OPTIMUM MIMO PRECODER DESIGNS FOR RICEAN FADING CHANNELS

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ABSTRACT

We consider the problem of space-time block code (STBC) design for Ricean fading channels in a multiple-input multiple-output (MIMO) communication system equipped with a linear minimum mean square error (MMSE) receiver. Two criteria, i.e., ergodic channel capacity and mean square error (MSE) are used for the design. Due to the complexity of the mathematics involved, no general solution has yet been proposed for a Ricean channel with arbitrary mean. In this paper, we present the necessary and sufficient conditions for the design of the optimal precoder applicable to any channel condition and any range of signal-to-noise ratio (SNR). Simulations verify that the performance of the MIMO system using the optimum precoder is indeed superior in performance.

Index Terms— MIMO, Ricean fading, channel capacity, MSE

I. INTRODUCTION

In MIMO communications, most research focus on STBC design for Rayleigh fading channels which are assumed to be IID Gaussian with zero mean. On the other hand, Ricean fading, being a more general model than Rayleigh fading, has important applications in wireless communications including the cases of having feedback channel state information (CSI) at the transmitter, and having a strong line of sight (LOS) path in the channels [1].

With Rayleigh fading, the STBC design problems can usually be solved by the *isotropic* property possessed by the channel fading coefficients. These design solutions include: 1) the optimal input signal covariance that maximizing the ergodic channel capacity [2] and 2) the optimal STBC that minimizing the detection error probability for a linear MMSE receiver [3]. However, for Ricean channels, the isotropic property no longer holds due to the arbitrary nonzero mean, resulting in difficulties to arrive at an optimum STBC design. Still, useful results on STBC designs for Rician fading channels in MIMO systems have been obtained: To maximize the ergodic channel capacity, the covariance matrix of the signal at the input to the Ricean channels has been considered, e.g., [4], [5]. The eigenvectors of the optimal covariance have been characterized [5]. However, no general close form solution on the optimal eigenvalues has been obtained. Instead, by observing that the objective is a concave function [6], two methods have been proposed seeking the optimal eigen-values: i) numerical method (e.g., [5]), and ii) maximizing an upper bound of the objective [4]. Nevertheless, method i) involves a huge amount of computation since each step of the iterations necessitates the evaluation of the expected value of a random function, and the upper bound derived in Method ii) is a coarse approximation of the original problem. On the other hand, there has been no explicit optimal STBC proposed for the problem of transmitting the signals through Ricean fading channels and received by a linear MMSE detector.

In this paper, we examine the problem of optimum STBC design in a MIMO system for Ricean fading channels. We approach the problem via two different criteria: 1) maximizing the ergodic capacity of the virtual channel, which is the transformed channel that contains both the original channel and the STBC, and 2) minimizing the MSE for a MIMO system equipped with a linear MMSE equalizer followed by a symbol-by-symbol detector. In both cases, we arrived at the respective optimum designs. We first derive the necessary and sufficient conditions for the optimal code in both cases. This is the first time that the optimal conditions are provided that describe how the transmission power should be allocated accordingly to the Ricean factor and an arbitrary channel mean. Based on the optimal conditions, we also unveil the channel states under which these two criteria converge. To apply the optimal conditions, we propose a close approximation for the complicated nonlinear functions involved, which enables a fast computation of the optimal code numerically.

II. SYSTEM MODEL

We consider a MIMO communication system with M transmitter and N ($N \geq M$) receiver antennas. The channel fading between the mth ($m = 1, \cdots, M$) transmitter and nth ($n = 1, \cdots, N$) receiver antenna is denoted by h_{nm}. Each of these coefficients h_{nm} is assumed to be Gaussian having a non-zero mean, and together they constitute an $N \times M$ channel matrix $\boldsymbol{H}$, which can be written as

$$\boldsymbol{H} = \sqrt{\frac{1}{K+1}}\boldsymbol{H}_0 + \sqrt{\frac{K}{K+1}}\tilde{\boldsymbol{H}} \tag{1}$$

