POWER-EFFICIENT RATE ALLOCATION FOR SLEPIAN-WOLF CODING OVER WIRELESS SENSOR NETWORKS

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ABSTRACT

Power consumption is a critical concern in communications over wireless sensor networks (WSN). In this paper, we address the rate-allocation problem for Slepian-Wolf coding of multiple correlated sources. The goal is to find the optimal rate-point that allows lossless reconstruction of the sources, while minimizing the overall transmission power consumption of the WSN under an exponential cost model. A novel water-filling algorithm to be performed by the receiver is proposed to solve the problem in a recursive manner. The feasibility and optimality of the proposed solution are analyzed mathematically and verified experimentally. Compared to the conventional Lagrangian-multiplier approach, our algorithm achieves dramatic reduction in computational complexity.

Index Terms—Slepian-Wolf coding, wireless sensor network, power optimization, rate-allocation, water-filling model.

1. INTRODUCTION

Significant efforts have been made in recent years in building wireless sensor networks (WSN) from computing devices with integrated sensing and wireless communication capabilities. Among others, video communication over WSN is envisioned for a wide range of applications, such as battlefield intelligence, surveillance, emergency response, and multimedia systems in consumer electronics [1]. Unlike many other wireless devices, the energy provisioned for a wireless sensor node is not expected to be easily renewable throughout its lifetime, which poses significant challenges to the design of WSN systems. In this context, we are interested in power-aware distributed source coding (DSC) [2], because its "simple encoding, complex decoding" principle is very suitable for video communication over WSN.

DSC can be categorized into lossless DSC and lossy DSC. Lossless DSC is also known as the Slepian-Wolf coding (SWC) [3], which can be treated as a channel coding problem [4]. In recent years, practical SWC's, e.g., [5][6], based on state-of-the-art channel codes such as the turbo codes and the LDPC codes have shown near-capacity coding performance. Further advances in practical SWC design make it possible to achieve any point inside the Slepian-Wolf (SW) region [7].

To minimize the sensor node's power consumption in transmitting the encoded bits over the WSN, careful rate-allocation (RA) is needed among the sources. In [8], separable cost functions with the linear and the exponential cost models are considered, and the RA problem is solved for the linear model. However, in wireless communications, the exponential model is more appropriate as suggested by Shannon's channel capacity formula. Solution for the exponential cost model has only been given for the

two-source case in [8]. In this paper, we address the problem for a general *M*-source case and propose a fast algorithm to search for the optimal rate point recursively. Compared to the conventional Lagrangian-multiplier approach, the proposed scheme reduces the computational complexity significantly. Simulation results demonstrate that significant power saving can be achieved using the proposed solution.

The rest of the paper is organized as follows. The powerefficient RA problem is formulated in Section 2. A fast waterfilling algorithm is proposed with its feasibility and optimality proved in Section 3. Simulation results are presented in Section 4. Section 5 concludes the paper.

2. PROBLEM FORMULATION

Let's consider a set of sources $X_1, ..., X_M$, each of which is i.i.d., takes values from a discrete alphabet and has a finite entropy. The sources are encoded separately in *M* different source nodes at rates $R_1, ..., R_M$, respectively. The encoded bits are transmitted over a WSN to a sink node, where joint decoding is performed. Lossless reconstruction¹ is possible iff the rate point $(R_1, ..., R_M)$ lies in the SW region defined by [9]:

$$R(\Phi) \ge H(X(\Phi) | X(\Phi^c))$$
⁽¹⁾

for all $\Phi \subseteq \{1, ..., M\}$, where

$$R(\Phi) = \sum_{k \in \Phi} R_k \tag{2}$$

and $X(\Phi) = \{X_k : k \in \Phi\}, X(\Phi^c) = \{X_k : k \notin \Phi\}$. For the sake of brevity, we also define $I_M = \{1, ..., M\}$.

As we have mentioned, the most essential cost metric in a WSN is the power consumption. During the transmission, power consumption is needed not only at the source nodes but also at the intermediate nodes. We are interested in minimizing the overall cost of the entire WSN.

