

Modern DSL networks suffer from crosstalk between different lines in the same cable bundle. By carefully choosing the transmit power spectra, the impact of crosstalk can be minimized leading to spectacular performance gains. This is also referred to as Dynamic Spectrum Management (DSM). This paper presents three novel low-complexity DSM algorithms with a different level of required message-passing. This level ranges from fully autonomous and distributed to semi-centralized execution. Simulations show good performances compared to existing state-of-the-art DSM algorithms.

Index Terms— Dynamic spectrum management, ADSL, multi-user, multi-carrier, distributed algorithms

1. INTRODUCTION

Digital Subscriber Line (DSL) technology remains by far the most popular broadband access technology. The increasing demand for higher data rates forces DSL systems to use higher frequencies. At these high frequencies, electromagnetic coupling becomes particularly harmful and causes interference, also called crosstalk, between lines operating in the same cable bundle. This crosstalk is a major obstacle for modern DSL systems towards reaching higher data rates.

Dynamic Spectrum Management (DSM) [2] refers to a set of solutions to the crosstalk problem. Basically these solutions consist of signal level coordination and/or spectrum level coordination amongst the different modems. In this paper the focus is on spectrum level coordination, which is also referred to as spectrum balancing or power control. Here the transmit power spectrum of each modem is designed to cause minimal disturbance to other modems, while preserving a high data rate. The problem of optimally choosing the transmit power spectra in order to maximize the data rates of the network can be formulated as an optimization problem [3] which is referred to as the spectrum management problem. Unfortunately this optimization problem is nonconvex and can have multiple local optima.

In this paper the focus is on DSM algorithms that are executed by the modems locally, also referred to as autonomous and/or distributed spectrum management algorithms.

One of the first autonomous algorithms is iterative water-filling (IW) [4]. Here each modem focuses on maximizing its own data rate without taking into account the damage caused to the other modems. In spite of its selfish nature, IW performs well for small crosstalk DSL scenarios. However for large crosstalk scenarios it can perform quite suboptimally. The ASB algorithm [5] is an alternative autonomous algorithm that removes this selfish behavior by incorporating the damage caused to a reference line. This leads to a more social behavior and so a better overall network performance. In this paper we propose an algorithm based on the same concept as ASB but provide an alternative way of solving the corresponding optimization problem. The main advantage of the proposed approach is that it is easier to incorporate multiple reference lines without impacting the computational complexity. Because of the similarity with the ASB algorithm, this algorithm is called ASB-2.

In [6] a spectrum management algorithm is proposed based on a combination of a local spectrum management algorithm in addition to a limited message-passing protocol. This algorithm is called SCALE and is based on an iterative convex approximation approach. In this paper we also propose a similar locally optimal algorithm based on another type of convex approximation, called distributed spectrum balancing (DSB). The main difference is that the convex approximation is chosen so that the resulting transmit spectrum update formula has a water-filling type of equation, which can be relevant from a practical implementation point of view.

The SCALE and DSB algorithms both converge to locally optimal solutions. As the spectrum management problem can have multiple local optima with significant difference in objective function value, these locally optimal algorithms can converge to suboptimal solutions. In this paper a multiple starting point approach is finally proposed to tackle this problem. This algorithm is called MS-DSB.

This paper is organized as follows. In section 2 the system model for the crosstalk environment is described. In section 3 the spectrum management problem is reviewed. In section 4 the three novel spectrum management algorithms are presented. Finally, in section 5 simulation results are shown.

2. SYSTEM MODEL

Most current DSL systems use Discrete Multi-Tone (DMT) modulation. For perfect tone synchronisation, the transmission for a binder of N modems can be modeled on each tone k by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \qquad , k = 1 \dots K.$$

The vector $\mathbf{x}_k = [x_k^1, x_k^2, \dots, x_k^N]^T$ contains the transmitted signals on tone k for all N modems. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n on tone k. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. $\mathbf{z}_k$ is the vector of additive noise on tone k, containing thermal noise, alien crosstalk, RFI,.... The vector $\mathbf{y}_k$ contains the received symbols.