where K is the Ricean factor, $\boldsymbol{H}_0$ is the channel mean known at the transmitter and it satisfies $tr\left(\boldsymbol{H}_0^H \boldsymbol{H}_0\right) = MN$, and $\tilde{\boldsymbol{H}}$ is a random matrix with i.i.d. Gaussian distributed elements of zero mean and unit variance. The matrix $\tilde{\boldsymbol{H}}$ remains constant for T time slots and may change to other states after the time elapses. We assume that only the statistical information of the random matrix $\tilde{\boldsymbol{H}}$ is available at the transmitter, while the receiver has perfect channel state information. For notation simplicity, we denote $\alpha = \sqrt{\frac{1}{K+1}}$, and $\bar{\alpha} = \sqrt{\frac{K}{K+1}}$ in the remainder of the paper.

We transmit an $MT \times 1$ signal vector $\boldsymbol{s}$ through the above-mentioned MIMO system during T time slots, where each signal symbol is independently chosen from a constellation with zero mean and unit variance, i.e.,

$$\boldsymbol{Y}=\sqrt{\frac{\rho}{M}}\boldsymbol{H}\boldsymbol{X}(\boldsymbol{s})+\boldsymbol{W} \tag{2}$$

where ρ is the signal-to-noise ratio (SNR) at each receiver, $\boldsymbol{X}(\boldsymbol{s})$ is the coded transmitted signal matrix with dimension $M\times T$, $\boldsymbol{Y}$ is the $N\times T$ received signal matrix, and the $M\times T$ matrix $\boldsymbol{W}$ is the additive white Gaussian noise with each element distributed as $\mathcal{CN}(0,1)$.

In this paper, we only consider linear STBC, i.e.,

$$\boldsymbol{X}(\boldsymbol{s})=\boldsymbol{B}\left(\sum_{i=1}^{MT}\boldsymbol{C}_i s_i\right) \tag{3}$$

where $\boldsymbol{B}$ is the $M\times M$ beamforming matrix, and $\boldsymbol{C}_i$ ($i=1,\cdots,MT$) is the coding matrix for the symbol s_i. Since the channel $\boldsymbol{H}$ is composed of a deterministic part $\boldsymbol{H}_0$ and an isotropic Gaussian random matrix $\tilde{\boldsymbol{H}}$, we can choose our coding strategy such that we employ $\boldsymbol{B}$ to take advantage of the deterministic part of the Ricean fading channel in the transmission of the signal, and choose $\boldsymbol{C}_i$ for optimally coding the signal in an isotopic random channel. These parameters satisfy the following power constraints

$$tr\left(\boldsymbol{B}\boldsymbol{B}^H\right)=M;\qquad \sum_{i=1}^{MT}tr\left(\boldsymbol{C}_i\boldsymbol{C}_i^H\right)=MT \tag{4}$$

Now, the optimal linear STBC for an isotropic random channel has been proved [3] to be of the unitary trace-orthogonal structure defined as

$$\boldsymbol{C}_i\boldsymbol{C}_i^H=\frac{1}{M}\boldsymbol{I};\quad \text{and}\quad tr\left(\boldsymbol{C}_i\boldsymbol{C}_j^H\right)=\delta_{ij} \tag{5}$$

Thus we can fix the matrices $\boldsymbol{C}_i$ to have these properties of Eqs. (5), and focus the design effort on the beamforming matrix $\boldsymbol{B}$. We approach the design from the prospect of two criteria: i) maximizing the ergodic channel capacity, and ii) minimizing the MSE at the output of a linear MMSE receiver.

