ASYMPTOTIC ERROR PERFORMANCE OF SPACE–TIME CODES OVER FULLY CORRELATED RICIAN FADING MIMO CHANNELS WITH IMPERFECT CSI*

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ABSTRACT

We analyze the asymptotic error performance of different receiver schemes based on block space–time codes over a fully correlated Rician fading MIMO channel. The receivers differ in the way they obtain the channel state information at the receiver (CSIR) required to detect the transmitted signal. The baseline is a *genie-aided* receiver, which has perfect CSIR knowledge without any power or rate expenditure. Alternatively, a *mismatched* receiver extracts CSIR by pilot symbol insertion and uses it in the perfect CSIR decision metric. Finally, an *optimum* receiver, avoiding direct CSIR detection, estimates the transmitted data from the whole received sample and from the knowledge of pilot symbols. Key results are the receiver diversity and asymptotic power gain.

***Index Terms*—** Correlated Rician MIMO channels, Pilot-aided receivers, Joint channel and data estimation.

1. INTRODUCTION

Early research studies on MIMO communications were based on two key assumptions: *rich scattering* and *perfect channel state information at the receiver (CSIR)* [2, 11]. Several subsequent studies criticized the rich scattering assumption, leading to uncorrelated Rayleigh fading, by providing more realistic channel models [3,4]. Separately-correlated (Kronecker) fading was proposed in [6] to give a better description of the MIMO channel fading vagaries. It was shown that one of the consequences of spatial correlation is the reduction of the MIMO channel achievable rate. More refined channel models were proposed in [7,12,13], deriving from physical and signal processing motivations, often with a considerable impact on the achievable rate. Though the precise choice of the channel model is still controversial, all proposals are encompassed by the *fully correlated Rician fading* channel. This model will be the focus of the current study.

The impact of the second key assumption, perfect receive CSIR, will also be addressed in this paper by considering three receiver architectures which differ in the way they obtain it for the purpose of signal detection.

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1. A baseline *genie-aided* receiver providing an error performance lower bound, as this receiver obtains CSIR from a genie without any power or rate expenditure.

2. A *mismatched* receiver, separating the channel and data detection parts by first obtaining an imperfect estimate of the CSIR through pilot-symbol channel estimation and then using this estimate in the perfect CSIR detection metric (thus creating a mismatch).

3. An *optimum* receiver, implementing a joint channel and data detection algorithm based on the knowledge of the whole received sample (including data and pilot symbols) and of the transmitted pilot symbols.

In a previous study [8] we derived the detection metrics of these three receivers and provided an error performance analysis based on the PEP. Here we build upon the earlier analysis and obtain the following two novel results.

1. All receivers attain the same *diversity order*.

2. Analytic expressions of the asymptotic gain of the receivers considered. These derivations are made possible by a substantial simplification (with respect to [8]) of the PEP expressions.

2. SYSTEM MODEL

We consider a narrowband MIMO block fading channel with n_T transmit and n_R receive antennas, characterized by the following matrix equation:

$$\mathbf{Y}_{n_\mathsf{R}\times N} = \mathbf{H}_{n_\mathsf{R}\times n_\mathsf{T}}\mathbf{X}_{n_\mathsf{T}\times N} + \mathbf{Z}_{n_\mathsf{R}\times N}. \tag{1}$$

Here, $\mathbf{X} \in \mathbb{C}^{n_\mathsf{T}\times N}$ is the transmitted signal matrix spanning N symbol intervals, $\mathbf{H} \in \mathbb{C}^{n_\mathsf{R}\times n_\mathsf{T}}$ is the channel matrix, $\mathbf{Z} \in \mathbb{C}^{n_\mathsf{R}\times N}$ is the additive noise matrix, and $\mathbf{Y} \in \mathbb{C}^{n_\mathsf{R}\times N}$ is the received signal matrix. We assume that the entries of $\mathbf{Z}$ are iid $\mathcal{N}_c(0, N_0)$ distributed. The channel matrix $\mathbf{H}$ is random with circularly-symmetric complex Gaussian entries. Since we assume a fully-correlated Rician fading MIMO channel, its joint pdf is given by $\mathrm{vec}(\mathbf{H}) \sim \mathcal{N}_c(\bar{\mathbf{h}}, \boldsymbol{\Sigma}_\mathbf{h})$ with $\bar{\mathbf{h}} \triangleq \mathrm{vec}(\bar{\mathbf{H}})$ and $\boldsymbol{\Sigma}_\mathbf{h}$ positive definite. The corresponding Rice factor is given by $K = \|\bar{\mathbf{H}}\|^2/\,\mathsf{Tr}(\boldsymbol{\Sigma}_\mathbf{h})$.