In this paper, we assume the encoded bits are transmitted over a packet switching network using unicast, and packets are routed along the shortest path. That is, a data flow is formed between each sensor and the sink. We also assume that the transmissions of different data flows do not interact with each other. For example, two packets arriving at an intermediate node are neither assembled together, nor subject to any processing such as network coding. This is typically the case in WSN as the sensor nodes are not designed to be so powerful in functionality. In this scenario, the cost function can be modeled as the sum of costs to communicate between the sensors and the sink

¹ It is worth noting that practical SWC schemes based on channel coding are not strictly lossless. It only means the decoding error probability can be arbitrarily small. However, we will use "lossless" in this paper for simplicity.

$$C = \sum_{k=1}^{M} c_k \left(R_k \right) \tag{3}$$

where c_k is a topology-dependent cost function. It is non-negative and non-decreasing in general.

A good model for cost functions is established in [8] and summarized as follows. Let $w_k > 0$ be the weighting factor (which, e.g., may reflect the noise level or the fading factor of a wireless link) assigned to the shortest path from X_k to the receiver, then the multiple cost functions are unified in the form of

$$c_k(R) = w_k \times c(R), \text{ for } k = 1, ..., M.$$
 (4)

where c(.) depends only on the rate value. Two typical examples for c(R) are the linear cost model with c(R) = R for wired networks, and the exponential model with $c(R) = \exp(R)$ for wireless networks. With the linear cost model, the min-cost rate point can be easily found and the result is presented in [8]. However, with the exponential cost model, which is more typical in a WSN, the problem has not been completely solved and will be addressed in this paper. The problem is formulated as: given the SW region defined in (1), find the rate point R^* in the SW region that minimizes the cost

$$C = \sum_{k=1}^{M} w_k \times \exp\left(R_k\right).$$
⁽⁵⁾

When M = 2, a closed-form solution is given in [8] using Lagrangian multipliers. One might want to extend the Lagrangian method to the more general case of M sources, but the computational complexity increases rapidly. In the next section, a novel approach is proposed to find the min-cost point in a recursive manner, based on a novel water-filling algorithm. The required complexity of this algorithm is much lower than the Lagrangian approach.

3. LOW-COMPLEXITY RATE-ALLOCATION

The SW region defined in (1) is the intersection of multiple halfspaces, therefore it is convex (not strictly). On the other hand, (5) defines a strictly-concave surface. Thus there is one and only one min-cost rate point R^* in the SW region, and R^* must lie on the boundary the SW region. That is, R^* must satisfy at least one of the equalities in (1). The main idea of the proposed algorithm is summarized as follow. According to Corollary 1 presented in subsection 3.1, the equality $R(I_M) = H(X(I_M))$ must hold for R^* . If none of the other equalities in (1) holds for R^* , it is straightforward to apply Lagrangian multipliers and the complexity is low. However that is not always the case. R^* might satisfy some other equalities in (1). If this happens, R^* can be achieved by first applying SWC to a subset of the sources independently from others, then using them as side information to decode other sources. In other words, we can treat the M-source RA problem recursively and reduce the number of sources in each recursion. Now the problem is how to find a suitable subset of the sources while still being able to achieve the minimum cost. We will introduce a water-filling model for this purpose.

3.1 A water-filling model for rate-allocation

Water-filling model has been used in conventional source coding of multiple correlated sources [10]. Extensions and modifications are needed to fit the model for our problem.

We use *M* tubes to represent the rate space of the *M* sources. We also introduce another $2^{M}-1-M$ virtual tubes, each of which holds the sum of the rates of a certain subset of sources. The total $2^{M}-1$ tubes represent the $2^{M}-1$ inequalities in (1). Now we can symbolize each tube by using a subset Φ of I_{M} , and a lower bound is marked at tube Φ as in (1). Besides this lower bound, an upper bound is also defined for tube Φ as

$$R(\Phi) \le H(X(\Phi)). \tag{6}$$

If the amount of water in a tube is less than the lower bound, we say there is an underflow; on the other hand if the amount of water in a tube is more than the upper bound, there is an overflow; when the amount of water equals the lower/upper bound, we say the tube is about to be underflowed/overflowed. A rate point is inside the SW region iff none of the tubes is underflowed.