To facilitate the analysis, we vectorize Eq. (2), combined with Eq. (3), and obtain

$$\boldsymbol{y}=\sqrt{\frac{\rho}{M}}(\boldsymbol{I}\otimes\boldsymbol{H}_B)\boldsymbol{F}\boldsymbol{s}+\boldsymbol{w} \tag{6}$$

where " $\otimes$ " denotes Kronecker product, $\boldsymbol{y}$ and $\boldsymbol{w}$ are the vectorized received signal and noise, $\boldsymbol{H}_B\triangleq\boldsymbol{H}\boldsymbol{B}$, and $\boldsymbol{F}\triangleq[\mathrm{vec}(\boldsymbol{C}_1),\cdots,\mathrm{vec}(\boldsymbol{C}_{\mathrm{MT}})]$. From Eq. (5), it is obvious that $\boldsymbol{F}$ is a unitary matrix. We are now ready to examine the design the matrix $\boldsymbol{B}$.

III. CHANNEL CAPACITY

In this section, we design the optimal beamforming matrix by maximizing the ergodic channel capacity of the virtual channel, which contains both the original channel and the STBC. The design problem can be formulated as [2], [7]

$$\max_{\boldsymbol{B}}: I_1=\mathrm{E}\left[\log\det\left(\boldsymbol{I}+\frac{\rho}{M}\boldsymbol{F}^H(\boldsymbol{I}\otimes\boldsymbol{H}_B^H)(\boldsymbol{I}\otimes\boldsymbol{H}_B)\boldsymbol{F}\right)\right] \tag{7}$$

subject to the power limits of Eq. (4), where "E" denotes expectation. Now, by employing matrix equality $\det(\boldsymbol{I}+\boldsymbol{A}\boldsymbol{B})=\det(\boldsymbol{I}+\boldsymbol{B}\boldsymbol{A})$ and noticing that $\boldsymbol{F}$ is unitary, I_1 can be simplified as

$$\begin{aligned}I_1&=\mathrm{E}\left[\log\det\left(\boldsymbol{I}+\frac{\rho}{M}(\boldsymbol{I}\otimes\boldsymbol{H}_B^H\boldsymbol{H}_B)\right)\right]\\&=T\mathrm{E}\left[\log\det\left(\boldsymbol{I}+\frac{\rho}{M}\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right)\right]\end{aligned}$$

where $\boldsymbol{Q}\triangleq\boldsymbol{B}\boldsymbol{B}^H$ with dimension $M\times M$. Then, the optimization problem in Eq. (7) is equivalent to

$$\max_{\boldsymbol{Q}}\ :\ I_2=\mathrm{E}\left[\log\det\left(\boldsymbol{I}+\frac{\rho}{M}\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right)\right] \tag{8a}$$

$$\text{s.t.}\ :\ tr(\boldsymbol{Q})=M \tag{8b}$$

The formulation of the problem in Eq. (8) is parallel to that [4], [5] which considered the optimization of the input covariance matrix for uncoded signals. This is a difficult problem since the objective contains a non-linear function of the non-central Wishart-distributed [8] random matrix $\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H$. Attempts to solve the problem have been made by: 1) Applying numerical method which is possible since the objective is a concave function. However, both the objective function and its gradient are expressed in terms of the expectation of non-linear functions, no close forms of which are available. Hence, each step of iteration has to involve the calculation of the expectation resulting in very intensive computation. 2) Interchanging the order of the expectation operator with the function log det in Eq. (8a), resulting in a deterministic upper bound on I_2. The optimal $\boldsymbol{\Lambda}_Q$ that maximizes the upper bound is given by the well known water-filling solution. Neither of these results is satisfactory in obtaining the true optimum solution of the original problem. In the following, we present our result in solving Eq. (8) as a theorem:

Theorem 1: For a Ricean fading MIMO communication system, when unitary trace-orthogonal code is employed, the ergodic channel capacity of the coded channel is maximized if and only if: 1) the eigenvector matrix of $\boldsymbol{Q}$ is equal to the right singular vector matrix of the channel mean, and 2) the eigenvalue matrix $\boldsymbol{\Lambda}_Q$ of $\boldsymbol{Q}$ ensures equal diagonal elements in the matrix $\boldsymbol{\Phi}\triangleq\mathrm{E}\left(\boldsymbol{\Lambda}_Q+\frac{M}{\rho}\left((\alpha\boldsymbol{\Lambda}_0+\bar{\alpha}\tilde{\boldsymbol{H}})^H(\alpha\boldsymbol{\Lambda}_0+\bar{\alpha}\tilde{\boldsymbol{H}})\right)^{-1}\right)^{-1}$ where $\boldsymbol{\Lambda}_0$ is the singular value of the channel mean.