From linear algebra results [5] we obtain a vectorized version of equation (1):

$$\mathbf{y} = (\mathbf{X}^{\mathsf{T}} \otimes \mathbf{I}_{n_{\mathsf{R}}})\mathbf{h} + \mathbf{z}. \tag{2}$$

Here, we defined $\mathbf{h} \triangleq \mathrm{vec}(\mathbf{H})$, $\mathbf{y} \triangleq \mathrm{vec}(\mathbf{Y})$, and $\mathbf{z} \triangleq \mathrm{vec}(\mathbf{Z})$. The vectorized channel equation allows to obtain the exact LMMSE estimate of the channel matrix $\mathbf{H}$.

In the following, we consider encoded frames consist of N_{p} pilot matrices and N_{d} data matrices $\mathbf{X}_i \in \mathbb{C}^{n_{\mathsf{T}} \times N}$ ($i = 1, \ldots, N_{\mathsf{p}} + N_{\mathsf{d}}$). Thus, pilot insertion reduces the effective rate by the factor $N_{\mathsf{p}}/(N_{\mathsf{p}} + N_{\mathsf{d}})$. Summarizing, we define $\widetilde{\mathbf{X}}_{\mathsf{p}} \triangleq (\mathbf{X}_1, \ldots, \mathbf{X}_{N_{\mathsf{p}}})$, $\widetilde{\mathbf{X}}_{\mathsf{d}} \triangleq (\mathbf{X}_{N_{\mathsf{p}}+1}, \ldots, \mathbf{X}_{N_{\mathsf{p}}+N_{\mathsf{d}}})$, $\widetilde{\mathbf{X}} \triangleq (\widetilde{\mathbf{X}}_{\mathsf{p}}, \widetilde{\mathbf{X}}_{\mathsf{d}})$, and related $\mathbf{Y}, \mathbf{Z}$ matrices. For future reference, we also define $\tilde{\boldsymbol{\mathfrak{X}}} \triangleq \widetilde{\mathbf{X}}^{\mathsf{T}} \otimes \mathbf{I}_{n_{\mathsf{R}}}$, $\tilde{\boldsymbol{\mathfrak{X}}}_{ij} \triangleq (\widetilde{\mathbf{X}}_i \widetilde{\mathbf{X}}_j^{\mathsf{H}})^* \otimes \mathbf{I}_{n_{\mathsf{R}}} = \tilde{\boldsymbol{\mathfrak{X}}}_i^{\mathsf{H}} \tilde{\boldsymbol{\mathfrak{X}}}_j$, $\widetilde{\boldsymbol{\mathfrak{D}}} \triangleq \tilde{\boldsymbol{\mathfrak{X}}}_{\mathsf{d}1} - \tilde{\boldsymbol{\mathfrak{X}}}_{\mathsf{d}2} = \widetilde{\boldsymbol{\Delta}}_{\mathsf{d}}^{\mathsf{T}} \otimes \mathbf{I}_{n_{\mathsf{R}}}$. We also assume that pilot matrices have orthogonal rows so that, defining the pilot symbol energy as $\mathcal{E}_{\mathsf{p}} \triangleq \|\widetilde{\mathbf{X}}_{\mathsf{p}}\|^2/(n_{\mathsf{T}} N_{\mathsf{p}})$, we have

$$\widetilde{\mathbf{X}}_{\mathsf{p}} \widetilde{\mathbf{X}}_{\mathsf{p}}^{\mathsf{H}} = N_{\mathsf{p}} \mathcal{E}_{\mathsf{p}} \mathbf{I}_{n_{\mathsf{T}}}. \tag{3}$$

We define the SNR by assuming iid transmitted symbols with zero mean and energy $\mathcal{E}_{\mathsf{s}}$:

$$\mathsf{SNR} \triangleq \frac{\mathbb{E}[\|\mathbf{H}\|^2]\mathcal{E}_{\mathsf{s}}}{n_{\mathsf{R}} N_0} = \frac{(\|\bar{\mathbf{H}}\|^2 + \mathsf{Tr}(\boldsymbol{\Sigma}_{\mathbf{h}}))\mathcal{E}_{\mathsf{s}}}{n_{\mathsf{R}} N_0}.$$