With the above water-filling model defined, we have the following proposition:

Proposition 1: if a rate point is inside the SW region, and two tubes Φ_1 and Φ_2 are about to be underflowed simultaneously, then the tubes $\Phi_1 \cap \Phi_2$ and $\Phi_1 \cup \Phi_2$ are both about to be underflowed.

Proof: By definition we have

$$R(\Phi_1) = H\left(X(\Phi_1) \mid X(\Phi_1^c)\right), R(\Phi_2) = H\left(X(\Phi_2) \mid X(\Phi_2^c)\right).$$
⁽⁷⁾

On the other hand,

$$R(\Phi_{1}) + R(\Phi_{2}) = \sum_{k \in \Phi_{1}} K_{k} + \sum_{k \in \Phi_{2}} K_{k}$$

$$= \sum_{k \in \Phi_{1} \cup \Phi_{2}} R_{k} + \sum_{k \in \Phi_{1} \cap \Phi_{2}} R_{k} = R(\Phi_{1} \cup \Phi_{2}) + R(\Phi_{1} \cap \Phi_{2})$$

$$\geq H \left[X(\Phi_{1} \cup \Phi_{2}) | X((\Phi_{1} \cup \Phi_{2})^{c}) \right] + H \left[X(\Phi_{1} \cap \Phi_{2}) | X((\Phi_{1} \cap \Phi_{2})^{c}) \right]$$
(8)

where the inequality is because the rate point is inside the SW region, so none of the tubes are underflowed. Combining (7) and (8), through some easy calculation based on the chain rule of conditional entropy, we have the following inequality:

$$H\left[X\left(\Phi_{1}\setminus\Phi_{2}\right)|X\left(\Phi_{1}^{c}\right)\right] \geq H\left[X\left(\Phi_{1}\setminus\Phi_{2}\right)|X\left(\left(\Phi_{1}\cup\Phi_{2}\right)^{c}\right)\right] \cdot {}^{(9)}$$

where $\Phi_1 | \Phi_2$ is the set difference of Φ_1 and Φ_2 . However, note that $(\Phi_1 \cup \Phi_2)^c \subseteq (\Phi_1)^c$, the left-hand side of (9) is in fact no greater than the right-hand side (chain rule). Hence the equality in (9) holds, and so does it in (8), from which the conclusion is drawn.

From Proposition 1 we know if a rate point is inside the SW region, we can find all the tubes that are about to be underflowed and derive the union of them, denoted as Φ_u . If $\Phi_u \neq I_M$, we can always find a source from $I_M \Phi_u$ (so that none of the tubes containing this source is about to be underflowed), decrease its bitrate by a small amount and still keep the rate point inside the SW region, however the overall cost in (5) is reduced. Hence the following two corollaries are in place:

Corollary 1: The min-cost point R^* must have $R(I_M) = H(X(I_M))$.

Proof: As shown above. Note that in this case, the tube I_M is about to be underflowed, and also about to be overflowed.

Corollary 2: The min-cost point R^* must have none of its tubes overflowed.

Proof: If a tube is overflowed, according to Corollary 1, its complementary tube is underflowed, meaning that R^* is out of the SW region.

Then the dual of Proposition 1 is stated:

Proposition 2: If R^* is the min-cost rate point, and two tubes Φ_1 and Φ_2 are about to be overflowed simultaneously, then the tubes $\Phi_1 \cap \Phi_2$ and $\Phi_1 \cup \Phi_2$ are both about to be overflowed.

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Proof: Similar to that of Proposition 1.

3.2 Rate-allocation algorithm based on the water-filling model

Now that we have established a water-filling model, we are in a position to describe an RA algorithm based on this model.