Outline of Proof: a) We perform SVD on $\boldsymbol{H}_0$ such that $\boldsymbol{H}_0=\boldsymbol{U}_L\boldsymbol{\Lambda}_0\boldsymbol{U}_R^H$, and using the isotropic property of $\tilde{\boldsymbol{H}}$, we can re-write I_2 as

$$I_2=\mathrm{E}\log\det\left(\boldsymbol{I}+\frac{\rho}{M}(\alpha\boldsymbol{\Lambda}_0+\bar{\alpha}\tilde{\boldsymbol{H}})\bar{\boldsymbol{Q}}(\alpha\boldsymbol{\Lambda}_0+\bar{\alpha}\tilde{\boldsymbol{H}})^H\right) \tag{9}$$

where $\bar{\boldsymbol{Q}}\triangleq\boldsymbol{U}_R^H\boldsymbol{Q}\boldsymbol{U}_R$.

b) Making use of the concavity of the function $\log\det(\cdot)$, and after some mathematical manipulations, an upper bound on I_2 in Eq. (9) is obtained such that:

$$I_2\le\mathrm{E}\log\det\left(\boldsymbol{I}+\frac{\rho}{M}(\alpha\boldsymbol{\Lambda}_0+\bar{\alpha}\tilde{\boldsymbol{H}})\boldsymbol{\Lambda}_Q(\alpha\boldsymbol{\Lambda}_0+\bar{\alpha}\tilde{\boldsymbol{H}})^H\right)\triangleq I_U \tag{10}$$

where $\boldsymbol{\Lambda}_Q\triangleq\mathrm{diag}(\mathrm{diag}(\bar{\boldsymbol{Q}}))$. The equality holds if and only if $\bar{\boldsymbol{Q}}$ is a diagonal matrix, i.e., $\boldsymbol{U}_R$ diagonalizes the matrix $\boldsymbol{Q}$.

c) Applying the method of Lagrange multipliers on I_U with the power constraint, we arrive at the conclusion that I_U is maximized if and only if the matrix $\boldsymbol{\Phi}$ has equal diagonal elements. $\square$

Remark 1: Theorem 1 holds for any distribution of the channel with a known mean. The result in Theorem 1 is a general condition that covers several special cases.

1) When $K \to \infty$, the channel reduces to Rayleigh fading. From the symmetry of matrix $\boldsymbol{\Phi}$, the optimal $\boldsymbol{\Lambda}_Q$ must be an identity matrix.
2) When $K = 0$, the channel becomes deterministic. The condition that $\boldsymbol{\Phi}$ has equal diagonal elements now reduces to the condition of $\boldsymbol{\Lambda}_Q + \frac{M}{\rho}\boldsymbol{\Lambda}_0^{-2}$ having equal diagonal elements. The solution in this case is water-filling.
3) The term $\frac{M}{\rho}\left((\alpha\boldsymbol{\Lambda}_0 + \bar{\alpha}\tilde{\boldsymbol{H}})^H(\alpha\boldsymbol{\Lambda}_0 + \bar{\alpha}\tilde{\boldsymbol{H}})\right)^{-1}$ is negligible compared with $\boldsymbol{\Lambda}_Q$ at high SNR. Thus, it is obvious that the optimal condition implies $\boldsymbol{\Lambda}_Q$ is an identity matrix.