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The transmit power is denoted as $s_k^n \triangleq \Delta_f E\{|x_k^n|^2\}$, the noise power as $\sigma_k^n \triangleq \Delta_f E\{|z_k^n|^2\}$. The vector containing the transmit power of modem n on all tones is $\mathbf{s}^n \triangleq [s_1^n, s_2^n, \ldots, s_K^n]^T$. The DMT symbol rate is denoted as f_s, the tone spacing as Δ_f.

When the number of interfering modems is large, the interference is well approximated by a Gaussian distribution. Under this assumption the achievable bit loading for modem n on tone k, given the transmit spectra $\mathbf{s}_k \triangleq [s_k^1, s_k^2, \ldots, s_k^N]^T$ of all modems in the system, is

$$b_k^n \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right), \qquad (1)$$

where Γ denotes the SNR-gap to capacity, which is a function of the desired BER, the coding gain and noise margin [7]. The total bit rate for modem n and the total power used by modem n are $R^n = f_s \sum_k b_k^n$ and $P^n = \sum_k s_k^n$ respectively.

Note that although this text focuses on the synchronous DSL transmission case explained above, it can straightforwardly be extended to the asynchronous DSL transmission case [8].

3. SPECTRUM MANAGEMENT PROBLEM

The problem of optimally balancing the transmit power spectra in order to maximize the data rates of the DSL network is referred to as the rate adaptive spectrum management problem. The objective is to find the optimal transmit spectra for a bundle of interfering DSL lines, maximizing a weighted bit rate, subject to per-modem total power constraints and spectral mask constraints. This can be formulated as follows:

$$\begin{array}{l} \max_{\mathbf{s}^1,\ldots,\mathbf{s}^N} \sum_{n=1}^N w_n R^n (= f_0) \\ \text{s.t.} \sum_k s_k^n \leq P^{n,\text{tot}} \quad , \ \forall n, \\ \text{s.t.}\ 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \ \forall n, \forall k, \end{array} \qquad (2)$$

where $P^{n,\text{tot}}$ denotes the total power budget for modem n and $s_k^{n,\text{mask}}$ denotes the spectral mask for modem n on tone k. The weights w_n are used to put more emphasis on some modems. In [3] it is explained how these weights can be adjusted if extra data rate constraints need to be satisfied.

4. DYNAMIC SPECTRUM MANAGEMENT ALGORITHMS

4.1. Distributed Spectrum Balancing (DSB)

In this section a distributed algorithm is proposed called distributed spectrum balancing (DSB). It has a similar approach as the algorithm from [9] but it requires less message-passing and it does not require solving a system of N equations. The derivation starts from a reformulation of the objective function of (2)

$$\begin{array}{l} f_0 = \sum_{n=1}^N w_n f_s \sum_k \log_2(\sum_{m=1}^N |\tilde{h}_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n) \\ \qquad \underbrace{-\sum_{n=1}^N w_n f_s \sum_k \log_2(\Gamma(\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m + \sigma_k^n))}_{\text{A=non-concave part}} \\ \text{with} \quad |\tilde{h}_k^{n,m}|^2 \begin{cases} = \Gamma|h_k^{n,m}|^2 & ,n \neq m \\ = |h_k^{n,n}|^2 & ,n = m. \end{cases} \end{array} \qquad (3)$$

The basic idea is to convexify this objective function by approximating its non-concave part A in a point $\mathbf{s}_{k,\text{ap}}$ by a lower bound hyperplane as follows

$$-\sum_{m\neq n} a_k^{m,n} s_k^m + c_k^n \leq -\log_2(\Gamma(\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m + \sigma_k^n)) \quad (4)$$

with equality in point $\mathbf{s}_{k,\text{ap}}$ and $a_k^{m,n} = \frac{\Gamma|h_k^{n,m}|^2/\log(2)}{\sum_{p\neq n}\Gamma|h_k^{n,p}|^2 s_{k,\text{ap}}^p + \Gamma\sigma_k^n}$.

Solving this modified (now convex) problem for fixed $a_k^{m,n}$ leads to a solution $\mathbf{s}_k, \forall k$. Using this solution as a new approximation point $\mathbf{s}_{k,\text{ap}} = \mathbf{s}_k$, it can be easily shown that the sequence of convex approximations produces a monotonically increasing objective function value which converges to a local optimum of the spectrum management problem (2).