Similarly, the received energy per bit will be given by

$$\mathcal{E}_{\mathsf{b}} = \frac{\mathbb{E}[\|\mathbf{H}\|^2](N_{\mathsf{d}} \mathcal{E}_{\mathsf{s}} + N_{\mathsf{p}} \mathcal{E}_{\mathsf{p}})}{N_{\mathsf{b}}}$$

where N_{b} is the number of information bits per code word. We also define the bit rate as $R_b \triangleq N_{\mathsf{b}}/N_{\mathsf{d}}$ bit/symbol and the power efficiency as the fraction of energy devoted to data transmission with respect to the total energy, due to data and pilot symbol transmission as

$$\eta \triangleq \frac{N_{\mathsf{d}} \mathcal{E}_{\mathsf{s}}}{N_{\mathsf{d}} \mathcal{E}_{\mathsf{s}} + N_{\mathsf{p}} \mathcal{E}_{\mathsf{p}}} = \frac{\gamma_{\mathsf{s}}}{\gamma_{\mathsf{s}} + \gamma_{\mathsf{p}}},$$

with $\gamma_{\mathsf{s}} \triangleq \frac{N_{\mathsf{d}} \mathcal{E}_{\mathsf{s}}}{N_0}$ and $\gamma_{\mathsf{p}} \triangleq \frac{N_{\mathsf{p}} \mathcal{E}_{\mathsf{p}}}{N_0}$. Hence we obtain

$$\frac{\mathcal{E}_{\mathsf{b}}}{N_0} = \frac{\mathbb{E}[\|\mathbf{H}\|^2]}{\eta R_b} \frac{\mathcal{E}_{\mathsf{s}}}{N_0} \implies \frac{\mathcal{E}_{\mathsf{s}}}{N_0} = \frac{\eta R_b}{\mathbb{E}[\|\mathbf{H}\|^2]} \frac{\mathcal{E}_{\mathsf{b}}}{N_0}.$$

Remark 2.1 In the following we study the system performance as a function of the $\mathcal{E}_{\mathsf{b}}/N_0$ ratio and of the power efficiency η. It is plain that if $\mathcal{E}_{\mathsf{s}} = \mathcal{E}_{\mathsf{p}}$, then η coincides with the uncoded system throughput $N_{\mathsf{d}}/(N_{\mathsf{d}} + N_{\mathsf{p}})$. However, the definition proposed allows for a more flexible interpretation in terms of resource exploitation. For example, for a given η, one can maximize the uncoded system throughput by setting N_{p} equal to its minimum value, n_{T} (to comply with (3)) by setting $\mathcal{E}_{\mathsf{p}} = (\eta^{-1} - 1) N_{\mathsf{d}} \mathcal{E}_{\mathsf{s}}/n_{\mathsf{T}}$, which can be considerably larger than $\mathcal{E}_{\mathsf{s}}$. The resulting transmitted signal becomes more *peaky* with obvious disadvantages in terms of linearity and pilot pollution (long-range interference).

3. RECEIVER ARCHITECTURES

As in [8], we consider a *genie-aided*, *mismatched* and *optimum* receiver. The receiver output is given by

$$\widehat{\mathbf{X}}_{\mathsf{d}} = \arg\min_{\widetilde{\mathbf{X}}_{\mathsf{d}}} \mu(\widetilde{\mathbf{X}}_{\mathsf{d}} \mid \widetilde{\mathbf{X}}_{\mathsf{p}}, \widetilde{\mathbf{Y}}) \tag{4}$$

with different decision metrics $\mu(\widetilde{\mathbf{X}}_{\mathsf{d}} \mid \widetilde{\mathbf{X}}_{\mathsf{p}}, \widetilde{\mathbf{Y}})$, characterizing the type of receiver. For the genie-aided receiver:

$$\mu(\widetilde{\mathbf{X}}_{\mathsf{d}} \mid \widetilde{\mathbf{X}}_{\mathsf{p}}, \widetilde{\mathbf{Y}}) = \|\widetilde{\mathbf{Y}}_{\mathsf{d}} - \mathbf{H}\widetilde{\mathbf{X}}_{\mathsf{d}}\|^2. \tag{5}$$

For the ML/LMMSE mismatched receiver:

$$\mu(\widetilde{\mathbf{X}}_{\mathsf{d}} \mid \widetilde{\mathbf{X}}_{\mathsf{p}}, \widetilde{\mathbf{Y}}) = \|\widetilde{\mathbf{Y}}_{\mathsf{d}} - \widehat{\mathbf{H}}_{\mathsf{ML/LMMSE}} \widetilde{\mathbf{X}}_{\mathsf{d}}\|^2 \tag{6}$$

with

$$\widehat{\mathbf{H}}_{\mathsf{ML}} = \widetilde{\mathbf{Y}}_{\mathsf{p}} \widetilde{\mathbf{X}}_{\mathsf{p}}^{\mathsf{H}} (\widetilde{\mathbf{X}}_{\mathsf{p}} \widetilde{\mathbf{X}}_{\mathsf{p}}^{\mathsf{H}})^{-1} \tag{7}$$

and, defining $\tilde{\boldsymbol{\mathfrak{X}}}_{\mathsf{p}} \triangleq \widetilde{\mathbf{X}}_{\mathsf{p}}^{\mathsf{T}} \otimes \mathbf{I}_{n_{\mathsf{R}}}$ and $\mathbf{R}_{\mathbf{h}} \triangleq \bar{\mathbf{h}}\bar{\mathbf{h}}^{\mathsf{H}} + \boldsymbol{\Sigma}_{\mathbf{h}}$,

