OPTIMUM COMBINING SYSTEMS IN THE PRESENCE OF RICIAN FADING: SINR AND CAPACITY ANALYSIS

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ABSTRACT

This paper analyzes the performance of adaptive antenna arrays employing linear combining techniques designed to maximize the SINR. In the communications literature, these systems are referred to as optimum combining (OC) systems. We consider the practical case where the desired signal undergoes Rician fading, and is corrupted by Rayleigh-faded interfering signals and noise. We first propose new closed-form Gamma approximations for the SINR distribution at the output of the OC combiner, which we show to be remarkably accurate. We then employ these approximations to derive new closed-form expressions for the ergodic capacity and simplified expressions for the high-SNR regime. These results reveal that the capacity improves monotonically with Rician K-factor.

***Index Terms*—Signal processing antennas, adaptive arrays, land mobile radio cellular systems, cochannel interference, interference suppression**

I. INTRODUCTION

Adaptive antenna arrays employing linear combining techniques provide an effective means of detecting a desired signal in the presence of interference and noise. In the communications literature, where adaptive arrays are commonly employed at the receiver side, the optimum linear combining strategy in terms of maximizing the signal-to-interference and noise ratio (SINR) is referred to as optimum combining (OC) [1].

Recently, the performance of OC systems has been studied under various assumptions on the channel propagation environment. In particular, when both the desired signal and the interfering signals exhibit Rayleigh fading, hereafter referred to as a Rayleigh-Rayleigh scenario, the performance of OC systems has been well-investigated. For this case, results are now available for the symbol error rate (SER) with various digital modulation formats, and the SINR probability density function (p.d.f.), cumulative distribution function (c.d.f.), and moment generating function (m.g.f.).

In practice, however, the desired signal often has a line-of-sight (LoS) component, in which case modeling the corresponding channel as Rician fading is more appropriate. The presence of LoS has been confirmed through physical measurements for a number of applications, such as micro-cellular mobile and indoor radio [2]. In contrast to the Rayleigh case, there are few performance results for OC systems where the desired signal has Rician fading. Moreover, of the results which are available, most restrict the number of interfering signals to be less than the number of antennas in the adaptive array and employ a simplified interference-limited model [2], where the effect of thermal noise is ignored. Only recently have some results been presented in [3, 4] which are not restricted to the interference-limited model and allow for arbitrary number of interferers and receive antennas. In that work, exact expressions were obtained for the SINR m.g.f. and the SER with M-PSK modulation (although these were not in closed-form), and for the SINR moments. The p.d.f. and c.d.f. of the SINR, however, still remain unknown.

In this paper, we consider OC systems where the desired signal has LoS, but the interferers do not, which we refer to as the Rician-Rayleigh scenario. We propose new closed-form Gamma approximations for the SINR p.d.f. and c.d.f. which we show to be remarkably accurate. Based on these new SINR statistical results, we derive expressions for the ergodic capacity of Rician-Rayleigh OC systems, as well as simplified expressions in the high SNR regime. These results reveal that LoS improves the ergodic capacity.

II. SYSTEM MODEL

We consider a system where the desired signal is corrupted by L interfering signals and thermal noise, and the receiver is equipped with an N_r-element antenna array. The received $N_r \times 1$ complex baseband signal vector is expressed as

$$\mathbf{z} = \sqrt{E_D}\mathbf{h}_0 x_0 + \sum_{j=1}^{L} \sqrt{E_I}\mathbf{h}_j x_j + \mathbf{n} \quad (1)$$

where E_D and E_I are the average transmit power from the desired and interferer signals respectively, $\mathbf{h}_0$ and $\mathbf{h}_j$ are the $N_r \times 1$ flat-fading channel vectors for the desired and jth interferer ($j = 1 \dots L$) signals respectively, and $\mathbf{n}$ is an $N_r \times 1$ additive white gaussian noise (AWGN) vector containing independent entries $\sim \mathcal{CN}(0, N_0)$. Also, x_0 and x_j are the desired and interfering data symbols respectively, modeled as zero mean random variables with unit variance. We assume that $\mathbf{h}_0$ and $\mathbf{h}_j$ ($j = 1 \dots L$) are known at the receiver.