According to Corollary 1, the total amount of water (rate budget) is $R(I_M) = H(X(I_M))$. We fill the water into the tubes as if the bit-rates are allocated. It is desirable that when the water is completely filled to the tubes, the obtained rate point is in the SW region with the minimum possible cost.

Supposedly, at some point, we have allocated R_i to X_i and R_j to X_j . To increase one of them by an arbitrarily small amount of bit-rate ΔR , one might introduce a cost increment of $w_i \exp(R_i) \Delta R$ or $w_j \exp(R_j) \Delta R$. The allocation scheme should pick the smaller of them, until the cost increments being the same, i.e., when the following relationship holds:

$$\ln w_i + R_i = \ln w_j + R_j. \tag{10}$$

After that, R_i and R_j should be increased evenly until some tube is about to be overflowed.

So the algorithm shall pre-fill the tubes to $-\ln w_1, \ldots, -\ln w_M$, respectively (without loss of generality, suppose none of the tubes is overflowed at this time). After that we still have $H(X(I_M)) + \sum_{k=1}^{M} \ln w_k$ rate budget in hand. Then we start to fill all the "real" tubes evenly, and we also keep close watch on both the real and the virtual tubes. The filling continues until an overflow is about to happen in a tube Φ_0 (Φ_0 can always be found because finally the upper bound of I_M will be reached). If there are multiple such tubes, we just randomly pick one. Then we state that the sources in Φ_0 can be SWC coded at the curent bit-rates allocated to $X(\Phi_0)$, without considering any other sources in $I_M \setminus \Phi_0$. That is because for any subset Φ of Φ_{0_2} .

$$R(\Phi) = R(\Phi_0) - R(\Phi_0 \setminus \Phi) = H(X(\Phi_0)) - R(\Phi_0 \setminus \Phi), (11)$$

$$\geq H\left(X\left(\Phi_{0}\right)\right) - H\left(X\left(\Phi_{0}\setminus\Phi\right)\right) = H\left[X\left(\Phi\right) \mid X\left(\Phi_{0}\setminus\Phi\right)\right]$$

where the inequality is because none of the subsets of Φ_0 is overflowed (Φ_0 is the first to reach its upper bound).

Now we can separate the *M*-source SWC into two phases: the sources in Φ_0 are encoded and decoded among themselves first, and then used as side information to decode others (others are encoded as if they knew the sources in Φ_0). This essentially reduces the problem to an $(M - ||\Phi_0||)$ -source case, where $||\Phi_0||$ denotes the cardinality of Φ_0 . The algorithm can be executed recursively until finally all the sources are coded. A more rigorous description of the algorithm is presented in the next subsection.

3.3 Proof of the optimality

From the previous subsection we know the RA algorithm based on the water-filling model always ends up with a rate point inside the SW region. However, it is not clear yet if this point is the min-cost point R^* . This issue is addressed in the following. For the sake of brevity, in this subsection, we assume $w_1=w_2=\ldots=w_M=1$.

We first have the following proposition:

Proposition 3: The min-cost point R^* must have each of its component rates $R_k \ge \eta$, where

$$\eta = \min_{\substack{\Phi \subset I_M\\\Phi \neq \phi}} \frac{H(X(\Phi))}{\|\Phi\|}.$$
(12)

where ϕ denotes the empty set.

Proof: If this is not true, without loss of generality, let $R_1 \le ... \le R_N < \eta \le R_{N+1} \le ... \le R_M$, where $1 \le N \le M$.

Now let's consider R_{N+1} . If we can reduce R_{N+1} by an arbitrarily small amount while still keeping the rate point inside

the SW region, then R^* cannot be the min-cost point. So some tube(s) containing X_{N+1} is about to be underflowed. Let Φ_{N+1} be the *intersection* of all those tubes. We have the following observations on Φ_{N+1} :

1) $(N+1) \in \Phi_{N+1}$.

2) Φ_{N+1} is about to be underflowed (Proposition 1).