Remark 2: From the expression of $\bar{\boldsymbol{Q}}$ and the condition of equality in Eq. (10), we conclude that the optimum eigenvector matrix of $\boldsymbol{Q}$ is the right singular vector matrix of $\boldsymbol{H}_0$. This result has been observed in [5].

Although Theorem 1 yields the condition for the optimum beamformer, the matrix $\boldsymbol{\Phi}$ governing this condition is still in the form of the expectation over a random function. The difficulty in calculating the expectation lies in that it needs twice the inverse of a matrix involving a non-central Wishart random matrix. Now, if we apply the matrix inverse lemma to the expression of $\boldsymbol{\Phi}$, we obtain

$$\boldsymbol{\Phi} = \boldsymbol{\Lambda}_Q^{-1} - \boldsymbol{\Lambda}_Q^{-1}\underbrace{\mathrm{E}\left\{\left(\boldsymbol{\Lambda}_Q^{-1} + \frac{\rho}{M}(\alpha\boldsymbol{\Lambda}_0 + \bar{\alpha}\tilde{\boldsymbol{H}})^H(\alpha\boldsymbol{\Lambda}_0 + \bar{\alpha}\tilde{\boldsymbol{H}})\right)^{-1}\right\}}_{\triangleq\boldsymbol{\Psi}}\boldsymbol{\Lambda}_Q^{-1} \tag{11}$$

There is no ready solution for $\boldsymbol{\Psi}$ in Eq. (11) which involves the expected inverse of a function containing a non-central Wishart matrix. To evaluate $\boldsymbol{\Psi}$, we will derive an approximate expression for it by calculating the first three terms of its Taylor expansion. The following lemma is introduced here to facilitate the development of the paper:

Lemma 1: For a noncentral Wishart distributed matrix [8] $\boldsymbol{W} \sim W_m(n, \boldsymbol{\Sigma}, \boldsymbol{\Sigma}^{-1}\boldsymbol{M}^H\boldsymbol{M})$, and a constant matrix $\boldsymbol{X}$, the expected value of the matrix $\mathrm{E}(\boldsymbol{W}+\boldsymbol{X})^{-1}$ is approximately given by

$$\begin{aligned}\mathrm{E}(\boldsymbol{W}+\boldsymbol{X})^{-1} \approx \boldsymbol{Z}_0^{-1} + \boldsymbol{Z}_0^{-1}\Big(&\boldsymbol{\Sigma}\boldsymbol{Z}_0^{-1}\boldsymbol{W}_0 + \boldsymbol{M}^H\boldsymbol{M}\boldsymbol{Z}_0^{-1}\boldsymbol{\Sigma} \\ &+ (tr\boldsymbol{Z}_0^{-1}\boldsymbol{\Sigma})\boldsymbol{W}_0 + (tr\boldsymbol{M}\boldsymbol{Z}_0^{-1}\boldsymbol{M}^H)\boldsymbol{\Sigma}\Big)\boldsymbol{Z}_0^{-1}\end{aligned} \tag{12}$$

with $\boldsymbol{W}_0 = \mathrm{E}\boldsymbol{W}$, and $\boldsymbol{Z}_0 = \boldsymbol{W}_0 + \boldsymbol{X}$. ■

Remark: This lemma is an extension of [9].

Apply Lemma 1 on $\boldsymbol{\Psi}$ by letting $\boldsymbol{\Sigma} = \bar{\alpha}^2\boldsymbol{I}$, $\boldsymbol{M} = \alpha\boldsymbol{\Lambda}_0$ and $\boldsymbol{X} = \boldsymbol{\Lambda}_Q^{-1}$, which are all diagonal, and we obtain