The receiver optimally combines the output from the N_r receive antennas, which results in a SINR given by [1]

$$\gamma_{\mathrm{SINR}} = \gamma_{\mathrm{SNR}} \mathbf{h}_0^{\dagger} \mathbf{R}^{-1} \mathbf{h}_0 \quad (2)$$

where $\gamma_{\rm SNR} = \frac{E_D}{N_0}$ is the average transmit SNR, $\mathbf{R} = \mathbf{I}_{N_r} + \gamma_{\rm INR}\mathbf{H}_I\mathbf{H}_I^\dagger$, $\gamma_{\rm INR} = \frac{E_I}{N_0}$ is the average interference-to-noise ratio (INR) and $\mathbf{H}_I = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_L]$.

We consider the case where the desired signal is subject to Rician fading and the interferer signals are subject to Rayleigh fading. The distribution of $\mathbf{h}_0$ is thus given by

$$\mathbf{h}_0 \sim \mathcal{CN}_{N_r,1}\left(\sqrt{\frac{K}{K+1}}\mathbf{m}, \frac{1}{K+1}\mathbf{I}_{N_r}\right) \tag{3}$$

where $\mathbf{m}$ is the channel mean vector, normalized to satisfy $\|\mathbf{m}\|^2 = N_r$, and K is the Rician K-factor. In addition, the Rayleigh interference channels are distributed according to

$$\mathbf{H}_I \sim \mathcal{CN}_{N_R,L}(\mathbf{0}_{N_r\times L}, \mathbf{I}_{N_r} \otimes \mathbf{I}_L) \ . \tag{4}$$

III. NEW ACCURATE APPROXIMATIONS FOR THE DISTRIBUTION OF THE SINR

The distribution of the SINR in (2) is particularly difficult to analyze since it is a random function of both the non-central complex normal vector $\mathbf{h}_0$ and the random matrix $\mathbf{R}$; and analytical expressions for the p.d.f. and c.d.f. are not forthcoming. Our approach is to approximate the SINR p.d.f. and c.d.f. as a Gamma random variable.

Our motivation for considering a Gamma distribution is that it has been shown previously to yield a good approximation (or exact fit in some scenarios) for a number of cases related to the SINR model in (2). These include among others, models with (i) Rayleigh-faded $\mathbf{R}$ and Rayleigh-faded $\mathbf{h}_0$ [5–7], (ii) $\mathbf{R} = \mathbf{I}_{N_r}$ (ie. no interference case) and Rayleigh-faded $\mathbf{h}_0$ [8], and (iii) $\mathbf{R} = \mathbf{I}_{N_r}$ and Rician-faded $\mathbf{h}_0$ with $N_r = 1$ (ie. scalar Rician fading) [9].

Here we present a closed-form Gamma approximation for the SINR model in (2) for the Rician-Rayleigh scenario. Our numerical results will demonstrate this simple approximation exhibits very good accuracy over a wide range of Rician-Rayleigh channel conditions, and therefore presents a valuable analytical tool for system performance analysis.

A Gamma distribution, in its most general form, has c.d.f. $F_Y(\cdot)$ and p.d.f. $f_Y(\cdot)$ given by

$$F_Y(y) = \frac{\gamma\left(k, \frac{y}{\theta}\right)}{\Gamma(k)}, \qquad f_Y(y) = \frac{y^{k-1}e^{-\frac{y}{\theta}}}{\Gamma(k)\theta^k} \tag{5}$$

with mean $k\theta$ and variance $k\theta^2$, where $\theta \in \mathcal{R}$ is the scale parameter and $k \in \mathcal{R}$ is the shape parameter. We obtain our new closed-form Gamma approximation for the SINR distribution by matching the Gamma mean and variance (thereby determining the scale and shape parameters also) with the Rician-Rayleigh SINR mean and variance. This is given in [3, Eq. 25], and after algebraic manipulation can be expressed as

$$\mathrm{E}[\gamma_{\rm SINR}] = \gamma_{\rm SNR}\vartheta_1 \tag{6}$$

$$\mathrm{Var}[\gamma_{\rm SINR}] = \gamma_{\rm SNR}^2\left(\left(1 - \frac{(K/(K+1))^2}{(N_r+1)}\right)\vartheta_2 - \vartheta_1^2\right) \tag{7}$$

where

$$\vartheta_\ell = \frac{\mu_\ell^{\rm Ray-Ray}}{\gamma_{\rm SNR}^\ell} \tag{8}$$

and $\mu_\ell^{\rm Ray-Ray}$ is the ℓth SINR moment in the Rayleigh-Rayleigh case given by [3, Eq. 26]. Thus, the scale parameter for our SINR Gamma approximation is evaluated as