Based on the KKT conditions of the convex approximation the following equivalent set of equations can be derived:

$$s_k^n = \left[\frac{w_n f_s/\log(2)}{\lambda_n + \underbrace{\sum_{m\neq n}\frac{w_m f_s \Gamma|h_k^{m,n}|^2}{\log(2)}\left(\frac{1}{\text{int}_k^m} - \frac{1}{\text{rec}_k^m}\right)}_{P_k^{\text{DSB},n}}} - \frac{int_k^n}{|h_k^{n,n}|^2}\right]_0^{s_k^{n,\text{mask}}} \qquad (5)$$

$$\lambda_n\left(\sum_{k=1}^K s_k^n - P^{n,\text{tot}}\right) = 0$$

where

$$\begin{array}{l} \text{int}_k^n = \sum_{m\neq n}\Gamma|h_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n \\ \text{rec}_k^n = \sum_{m=1}^N |\tilde{h}_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n \end{array} \qquad (6)$$

The quantities int_k^n and rec_k^n are the received interference and noise of modem n on tone k and the received signal of modem n on tone k respectively. Both quantities are already measured by current state-of-the-art modems. This leads to **Algorithm 1**. Every modem iteratively applies formula (5) where $P_k^{\text{DSB},n}$ is a constant and bisects on λ_n. Note that only one fixed point iteration is performed for every λ_n. In addition, the term $P_k^{\text{DSB},n}$ is updated infrequently (4) by the spectrum management center (SMC). Note that formula (5) is a water-filling type of equation with a tone dependent penalty which can be relevant from a practical implementation point of view.

Algorithm 1 Distributed Spectrum Balancing (DSB)

```
Modem n algorithm:
Initialize all s_k^n = s_k^{n,mask}/2, ∀k
loop {execute at regular intervals}
    λ_n^min = 0, λ_n^max = Λ^max , λ_n = (λ_n^max + λ_n^min)/2
    Receive messages (P_k^{DSB,n}) from SMC, ∀k
    while |Σ_k s_k^n − P^{n,tot}| > δ and λ_n > γ do
        λ_n = (λ_n^max + λ_n^min)/2
        Update s_k^n using (5), ∀k
        if Σ_k s_k^n > P^{n,tot} then
            λ_n^min = λ_n
        else
            λ_n^max = λ_n
        end if
    end while
    Measure int_k^n = Γ(Σ_{p≠n} |h_k^{n,p}|^2 s_k^p + σ_k^p), ∀k
    Compute messages (1/int_k^n − 1/rec_k^n) ,∀k, and send to SMC
end loop
SMC algorithm:
loop
    Receive messages (1/int_k^n − 1/rec_k^n) from modems n, ∀k
    Compute messages (P_k^{DSB,n}) and send to each modem n, ∀k.
end loop
```

4.2. Autonomous Spectrum Balancing 2 (ASB-2)

The ASB algorithm [5] introduced the concept of a reference line. Each modem chooses its transmit spectrum so that it maximizes its own data rate and that it takes the damage into account caused to a reference line. By taking multiple reference lines into account the performance can be improved.

Every modem n has to solve the following optimization problem where $h_k^{p,\mathrm{ref},n}, h_k^{p,\mathrm{ref}}, s_k^{p,\mathrm{ref}}, \sigma_k^{p,\mathrm{ref}}, w_p^{\mathrm{ref}}$ are constants representing respectively the crosstalk channel from the n-th modem into reference line p, the channel attenuation, the power, the noise and the weight of the reference line p for all M reference lines.

$$
\begin{aligned}
&\max_{\mathbf{s}_1^n,\ldots,\mathbf{s}_K^n} w_n f_s \sum_k \log_2\left(1+\frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m+\sigma_k^n}\right)\\
&\quad+\sum_{p=1}^{M} w_p^{\mathrm{ref}} f_s \sum_k \log_2\left(1+\frac{1}{\Gamma}\frac{|h_k^{p,\mathrm{ref}}|^2 s_k^{p,\mathrm{ref}}}{|h_k^{p,\mathrm{ref},n}|^2 s_k^n+\sigma_k^{p,\mathrm{ref}}}\right)\\
&\text{s.t.} \sum_k s_k^n \le P^{n,\mathrm{tot}},\\
&\text{s.t. } 0 \le s_k^n \le s_k^{n,\mathrm{mask}}, \forall k,
\end{aligned}
\tag{7}
$$