$$\mathrm{vec}(\widehat{\mathbf{H}}_{\mathsf{LMMSE}}) = (\tilde{\boldsymbol{\mathfrak{X}}}_{\mathsf{p}}^{\mathsf{H}} \tilde{\boldsymbol{\mathfrak{X}}}_{\mathsf{p}} + N_0 \mathbf{R}_{\mathbf{h}}^{-1})^{-1} \tilde{\boldsymbol{\mathfrak{X}}}_{\mathsf{p}}^{\mathsf{H}} \mathrm{vec}(\widetilde{\mathbf{Y}}_{\mathsf{p}}). \tag{8}$$

For the optimum receiver,

$$\mu(\widetilde{\mathbf{X}}_{\mathsf{d}} \mid \widetilde{\mathbf{X}}_{\mathsf{p}}, \widetilde{\mathbf{Y}}) = \ln\det \boldsymbol{\Psi}(\widetilde{\mathbf{X}}) - \mathbf{a}(\widetilde{\mathbf{X}})^{\mathsf{H}} \boldsymbol{\Psi}(\widetilde{\mathbf{X}})^{-1} \mathbf{a}(\widetilde{\mathbf{X}}) \tag{9}$$

where, setting $\tilde{\boldsymbol{\mathfrak{X}}} \triangleq \widetilde{\mathbf{X}}^{\mathsf{T}} \otimes \mathbf{I}_{n_{\mathsf{R}}}$,

$$\left\{ \begin{array}{lcl} \mathbf{a}(\widetilde{\mathbf{X}}) & \triangleq & \boldsymbol{\Sigma}_{\mathbf{h}}^{-1}\bar{\mathbf{h}} + \tilde{\boldsymbol{\mathfrak{X}}}^{\mathsf{H}} \mathrm{vec}(\widetilde{\mathbf{Y}})/N_0 \\ \boldsymbol{\Psi}(\widetilde{\mathbf{X}}) & \triangleq & \boldsymbol{\Sigma}_{\mathbf{h}}^{-1} + \tilde{\boldsymbol{\mathfrak{X}}}^{\mathsf{H}} \tilde{\boldsymbol{\mathfrak{X}}} \end{array} \right. , \tag{10}$$

after some simplification of the results in [8].

4. ERROR PERFORMANCE

The error performance is analyzed through the PEP $P(\widetilde{\mathbf{X}}_{\mathsf{d}1} \to \widetilde{\mathbf{X}}_{\mathsf{d}2}) = P(\Delta < 0)$ for some properly defined random variable Δ. The exact PEP (or an upper bound to it) can be derived as summarized in [8, App. D]. The derivation is based on the moment generating function (MGF) $\Phi_\Delta(s) = \mathbb{E}[\exp(-s\Delta)]$. Assuming that c is in the region of convergence of $\Phi_\Delta(s)$, we have:

$$P(\Delta < 0) = \frac{1}{\mathrm{j}\,2\pi} \int_{c-\mathrm{j}\,\infty}^{c+\mathrm{j}\,\infty} \Phi_\Delta(s) \frac{ds}{s}.$$

Summarizing and simplifying the results from [8], we can write the MGF $\Phi_\Delta(s)$ for the three receivers considered as follows:

$$\Phi_\Delta(s) = K^s \frac{\exp\{-\boldsymbol{\mu}^{\mathsf{H}} \mathbf{M}(s)[N_0\mathbf{I} + \boldsymbol{\Sigma}(s)\mathbf{M}(s)]^{-1}\boldsymbol{\mu}\}}{\det[\mathbf{I} + \boldsymbol{\Sigma}(s)\mathbf{M}(s)/N_0]} \tag{11}$$

where $K, \boldsymbol{\mu}, \boldsymbol{\Sigma}(s), \mathbf{M}(s)$ depend on the receiver considered. For the genie-aided receiver, setting $\widetilde{\boldsymbol{\mathfrak{D}}} \triangleq (\widetilde{\mathbf{X}}_{\mathsf{d}1} - \widetilde{\mathbf{X}}_{\mathsf{d}2})^{\mathsf{T}} \otimes \mathbf{I}_{n_{\mathsf{R}}}$,

$$K = 1, \;\; \boldsymbol{\mu} = \bar{\mathbf{h}}, \;\; \boldsymbol{\Sigma}(s) = \boldsymbol{\Sigma}_{\mathbf{h}}, \;\; \mathbf{M}(s) = s(1-s)\widetilde{\boldsymbol{\mathfrak{D}}}^{\mathsf{H}} \widetilde{\boldsymbol{\mathfrak{D}}}.$$