$$\theta_{\rm SINR}(K) = \gamma_{\rm SNR}\theta_{\rm INR}(K) \tag{9}$$

with

$$\theta_{\rm INR}(K) = \left(1 - \frac{(K/(K+1))^2}{(N_r+1)}\right)\frac{\vartheta_2}{\vartheta_1} - \vartheta_1 \ , \tag{10}$$

and the shape parameter as

$$k_{\rm INR}(K) = \frac{\vartheta_1^2}{\left(1 - \frac{(K/(K+1))^2}{(N_r+1)}\right)\vartheta_2 - \vartheta_1^2} \ . \tag{11}$$

Our desired analytical Rician-Rayleigh SINR c.d.f. and p.d.f. approximations are now directly obtained by making the following substitutions into (5): $\theta = \theta_{\rm SINR}(K)$, $k = k_{\rm INR}(K)$, and $y = \gamma$.

Fig. 1 presents a comparison of the Gamma-approximated SINR c.d.f. curves with Monte Carlo simulated c.d.f. curves considering different numbers of interferers. We see a very accurate match between our new analytical Gamma approximation and the true SINR c.d.f. Moreover, the outage probability is seen to degrade significantly as the number of interferers increase.

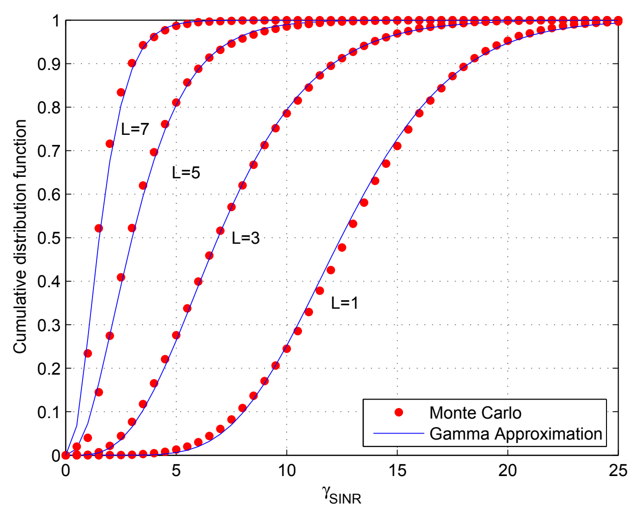

Fig. 1. C.d.f. of the SINR of Rician-Rayleigh OC systems; comparison of analytical Gamma approximation and Monte Carlo simulated c.d.f.s for different numbers of interferers L. $N_r = 5$, $K = 7$dB, and $\gamma_{\rm SNR} = \gamma_{\rm INR} = 5$dB.

Fig. 2 compares Gamma-approximated SINR p.d.f. curves with Monte-Carlo simulated p.d.f. curves for different numbers of interferers. We see an accurate match between our new analytical Gamma approximation and the true SINR p.d.f. Note that both our p.d.f. and c.d.f. approximations are accurate for other system configurations, but are not shown due to space limitations.

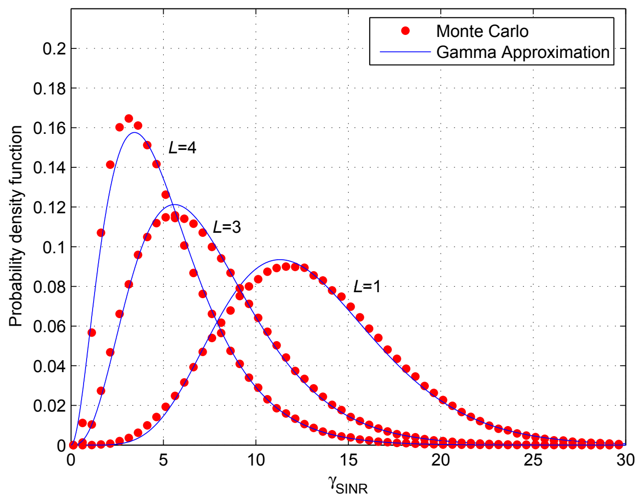

Fig. 2. P.d.f. of the SINR of Rician-Rayleigh OC systems; comparison of analytical Gamma approximation and Monte Carlo simulated p.d.f.s for different numbers of interferers L. $N_r = 5$, $K = 7$dB, and $\gamma_{\rm SNR} = \gamma_{\rm INR} = 5$dB.