$$\begin{aligned}\boldsymbol{\Psi} \approx \boldsymbol{R}_0^{-1} &+ \frac{\bar{\alpha}^2\rho}{M}(2\alpha^2\boldsymbol{\Lambda}_0^2 + \bar{\alpha}^2N\boldsymbol{I})\boldsymbol{R}_0^{-3} + \frac{\bar{\alpha}^2\rho}{M}(tr\boldsymbol{R}_0^{-1}) \\ &(\alpha^2\boldsymbol{\Lambda}_0^2 + \bar{\alpha}^2N\boldsymbol{I})\boldsymbol{R}_0^{-2} + \frac{\alpha^2\bar{\alpha}^2\rho}{M}(tr\boldsymbol{R}_0^{-1}\boldsymbol{\Lambda}_0^2)\boldsymbol{R}_0^{-2}\end{aligned} \tag{13}$$

where $\boldsymbol{R}_0 = \boldsymbol{\Lambda}_Q^{-1} + \frac{\rho}{M}\mathrm{E}(\alpha\boldsymbol{\Lambda}_0 + \bar{\alpha}\tilde{\boldsymbol{H}})^H(\alpha\boldsymbol{\Lambda}_0 + \bar{\alpha}\tilde{\boldsymbol{H}}) = \boldsymbol{\Lambda}_Q^{-1} + \frac{\rho}{M}\left(\alpha^2\boldsymbol{\Lambda}_0^2 + \bar{\alpha}^2N\boldsymbol{I}\right)$. Here we observe that the first term $\boldsymbol{R}_0^{-1}$ is obtained by interchanging the order of the expectation and the matrix inverse in $\boldsymbol{\Psi}$. Thus, if we interchange the order of expectation with log det in Eq. (10), followed by applying the expectation on the terms inside the brackets and maximizing the result, this will yield $\boldsymbol{R}_0^{-1}$ for which the second and third order information in Eq. (13) will be lost.

Substituting Eq. (13) into Eq. (11), we obtain the approximate expression of $\boldsymbol{\Phi}$ which involves higher order polynomials of $\boldsymbol{\Lambda}_Q$. Since the matrices are all deterministic, it can be efficiently calculated numerically.

IV. MINIMIZATION OF MSE

We now consider the precoder design for Ricean channels in a MIMO system equipped with an MMSE receiver. In this case, the received signal $\boldsymbol{y}$ in Eq. (6) is processed by a linear MMSE equalizer followed by a symbol-by-symbol detector. It is known [10] that the MSE associated with the equalized signals is

$$\begin{aligned}\bar{\epsilon}^2 &= \mathrm{E}tr\Big(\boldsymbol{I} + \frac{\rho}{M}\boldsymbol{F}^H(\boldsymbol{I}\otimes\boldsymbol{H}_B)^H(\boldsymbol{I}\otimes\boldsymbol{H}_B)\boldsymbol{F}\Big)^{-1} \\ &= T\mathrm{E}tr\left(\boldsymbol{I} + \frac{\rho}{M}\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right)^{-1}\end{aligned} \tag{14}$$

Hence, we seek to solve the following optimization problem:

$$\min_{\boldsymbol{Q}} : \mathrm{E}tr\left(\boldsymbol{I} + \frac{\rho}{M}\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right)^{-1} \tag{15}$$

subject to the same power constraint as shown in Eq. (8b). The solution is provided by the following theorem:

Theorem 2: For a MIMO communication system with Ricean fading channels, when the unitary trace-orthogonal STBC is employed, and the signals are received by a linear MMSE equalizer followed by a symbol-by-symbol detector, the detection MSE is minimized if and only if 1) The eigenvector matrix of $\boldsymbol{Q}$ is the right singular value matrix of the channel mean, and 2) The eigenvalue matrix of $\boldsymbol{Q}$ is the one that enables the matrix $\boldsymbol{\Omega} \triangleq \frac{\rho}{M}\mathrm{E}\boldsymbol{H}_\Lambda^H\left(\boldsymbol{I} + \frac{\rho}{M}\boldsymbol{H}_\Lambda\boldsymbol{\Lambda}_Q\boldsymbol{H}_\Lambda^H\right)^{-2}\boldsymbol{H}_\Lambda$ to have equal diagonal elements, where $\boldsymbol{H}_\Lambda \triangleq \boldsymbol{\Lambda}_0 + \tilde{\boldsymbol{H}}$.