For the ML/LMMSE mismatched receiver, $K = 1$, $\boldsymbol{\mu} = \binom{\bar{\mathbf{h}}}{\mathbf{0}}$,

$$\boldsymbol{\Sigma}(s) = \begin{pmatrix} \boldsymbol{\Sigma}_{\mathbf{h}} & \mathbf{0} \\ \mathbf{0} & \gamma_{\mathsf{p}}^{-1}\mathbf{I} \end{pmatrix}, \quad \mathbf{M}(s) = \begin{pmatrix} \mathbf{M}_{11}(s) & \mathbf{M}_{12}(s) \\ \mathbf{M}_{21}(s) & \mathbf{M}_{22}(s) \end{pmatrix},$$

with $\mathfrak{P}_{\mathsf{ML}} = \mathbf{I}_{n_\mathsf{T} n_\mathsf{R}}$, $\mathfrak{P}_{\mathsf{LMMSE}} = (\mathbf{I}_{n_\mathsf{T} n_\mathsf{R}} + \gamma_{\mathsf{p}}^{-1}\mathbf{R}_{\mathbf{h}}^{-1})^{-1}$, and

$$\begin{cases} \mathbf{M}_{11}(s) = 2s\mathcal{H}[(\mathbf{I}-\mathfrak{P})^{\mathsf{H}}\tilde{\mathfrak{X}}_{\mathsf{d}1}^{\mathsf{H}}\widetilde{\mathfrak{D}}\mathfrak{P}] + s(1-s)\mathfrak{P}^{\mathsf{H}}\widetilde{\mathfrak{D}}^{\mathsf{H}}\widetilde{\mathfrak{D}}\mathfrak{P} \\ \mathbf{M}_{12}(s) = \mathbf{M}_{11}(s) - s\mathfrak{P}^{\mathsf{H}}\widetilde{\mathfrak{D}}^{\mathsf{H}}\tilde{\mathfrak{X}}_{\mathsf{d}1} \\ \mathbf{M}_{21}(s) = \mathbf{M}_{11}(s) - s\tilde{\mathfrak{X}}_{\mathsf{d}1}^{\mathsf{H}}\widetilde{\mathfrak{D}}\mathfrak{P} \\ \mathbf{M}_{22}(s) = \mathbf{M}_{11}(s) - 2s\mathcal{H}(\tilde{\mathfrak{X}}_{\mathsf{d}1}^{\mathsf{H}}\widetilde{\mathfrak{D}}\mathfrak{P}) \end{cases}$$

For the optimum receiver,

$$K = \frac{\det(\widetilde{\mathbf{X}}_{11} + N_0\boldsymbol{\Sigma}_{\mathbf{h}}^{-1})}{\det(\widetilde{\mathbf{X}}_{22} + N_0\boldsymbol{\Sigma}_{\mathbf{h}}^{-1})}, \; \boldsymbol{\mu} = \begin{pmatrix} \widetilde{\mathbf{X}}_{11} + N_0\boldsymbol{\Sigma}_{\mathbf{h}}^{-1} \\ \widetilde{\mathbf{X}}_{21} + N_0\boldsymbol{\Sigma}_{\mathbf{h}}^{-1} \end{pmatrix}\bar{\mathbf{h}},$$

$$\boldsymbol{\Sigma}(s) = \begin{pmatrix} \widetilde{\mathbf{X}}_{11}\boldsymbol{\Sigma}_{\mathbf{h}}\widetilde{\mathbf{X}}_{11} + N_0\widetilde{\mathbf{X}}_{11} & \widetilde{\mathbf{X}}_{11}\boldsymbol{\Sigma}_{\mathbf{h}}\widetilde{\mathbf{X}}_{12} + N_0\widetilde{\mathbf{X}}_{12} \\ \widetilde{\mathbf{X}}_{21}\boldsymbol{\Sigma}_{\mathbf{h}}\widetilde{\mathbf{X}}_{11} + N_0\widetilde{\mathbf{X}}_{21} & \widetilde{\mathbf{X}}_{21}\boldsymbol{\Sigma}_{\mathbf{h}}\widetilde{\mathbf{X}}_{12} + N_0\widetilde{\mathbf{X}}_{22} \end{pmatrix},$$

$$\mathbf{M}(s) = s\begin{pmatrix} (\widetilde{\mathbf{X}}_{11} + N_0\boldsymbol{\Sigma}_{\mathbf{h}}^{-1})^{-1} & \mathbf{0} \\ \mathbf{0} & -(\widetilde{\mathbf{X}}_{22} + N_0\boldsymbol{\Sigma}_{\mathbf{h}}^{-1})^{-1} \end{pmatrix},$$

where $\tilde{\mathfrak{X}}_i \triangleq \widetilde{\mathbf{X}}_i^* \otimes \mathbf{I}_{n_\mathsf{R}}$ and $\widetilde{\mathbf{X}}_{ij} \triangleq (\tilde{\mathfrak{X}}_i\tilde{\mathfrak{X}}_j^{\mathsf{H}})^*$.