Note that the Gamma distribution has the same form as the Nakagami-m fading distribution, for which various results are available in the literature. Thus, by employing these prior results, we can easily obtain new simple closed-form approximations for various performance measures of interest to Rician-Rayleigh OC systems, which would not otherwise be possible via exact methods. We will demonstrate this in the following section when considering ergodic capacity.

IV. ERGODIC CAPACITY ANALYSIS OF RICIAN-RAYLEIGH OC SYSTEMS

We now utilize our new Gamma approximated SINR distributions proposed in the previous section to analyze the ergodic capacity of Rician-Rayleigh OC systems, which is given by

$$C(\gamma_{\rm SNR}) = {\rm E}_X\left[\log_2(1+\gamma_{\rm SNR}X)\right] \qquad (12)$$

where $X = \mathbf{h}_0^\dagger \mathbf{R}^{-1}\mathbf{h}_0$. Due to the difficulty in obtaining exact expressions for the SINR p.d.f. and c.d.f. (equivalently X), exact ergodic capacity solutions are not forthcoming. By employing the closed-form Gamma approximation presented in Section III however, we will see that very accurate closed-form capacity approximations are possible for the entire range of SNR levels. To gain further insights, we also investigate the ergodic capacity in the high SNR regime.

IV-A. Accurate Capacity Approximation Based on Gamma Distribution

Substituting our closed-form SINR Gamma approximation given by (5), (9), and (11) into the expectation (12), integrating using Nakagami-m results from [10] and applying [11, Eq. 9.31.5] and [11, Eq. 9.34.8], leads to

$$\begin{aligned} C(\gamma_{\rm SNR}) &\approx \log_2 e\, \theta_{\rm INR}(K) k_{\rm INR}(K)\gamma_{\rm SNR} \\ &\times {}_3F_1([k_{\rm INR}(K)+1,1,1];2;-\theta_{\rm INR}(K)\gamma_{\rm SNR}) \end{aligned} \qquad (13)$$

where $.F.(\cdot;\cdot;\cdot)$ is a hypergeometric function.

The accuracy of (13) is shown in Fig. 3, where it is compared with Monte Carlo simulated results for different numbers of interferers. In all cases, the difference between the capacity curves for the Gamma approximation and the Monte Carlo simulated curves is almost negligible.

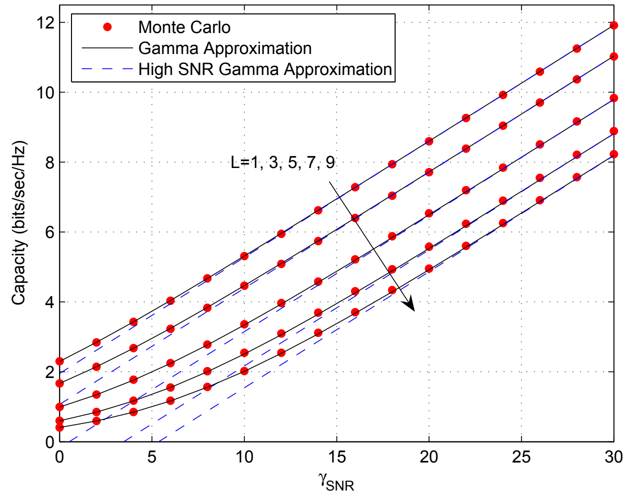

Fig. 3. Ergodic capacity of Rician-Rayleigh OC systems; comparison of analytical Gamma approximation, analytical high SNR Gamma approximation, and Monte Carlo simulated capacity for different numbers of interferers L. Results are shown for $N_r = 5$, $K = 5$dB, and $\gamma_{\rm INR} = 5$dB.

IV-B. High SNR Analysis

At high SNR, the ergodic capacity (12) takes the general form [12]

$$C(\gamma_{\rm SNR}) = \mathcal{S}_\infty\left(\frac{\gamma_{\rm SNR}|_{\rm dB}}{3\ {\rm dB}} - \mathcal{L}_\infty\right) + o(1) \qquad (14)$$

where $\mathcal{S}_\infty$ denotes the high-SNR slope in bits/s/Hz/(3 dB) given by

$$\mathcal{S}_\infty \triangleq \lim_{\gamma_{\rm SNR}\to\infty} \frac{C(\gamma_{\rm SNR})}{\log_2(\gamma_{\rm SNR})} \qquad (15)$$

and $\mathcal{L}_\infty$ is the high SNR power offset (3 dB units) given by

$$\mathcal{L}_\infty \triangleq \lim_{\gamma_{\rm SNR}\to\infty}\left(\log_2(\gamma_{\rm SNR}) - \frac{C(\gamma_{\rm SNR})}{\mathcal{S}_\infty}\right). \qquad (16)$$