Proof: Similar logic in developing Theorem 1 can be applied here. □

Similar to Section III, the condition described in Theorem 2 is in the form of the expectation of a random non-linear function which needs to be calculated. However, here we observe that the matrix $\boldsymbol{\Omega}$ contains a second order matrix inverse making the calculation even more complicated than that for $\boldsymbol{\Phi}$. In the following, we find an approximation for $\boldsymbol{\Omega}$ in terms of the SNR ρ.

First, by applying matrix inverse lemma, and after some mathematical manipulations, the matrix $\boldsymbol{\Omega}$ can be written as

$$\begin{aligned}\boldsymbol{\Omega} = 2\boldsymbol{\Phi} - \boldsymbol{\Lambda}_Q^{-2}\boldsymbol{\Psi} + \mathrm{E}\boldsymbol{\Lambda}_Q^{-1}&\left(\frac{\rho}{M}\boldsymbol{H}_\Lambda^H\boldsymbol{H}_\Lambda + \boldsymbol{\Lambda}_Q^{-1}\right)^{-1} \\ \boldsymbol{\Lambda}_Q^{-2}&\left(\frac{\rho}{M}\boldsymbol{H}_\Lambda^H\boldsymbol{H}_\Lambda + \boldsymbol{\Lambda}_Q^{-1}\right)^{-1}\end{aligned} \tag{16}$$

The first two terms in Eq. (16) concern with the matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$. From Eq. (11), we observe that while the SNR associated with these two matrices are in the first order of ρ, the third term contains the second order. The higher is the SNR, the less impact the third them has on the whole function $\boldsymbol{\Omega}$. The approximate expressions for the first two terms have already been obtained in Section III. Now, we need to find an approximation for the third term. We apply Taylor expansion around $\boldsymbol{R}_0^{-2}\boldsymbol{\Lambda}_Q^{-3}$ resulting in a series of term containing $\{\rho^{-2}, \rho^{-3}, \cdots\}$. Ignoring the effects of ρ^{-3} and negatively higher order terms under moderately high SNR, we obtain the following approximate expression of $\boldsymbol{\Omega}$,

$$\boldsymbol{\Omega} \approx 2\boldsymbol{\Phi} - \boldsymbol{\Lambda}_Q^{-2}\boldsymbol{\Psi} + \boldsymbol{R}_0^{-2}\boldsymbol{\Lambda}_Q^{-3} \tag{17}$$

All the matrices in Eq. (17) are now deterministic. By applying Theorem 2, the optimal $\boldsymbol{\Lambda}_Q$ can be numerically computed. At lower SNR, we may incorporate more terms in the series to compute the value of $\boldsymbol{\Omega}$. The terms are still deterministic and presents no problem in computation.

V. ERGODIC CAPACITY AND MSE

Theorem 1 and 2 provide us with the optimal conditions for the beamforming matrix to achieve either the the maximum ergodic channel capacity or the minimum MSE. In this section, we will examine the conditions under which these two criteria yield the same optimum code. The following corollary indicates the conditions:

Corollary 1: For a MIMO system with Ricean fading channels, maximizing ergodic channel capacity and minimizing MSE results in the same optimal code if and only if the channel mean matrix has equal eigenvalues (including zeros).