5. ASYMPTOTIC PERFORMANCE

From MacLaurin's expansion of the MGF,

$$\Phi_\Delta(s; N_0) = \sum_{k=\mathsf{d_o}}^{\infty} \Phi_{\Delta,k}(s) N_0^k, \tag{12}$$

we obtain the diversity order $\mathsf{d_o}$ (lowest degree in N_0) and the corresponding coefficient

$$\kappa \triangleq \frac{1}{\mathrm{j}\,2\pi}\int_{c-\mathrm{j}\,\infty}^{c+\mathrm{j}\,\infty} \Phi_{\Delta,\mathsf{d_o}}(s)\frac{ds}{s}$$

allows us to express the PEP as follows:

$$P(\widetilde{\mathbf{X}}_{\mathsf{d}1} \to \widetilde{\mathbf{X}}_{\mathsf{d}2}) = \kappa N_0^{\mathsf{d_o}}[1 + O(N_0)].$$

The following theorem shows that the diversity order achieved is the same by the three receivers considered and the corresponding values of the coefficient κ are evaluated to provide the relative asymptotic power gain/loss of the receivers.

Theorem 1 *In the asymptotic SNR regime ($N_0 \downarrow 0$), the PEPs of the genie-aided (*G*), mismatched (*M*), and optimum receivers (*O*) are given as follows:*

$$P(\widetilde{\mathbf{X}}_{\mathsf{d}1} \to \widetilde{\mathbf{X}}_{\mathsf{d}2}) = \kappa_{\mathsf{G/M/O}} N_0^{\mathsf{d_o}}[1 + o(N_0)] \tag{13}$$

where $\mathsf{d_o} = n_\mathsf{R}\,\mathsf{rank}(\widetilde{\boldsymbol{\Delta}}_\mathsf{d} \triangleq \widetilde{\mathbf{X}}_{\mathsf{d}1} - \widetilde{\mathbf{X}}_{\mathsf{d}2})$ *represents the diversity order of all three receivers and the constant* $\kappa_{\mathsf{G/M/O}}$ *depends on the receiver considered and is given as follows.*

- *For the genie-aided receiver,*

$$\kappa_\mathsf{G} = \binom{2\mathsf{d_o}-1}{\mathsf{d_o}} \frac{\exp(-\|\mathbf{I}_{\widetilde{\boldsymbol{\Lambda}}_\mathsf{G}}\widetilde{\mathbf{U}}_\mathsf{G}\boldsymbol{\Sigma}_{\mathbf{h}}^{-1/2}\bar{\mathbf{h}}\|^2)}{\det_+(\widetilde{\mathfrak{D}}\boldsymbol{\Sigma}_{\mathbf{h}}\widetilde{\mathfrak{D}}^{\mathsf{H}})}$$

  where $\widetilde{\mathbf{U}}_\mathsf{G}$ *and* $\widetilde{\boldsymbol{\Lambda}}_\mathsf{G}$ *are unitary and diagonal matrices, respectively, deriving from the orthogonal factorization* $\boldsymbol{\Sigma}_{\mathbf{h}}^{1/2}\widetilde{\mathfrak{D}}^{\mathsf{H}}\widetilde{\mathfrak{D}}\boldsymbol{\Sigma}_{\mathbf{h}}^{1/2} = \widetilde{\mathbf{U}}_\mathsf{G}^{\mathsf{H}}\widetilde{\boldsymbol{\Lambda}}_\mathsf{G}\widetilde{\mathbf{U}}_\mathsf{G}$. [1]

- *For both the ML and the LMMSE mismatched receivers,* κ_M *can be obtained from*

$$\begin{aligned} \Phi_{\Delta,\mathsf{d_o}}(s) \;=\; & \frac{\exp(-\|\mathbf{I}_{\widetilde{\boldsymbol{\Lambda}}_\mathsf{M}(s)}\widetilde{\mathbf{U}}_\mathsf{M}(s)^{\mathsf{H}}\boldsymbol{\Sigma}_{\mathbf{h}}^{-1/2}\bar{\mathbf{h}}\|^2)}{[s(1-s)]^{\mathsf{d_o}}\det_+(\widetilde{\boldsymbol{\Lambda}}_\mathsf{G})} \\ & \det\left\{\mathbf{I}_{n_\mathsf{T}n_\mathsf{R}} - \frac{s}{1-s}\frac{\tilde{\mathfrak{X}}_{\mathsf{d}1}^{\mathsf{H}}\widetilde{\mathfrak{D}}(\widetilde{\mathfrak{D}}^{\mathsf{H}}\widetilde{\mathfrak{D}})^{-1}\widetilde{\mathfrak{D}}\tilde{\mathfrak{X}}_{\mathsf{d}1}}{N_\mathsf{p}\mathcal{E}_\mathsf{p}}\right\}^{-1} N_0^{\mathsf{d_o}} \end{aligned}$$