For M reference lines, the ASB algorithm needs to solve a polynomial equation of degree $2M+1$ for each modem n on each tone k. To reduce this complexity, an alternative solution is proposed which will be referred to as ASB-2. Based on the KKT stationarity condition of (7) the following fixed point equation can be derived:

$$
s_k^n = \left[\frac{w_n f_s/\log(2)}{\lambda_n + P_k^{\mathrm{ASB}-2,n}} - \frac{\sum_{m\neq n}\Gamma|h_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n}{|h_k^{n,n}|^2}\right]_0^{s_k^{n,mask}}
\tag{8}
$$

with $P_k^{\mathrm{ASB}-2,n} =$
$\sum_{p=1}^{M} \frac{w_p^{\mathrm{ref}} f_s |h_k^{p,\mathrm{ref}}|^2 s_k^{p,\mathrm{ref}} \Gamma |h_k^{p,\mathrm{ref},n}|^2/\log(2)}{(\Gamma|h_k^{p,\mathrm{ref},n}|^2 s_k^n + \Gamma\sigma_k^{p,\mathrm{ref}})(|h_k^{p,\mathrm{ref}}|^2 s_k^{p,\mathrm{ref}} + \Gamma|h_k^{p,\mathrm{ref},n}|^2 s_k^n + \Gamma\sigma_k^{p,\mathrm{ref}})}$.

The ASB-2 algorithm uses this fixed point equation for updating its transmit powers. Note that this approach does not necessarily converge to the same solution as the ASB algorithm.

This leads to the same algorithm as Algorithm 1 where the lines 5,16,19-22 are removed as it is an autonomous algorithm. Furthermore the updating formula in line 8 is replaced by (8).

4.3. Multiple Starting Point Distributed Spectrum Balancing (MS-DSB)

The DSB algorithm from section 4.1 converges to a locally optimal solution. As the spectrum management problem (2) is nonconvex with multiple local optima, this local optimum is not necessarily the global optimum. Note that DSB starts from a set of initial transmit powers as can be seen in Algorithm 1 line 2. In the case of multiple local optima, a different set of initial transmit powers can converge to a different local optimum. It is not possible to find a set of initial transmit powers converging always to the globally optimal solution for all possible DSL scenarios. This is a general constraint for locally optimal DSM algorithms.

Based on this observation, a third DSM algorithm is proposed combining DSB with a multiple starting point approach. For each tone, multiple initial transmit powers are chosen and for each choice the transmit powers are updated iteratively so as to converge to a local optimum. The best of the converged results is then chosen for each tone independently. By using multiple initial transmit powers it is more likely that the global optimum will be found.

In order to keep the complexity low, the number of initial transmit power sets is fixed to $N+2$ sets. The $N+2$ initial transmit power sets are given in **Table 1** where **0** is the zero vector of dimension N, $\mathbf{s}_k^{\mathrm{mask}}$ is a vector of dimension N with the corresponding spectral masks on tone k for the N modems and $\mathbf{e_n}.s_k^{n,\mathrm{mask}}$ is the unit vector in the n-th dimension multiplied by the corresponding spectral mask. The first transmit power set corresponds to the initial transmit power set of SCALE [6]. The second transmit power set corresponds to the initial transmit power set of DSB. This choice ensures that the obtained local optimum on tone k is at least as good as those obtained by the SCALE and DSB algorithms. The remaining N transmit power sets are based on the observation that in the case of large crosstalk, the optimal solution converges to an FDMA solution where only one modem is active in each tone k. This fact was recently proved in [10]. This leads to **Algorithm 2**.

Table 1. $N+2$ initial transmit powers

1	$\mathbf{0}$
2	$\mathbf{s}_k^{\mathrm{mask}}/2$
$3\ldots(N+2)$	$\mathbf{e_n}.s_k^{n,\mathrm{mask}}$, for $n=1\ldots N$