Outline of proof: a) After some manipulations, the matrix $\boldsymbol{\Omega}$ can be written as:

$$\boldsymbol{\Omega} = \boldsymbol{\Phi} + \mathrm{E}\left(\frac{M}{\rho}(\boldsymbol{H}_\Lambda^H \boldsymbol{H}_\Lambda)^{-1} + \boldsymbol{\Lambda}_Q\right)^{-2}\boldsymbol{\Lambda}_Q \tag{18}$$

b) For maximization of ergodic capacity, $\boldsymbol{\Phi}$ must have equal diagonal elements. For minimization of MSE, $\boldsymbol{\Omega}$ must also have equal diagonal elements. Substituting the expression of $\boldsymbol{H}_\Lambda$ and comparing both sides of Eq. (18), the conclusion that the channel mean has equal eigenvalues can be reached. □

VI. SIMULATION

We consider a MIMO system with $M = N = T = 2$, where the channel has the mean $\boldsymbol{H}_0 = \begin{pmatrix} 0.5620 & 0.1873 \\ 0.3746 & 1.8731 \end{pmatrix}$, and Ricean factor $K = 0.5$. The input signals are randomly chosen from a 4-QAM constellation and coded with unitary trace-orthogonal code and the beamforming matrix is obtained numerically by applying Theorem 2 and Eq. (17). At the receiver, the signals are processed by a linear MMSE equalizer followed by a symbol-by-symbol detector. The bit-error-rate (BER) performance under different SNR is plotted in Fig. 1. We also simulate two cases where the signals are processed through the same system except with different beamformers: 1) $\boldsymbol{B} = \boldsymbol{I}$ and 2) $\boldsymbol{B}$ is obtained by minimizing the lower bound resulted from interchanging the expectation with $tr(\cdot)^{-1}$ in Eq. (15) so that the expectation applies directly to the random channel matrix term inside the bracket. The superior performance of the proposed optimum beamformer can be clearly observed.

VII. CONCLUSION

In this paper, we have examined the problem of optimum precoder designs for MIMO systems under Ricean fading. We have approached the problem using two different criteria and presented the necessary and sufficient conditions for optimum precoders to satisfy them. To apply the optimal conditions, we expanded the expectation of the non-linear functions into their deterministic Taylor series and approximated them by choosing the dominant terms. From these, the STBC can be obtained efficiently. The accuracy of the aprroximate solution can be increased by including more higher order terms.

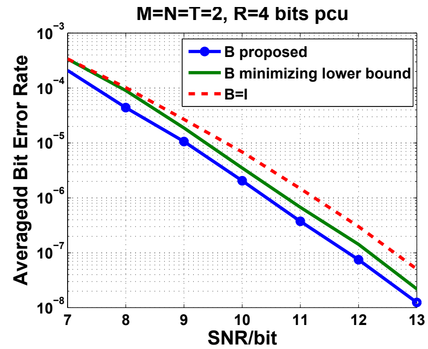

Fig. 1. BER performance comparisons.

VIII. REFERENCES

[1] S. Haykin and M. Moher, *Modern Wireless Communications*. Pearson Prentice Hall, 2005.

[2] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Euro. Trans. Telecomm.*, vol. 10, pp. 585–595, Nov./Dec. 1999.

[3] J. Liu, J.-K. Zhang, and K. M. Wong, "On the design of minimum BER linear space-time block codes for MIMO systems with MMSE receivers," *IEEE Trans. Signal Processing*, vol. 54, no. 8, pp. 3147–3158, Aug. 2006.

[4] S. K. Jayaweera and H. V. Poor, "On the capacity of multiple-antenna systems in rician fading," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 1102–1111, May 2005.

[5] A. M. Tulino, A. Lozano, and S. Verdu, "Capacity-achieving input covariance for single-user multi-antenna channels," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 662–671, Mar. 2006.

[6] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[7] J. Zhang, J. Liu, and K. M. Wong, "Trace-orthonormal full diversity cyclotomic space-time codes," *IEEE Trans. Signal Processing*, vol. 55, pp. 618–630, Feb. 2007.

[8] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*. John Wiley & Sons, 1982.

[9] H. Neudecker, "The expected value of the inverse of a non-central Wishart matrix," *Statistical Papers*, vol. 46, pp. 469–472, 2005.

[10] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant filterbank precoders and equalizers Part I: Unification and optimal designs," *IEEE Trans. Signal Processing*, vol. 47, pp. 1988–2006, July 1999.