For Rician-Rayleigh OC systems, we again use our Gamma SINR distribution approximations proposed in the previous section to accurately approximate S_∞ and $\mathcal{L}_\infty$ respectively as follows

$$S_\infty \approx 1 \tag{17}$$

$$\mathcal{L}_\infty \approx -\log_2(\theta_{\rm INR}(K)) - \psi(k_{\rm INR}(K))\log_2 e \tag{18}$$

where $\psi(\cdot)$ is the digamma function, and $\theta_{\rm INR}(\cdot)$ and $k_{\rm INR}(\cdot)$ are defined in (10) and (11) respectively. The proof is omitted due to space limitations.

The accuracy of this result is confirmed in Fig. 3, where we compare the high SNR capacity approximation obtained by combining (14), (17) and (18) with the Monte Carlo simulated ergodic capacity for different numbers of interferers. We see that in all cases there is almost negligible error between the capacity approximation curves and the exact (simulated) capacity curves in the high SNR regime.

It is interesting to note that the Rician K-factor affects the high SNR power offset $\mathcal{L}_\infty$, but not the capacity slope S_∞. Indeed, in the Appendix we prove that $\mathcal{L}_\infty$ is a decreasing function of K; thereby demonstrating that LoS improves the ergodic capacity in the high SNR regime.

Appendix

We start by taking the derivative of $\mathcal{L}_\infty$ w.r.t. K, which after some algebraic manipulation results in

$$\frac{\mathrm{d}\mathcal{L}_\infty}{\mathrm{d}K} = \frac{\log_2 e \frac{2K/(K+1)^3}{(N_r+1)}\vartheta_2}{\left(1-\frac{(K/(K+1))^2}{(N_r+1)}\right)\vartheta_2 - \vartheta_1^2} \tag{19}$$

$$\times\left(1-\frac{\vartheta_1^2\psi\left(1,\frac{\vartheta_1^2}{\left(1-\frac{(K/(K+1))^2}{(N_r+1)}\right)\vartheta_2-\vartheta_1^2}\right)}{\left(1-\frac{(K/(K+1))^2}{(N_r+1)}\right)\vartheta_2-\vartheta_1^2}\right) \tag{20}$$

where $\theta_{\rm INR}(K)$ and $k_{\rm INR}(K)$ are given in (10) and (11) respectively and $\psi(\cdot,\cdot)$ is the polygamma function [11]. Since (19) is always positive, to prove $\mathcal{L}_\infty$ is a decreasing function of K, we are therefore required to show that

$$1-\varphi\psi(1,\varphi)\leq 0 \quad\rightarrow\quad \psi(1,\varphi)-\frac{1}{\varphi}\geq 0 \tag{21}$$

where $\varphi\in\mathcal{R}^+$ is the second argument of the polygamma function in (20). To proceed, we first consider the case when $0\leq\varphi\leq 1$. By replacing $\psi(1,\varphi)$ in (21) with its series expansion, we obtain the following equivalent condition

$$\sum_{p=0}^{\infty}\frac{1}{(p+\varphi)^2}-\frac{1}{\varphi}=\frac{1}{\varphi^2}-\frac{1}{\varphi}+\sum_{p=1}^{\infty}\frac{1}{(p+\varphi)^2}\geq 0 \tag{22}$$

which clearly holds when $0\leq\varphi\leq 1$. Now consider the case when $\varphi>1$. We first lower bound $\psi(1,\varphi)$ in (21) using [13, Theorem 2.1] which gives us, after algebraic manipulation, the following sufficient condition for which the inequality in (21) holds

$$1+\frac{1}{\varphi^2}-e^{-\frac{1}{\varphi}}-\frac{1}{\varphi}=\frac{1}{\varphi^2}+\frac{1}{2\varphi^2}-\frac{1}{3!\varphi^3}+\frac{1}{4!\varphi^4}+\cdots\geq 0 \tag{23}$$

It is clear that (23) will converge to a positive value when $\varphi>1$, hence the condition in (23) always holds for $\varphi>1$.

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