Algorithm 2 MS-DSB

1: **SMC algorithm:**
2: **loop** {Execute at regular intervals}
3: Receive messages $\left(\frac{1}{\mathrm{int}_k^n}-\frac{1}{\mathrm{rec}_k^n}\right)$ from modems $n, \forall k$
4: Compute messages $(P_k^{\mathrm{DSB},n})\ \forall n, k$.
5: **while** total power constraints not satisfied **do**
6: update Lagrange multipliers using subgradient approach [3]
7: **for** all tones k **do**
8: **for** each initial transmit power set **do**
9: **for** iterations **do**
10: **for** each modem **do**
11: Apply formula (5)
12: **end for**
13: **end for**
14: **end for**
15: Take best of all converged solutions
16: **end for**
17: **end while**
18: Compute per-tone dependent penalties $P_k^{\mathrm{DSB},n}(5), \forall n, k$
19: Send $P_k^{\mathrm{DSB},n}$ to all modems n
20: **end loop**
21: **Modem n algorithm:**
22: **loop**
23: Apply modem part of DSB algorithm
24: **end loop**

Each modem executes the modem part of the DSB algorithm locally. The spectrum management center (SMC) executes the multiple starting point extension. The SMC regularly receives messages $\left(\frac{1}{\mathrm{int}_k^n}-\frac{1}{\mathrm{rec}_k^n}\right)$ from the N modems and computes the quantities $P_k^{\mathrm{DSB},n}$. The while loop at line 5 will search for the Lagrange multipliers so that the active power constraints are satisfied, using a subgradient approach at line 6 (see also [3] [11]). Note that all the Lagrange multipliers are updated in parallel. Then for each tone and for each initial transmit power set, the modems iterate using the DSB formula (5). The best of the converged results is used. When the while loop is converged the quantities $P_k^{\mathrm{DSB},n}$ are communicated to the modems so that they can steer their transmit spectra to improve the network performance. This algorithm will be referred to as MS-DSB, i.e. multiple starting point distributed spectrum balancing. As most of the computation happens in the SMC, this algorithm can be viewed as a semi-centralized DSM algorithm.

5. SIMULATION RESULTS

This section presents simulation results of the dynamic spectrum management algorithms presented in section 4. The ADSL downstream scenario is shown in Figure 1. The simulations are performed for a two-modem case ($N = 2$) up to a seven-modem case ($N = 7$). The four-modem scenario, for example, consists of active modems 1,2,3,4 where modems 5,6,7 are inactive. The twisted pair lines have a diameter of 0.5 mm (24 AWG). The maximum transmit power is 20.4 dBm [12]. The SNR gap Γ is 12.9 dB, corresponding to a coding gain of 3 dB, a noise margin of 6 dB and a target symbol error probability of 10^{-7}. The tone spacing Δ_f is 4.3125 kHz. The DMT symbol rate f_s is 4 kHz. The simulations are performed in Matlab on a TravelMate 4002WLMi with 768 MB of RAM and an Intel Pentium M processor 1.60 GHz.

The results are summarized in Table 2. For each number of modems and for each spectrum management algorithm there are two cells representing the simulation time and the performance in weighted rate sum with respect to the performance of the MS-DSB algorithm. It can be seen that ASB-2 has a better performance than IW. This is because ASB-2 is less selfish leading to a better overall network performance.

The performance of SCALE and DSB can differ as they can converge to different local optima. For the 5-modem case SCALE performs better than DSB, whereas DSB performs better for the 6- and 7-modem cases. Note that for the same initial transmit powers, SCALE and DSB generally converge to exactly the same solutions. This is confirmed by many simulations.

The MS-DSB algorithm performs better than SCALE and DSB because of its multiple starting point approach. Note that although the likelihood that MS-DSB is global optimal is larger than for SCALE and DSB, it does not necessarily always converge to the global optimal solution. If the number of initial starting points is increased the likelihood of global optimality will also increase.

A complete convergence analysis of the proposed algorithms is outside the scope of this paper. However it should be mentioned that convergence problems have never been encountered during extensive simulations of multi-user ADSL and VDSL scenarios.

6. CONCLUSION

In this paper three novel dynamic spectrum management algorithms are proposed with a different level of required message-passing, ranging from fully autonomous and distributed to semi-centralized execution. It is shown that the likelihood of attaining global optimality, increases with the level of message-passing. Simulation results show that these algorithms perform very well in mitigating the effect of crosstalk.