  where $\widetilde{\mathbf{U}}_\mathsf{M}(s), \widetilde{\boldsymbol{\Lambda}}_\mathsf{M}(s)$ *are obtained from the unitary factorization*

$$\begin{aligned} \widetilde{\mathbf{U}}_\mathsf{M}(s)^{\mathsf{H}}\widetilde{\boldsymbol{\Lambda}}_\mathsf{M}(s)\widetilde{\mathbf{U}}_\mathsf{M}(s) \;=\; & \boldsymbol{\Sigma}_{\mathbf{h}}^{1/2}\Big\{\mathbf{M}_{11}(s) - \mathbf{M}_{12}(s) \\ & \cdot[N_\mathsf{p}\mathcal{E}_\mathsf{p}\mathbf{I}_{n_\mathsf{T}n_\mathsf{R}} + \mathbf{M}_{22}(s)]^{-1}\mathbf{M}_{21}(s)\Big\}\boldsymbol{\Sigma}_{\mathbf{h}}^{1/2} \end{aligned}$$

  where

$$\begin{cases} \mathbf{M}_{11}(s) = s(1-s)\widetilde{\mathfrak{D}}^{\mathsf{H}}\widetilde{\mathfrak{D}} \\ \mathbf{M}_{12}(s) = s(1-s)\widetilde{\mathfrak{D}}^{\mathsf{H}}\widetilde{\mathfrak{D}} - s\widetilde{\mathfrak{D}}^{\mathsf{H}}\tilde{\mathfrak{X}}_{\mathsf{d}1} \\ \mathbf{M}_{21}(s) = s(1-s)\widetilde{\mathfrak{D}}^{\mathsf{H}}\widetilde{\mathfrak{D}} - s\tilde{\mathfrak{X}}_{\mathsf{d}1}^{\mathsf{H}}\widetilde{\mathfrak{D}} \\ \mathbf{M}_{22}(s) = s(1-s)\widetilde{\mathfrak{D}}^{\mathsf{H}}\widetilde{\mathfrak{D}} - 2s\mathcal{H}(\tilde{\mathfrak{X}}_{\mathsf{d}1}^{\mathsf{H}}\widetilde{\mathfrak{D}}) \end{cases}.$$

- *For the optimum receiver, under the mild assumption that the code words* $\widetilde{\mathbf{X}}_{\mathsf{d}1}, \widetilde{\mathbf{X}}_{\mathsf{d}2}$ *have full row rank and* $\widetilde{\mathbf{X}}_1\widetilde{\mathbf{X}}_2^{\mathsf{H}} = N_\mathsf{p}\mathcal{E}_\mathsf{p}\mathbf{I}_{n_\mathsf{T}} + \widetilde{\mathbf{X}}_{\mathsf{d}1}\widetilde{\mathbf{X}}_{\mathsf{d}2}^{\mathsf{H}}$ *is not singular,*

$$\kappa_\mathsf{O} = \binom{2\mathsf{d_o}-1}{\mathsf{d_o}} \frac{\exp(-\bar{\mathbf{h}}^{\mathsf{H}}\boldsymbol{\Sigma}_{\mathbf{h}}^{-1}\bar{\mathbf{h}})}{\det_+\Big\{\tilde{\mathfrak{X}}_{11}\boldsymbol{\Sigma}_{\mathbf{h}}[\tilde{\mathfrak{X}}_{22} - \tilde{\mathfrak{X}}_{21}\tilde{\mathfrak{X}}_{11}^{-1}\tilde{\mathfrak{X}}_{12}]\tilde{\mathfrak{X}}_{22}^{-1}\Big\}}.$$

Proof omitted for space limitations see [9].

In the case of full diversity order $\mathsf{d_o} = n_\mathsf{T}n_\mathsf{R}$, we have the following result.

Corollary 1 *With full rank* $\widetilde{\boldsymbol{\Delta}}_\mathsf{d}$ *and hence full diversity order* $\mathsf{d_o} = n_\mathsf{T}n_\mathsf{R}$*, the asymptotic power loss of the optimum receiver with respect to the genie-aided is given by*

$$\lambda_\mathsf{O} = \left(\frac{\det(\widetilde{\mathfrak{D}}^{\mathsf{H}}\widetilde{\mathfrak{D}})}{\det\Big\{\tilde{\mathfrak{X}}_{11} - \tilde{\mathfrak{X}}_{12}\tilde{\mathfrak{X}}_{22}^{-1}\tilde{\mathfrak{X}}_{21}\Big\}}\right)^{1/\mathsf{d_o}}. \tag{14}$$

[1] $\mathbf{I}_{\mathbf{A}}$ is defined by $(\mathbf{I}_{\mathbf{A}})_{ij} = 0$ if $(\mathbf{A})_{ij} = 0$ and 1, otherwise.