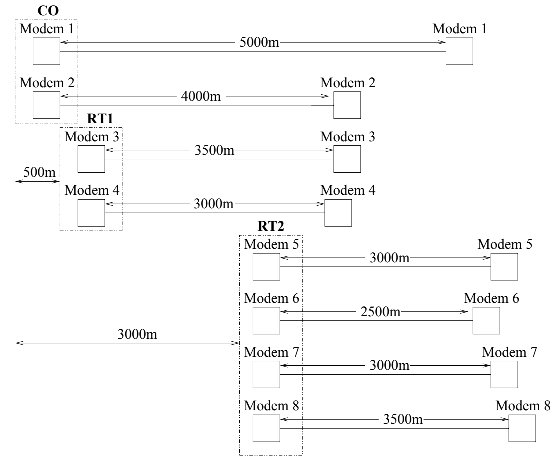

Fig. 1. Asymmetric ADSL Downstream scenario

Table 2. Performance comparison of distributed algorithms

Modems	IWF	ASB-2	SCALE	DSB	MS-DSB
2	0.03s	0.04s	0.56s	0.05s	1.18s
	99.9%	99.9%	100.0%	100.0%	100.0%
3	0.08s	0.08s	1.09s	0.1s	1.44s
	99.9%	99.9%	100.0%	100.0%	100.0%
4	0.13s	0.08s	1.69s	0.19s	2.19s
	99.5%	99.6%	100.0%	100.0%	100.0%
5	0.24s	0.66s	2.75s	1.66s	5.20s
	71.8%	86.9%	97.9%	92.8%	100.0%
6	0.28s	0.94s	3.48s	2.61s	7.30s
	86.9%	98.3%	89.5%	100.0%	100.0%
7	0.36s	0.93s	7.15s	3.24s	6.21s
	92.7%	99.8%	83.6%	100.0%	100.0%

7. REFERENCES

[1] P. Tsiaflakis, M. Diehl, M. Moonen, "Low-Complexity Distributed Spectrum Management Algorithms for Multi-User DSL Networks," *Internal Report 07-145, Submitted for publication*, 2007.

[2] K. B. Song et al., "Dynamic Spectrum Management for Next-Generation DSL Systems," *IEEE Communications Magazine*, vol. 40, no. 10, pp. 101–109, Oct. 2002.

[3] P. Tsiaflakis, J. Vangorp, M. Moonen, J. Verlinden, "A Low Complexity Optimal Spectrum Balancing Algorithm for Digital Subscriber Lines," *Elsevier Signal Processing*, vol. 87, no. 7, July 2007.

[4] W. Yu, G. Ginis and J. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines," *IEEE J. Sel. Area. Comm.*, vol. 20, no. 5, pp. 1105–1115, Jun. 2002.

[5] R. Cendrillon, J. Huang, M. Chiang, M. Moonen, "Autonomous Spectrum Balancing for Digital Subscriber Lines," *IEEE Transactions on Signal Processing*, vol. 55, no. 8, 2007.

[6] J. Papandriopoulos and J. S. Evans, "Low-Complexity Distributed Algorithms for Spectrum Balancing in Multi-User DSL Networks," in *IEEE International Conference on Communications (ICC)*, June 2006, vol. 7, pp. 3270 – 3275.

[7] Thomas Starr, John M. Cioffi, Peter J. Silverman, *Understanding Digital Subscriber Lines*, Prentice Hall, 1999.

[8] V. M. K. Chan, W. Yu, "Multiuser Spectrum Optimization for Discrete Multitone Systems With Asynchronous Crosstalk," *IEEE Transactions on Signal Processing*, vol. 55, no. 11, pp. 5425–5435, Nov. 2007.

[9] P. Tsiaflakis, J. Vangorp, M. Moonen, J. Verlinden, "Convex Relaxation Based Low-Complexity Optimal Spectrum Balancing for Multi-User DSL," in *ICASSP*, April 2007, vol. 3, pp. III–349 – III–352.

[10] S. Hayashi, Z. Q. Luo, "Dynamic Spectrum Management: When is FDMA sum-rate optimal?," in *ICASSP*, April 2007, vol. 3, pp. 609–612.

[11] W. Yu and R. Lui, "Dual Methods for Nonconvex Spectrum Optimization of Multicarrier Systems," *IEEE Transactions on Communications*, vol. 54, no. 7, July 2006.

[12] ITU G.992.1, "Asymmetrical digital subscriber line (ADSL) transceivers," 1999.