Remark 5.1 Theorem 1 suggests an alternative code design criterion for optimum receivers, which is based on the maximization of a different determinant than the one suggested in [10], namely, $\det\left\{\tilde{\mathfrak{X}}_{11}-\tilde{\mathfrak{X}}_{12}\tilde{\mathfrak{X}}_{22}^{-1}\tilde{\mathfrak{X}}_{21}\right\}$. It is worth noting that this design criterion holds regardless of the spatial correlation of the MIMO Rician fading channel, provided that the rank has already been maximized.

6. NUMERICAL RESULTS

We apply the results presented to the fully correlated Rician fading MIMO channel with channel matrix

$$\mathbf{H} = \bar{\mathbf{H}} + \mathbf{R}_1^{1/2}\mathbf{H}_{w,1}\mathbf{T}_1^{1/2} + \mathbf{R}_2^{1/2}\mathbf{H}_{w,2}\mathbf{T}_2^{1/2}.$$

We assume that $n_\mathsf{T} = 2, n_\mathsf{R} = 2, \bar{\mathbf{H}}_{ij} = 1, (\mathbf{R}_k)_{ij} = \alpha_k^{|i-j|}$, $(\mathbf{T}_k)_{ij} = \beta_k^{|i-j|}$, for $k = 1, 2$, $\alpha_1 = 0.1$, $\alpha_2 = 0.7$, $\beta_1 = 0.7$, $\beta_2 = 0.1$, and $\mathbf{H}_{w,1}, \mathbf{H}_{w,2}$ have iid $\mathcal{N}_c(0, 1)$ entries. Figure 1 shows the PEP corresponding to a pair of code words from a space–time code obtained by concatenating a 4-state QPSK trellis code with the Alamouti code. The code words are:
$\mathbf{X}_1 = \text{Alam}(2, 1, 3, 1, 0, 1, 3, 1, 0, 1, 1, 0, 1, 3, 3, 1, 2, 0, 0, 0)$,
$\mathbf{X}_2 = \text{Alam}(2, 1, 3, 3, 3, 3, 3, 1, 0, 1, 1, 0, 1, 3, 3, 1, 2, 0, 0, 0)$,
where $\text{Alam}(\alpha_1, \ldots, \alpha_N) = \begin{pmatrix} s_1 & -s_2^* \ldots & s_{N-1} & -s_N^* \\ s_2 & +s_1^* \ldots & s_N & +s_{N-1}^* \end{pmatrix}$ with $s_k \triangleq \exp(\mathrm{j}\,(2\alpha_k + 1)\pi/4)$.

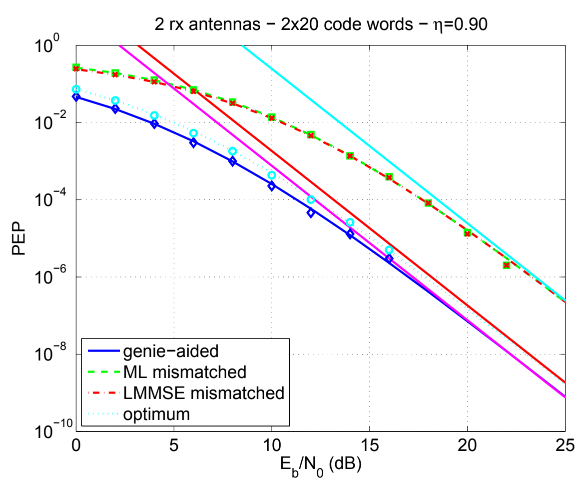

Fig. 1. Plot of $P(\mathbf{X}_1 \to \mathbf{X}_2)$ with power efficiency of 90% and three types of receivers: genie-aided, ML and LMMSE mismatched, and optimum receiver (*lines*: analytic results; *dots*: simulation; *straight lines*: asymptotic results).

Besides the analytic PEPs, the figure contains simulation (dots) and asymptotic results (dots and straight lines, respectively), which confirm the validity of Theorem 1. It is shown that, with a 90% power efficiency, the asymptotic loss of the optimum receiver to the genie-aided (1.0 dB) is considerably smaller than that of the mismatched receivers (7.3 dB).

7. CONCLUSIONS

The focus of this work is the study of several receivers over the fully correlated Rician fading MIMO channel. Building on earlier results [8], we derived the asymptotic PEP of the mismatched, optimum, and genie-aided receivers. We showed that all receivers attain the same diversity order and we gave closed-form expression of the asymptotic power gains. A code design criterion is given in Corollary 1.

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