3) $\Phi_{N+1} \cap I_N = \phi$, where $I_N = \{1, ..., N\}$. Otherwise, for example,

if $1 \in \Phi_{N+1}$, let's consider replacing the pair (R_1, R_{N+1}) with $(R_1 + \Delta R, R_{N+1} - \Delta R)$, where ΔR is an arbitrarily small positive number. This operation still keeps the rate point inside the SW region², but the overall cost is decreased:

 $\exp(R_1) + \exp(R_{N+1}) > \exp(R_1 + \Delta R) + \exp(R_{N+1} - \Delta R)$ (13) according to Jensen's inequality (note that $R_{N+1} > \Delta R$). This contradicts that R^* is the min-cost point. So none of the sources in I_N is in Φ_{N+1} .

In the same way we define Φ_{N+2} , ..., Φ_M . Now we consider the union of them: $\Phi = \Phi_{N+1} \cup ... \cup \Phi_M$. Firstly, $\Phi \cap I_N = \phi$ because none of $\Phi_{N+1}, ..., \Phi_M$ has a non-empty intersection with I_N ; secondly, $\{N+1, ..., M\} \subseteq \Phi$ because $(N+1) \in \Phi_{N+1}$, etc. Combining the two facts we conclude that

$$\Phi = \{N+1, \dots, M\}.$$
 (14)

According to Proposition 1, Φ is about to be underflowed, so its complementary, $\Phi^c = I_N$, is about to be overflowed. Thus

$$N\eta > R(I_N) = H(X(I_N)) \ge ||I_N||\eta = N\eta .$$
⁽¹⁵⁾

where the first inequality is because $R_1 \leq ... \leq R_N < \eta$, the first equality is because I_N (or Φ^c) is about to be overflowed, and the second inequality is from (12). Now we can see the contradiction, by which Proposition 3 is proved.

Now the optimality of our algorithm is stated:

Theorem 1: The RA algorithm based on the water-filling model result in the min-cost rate point R^* .

Proof: This can be proved by induction. If there is only one source, the statement is trivially true. If we assume that it is true for any *m* sources where m < M, then for the *M*-source case, in the water-filling algorithm, the first tube to be overflowed, Φ_0 , achieves the minimum in (12). At this time, $R_k = H(X(\Phi_0))/||\Phi_0|| = \eta$, for any $k \in \Phi_0$. According to Proposition 3, R^* must have each of the component rate $R_k \ge \eta$; on the other hand, according to Corollary 2, $R(\Phi_0) \le H(X(\Phi_0)) = ||\Phi_0||\eta$ is necessary. Consequently, the min-cost rate point R^* must have $R_k = \eta$, for any $k \in \Phi_0$, which is the same as the result in the water-filling algorithm. After that the problem is reduced to an $(M - ||\Phi_0||)$ -source case and is confirmed from the induction hypothesis.

3.4 Summary of the Algorithm and Complexity Analysis

The water-filling algorithm will be performed by the receiver node which is assumed less constrained by power than the source sensor nodes, and the results of the optimal rate allocation will be fed back to each source node for compression. The algorithm is summarized as follow:

1) Let m = M, $\Phi_1 = \phi$.

2) Calculate the $2^m - 1$ entropy values $H[X(\Phi)|X(\Phi_1)]$, where Φ is any non-empty subset of $I_{M}(\Phi_1)$.

² Increasing R_1 does not affect any inequality in (1); and decreasing R_{N+1} may cause an about-to-underflow tube to be actually underflowed, but those tubes all contain R_1 , so the amount of water of each of those tubes is not changed.

- 3) Find Φ with the minimum $\{H[X(\Phi)|X(\Phi_1)] + \sum_{j \in \Phi} \ln w_j\} / ||\Phi||,$ denote it as Φ_0 .
- 4) Set $R_k = \{H[X(\Phi)|X(\Phi_1)] + \sum_{j \in \Phi} \ln w_j\} / ||\Phi_0|| \ln w_k$ to each X_k for all $k \in \Phi_0$.
- 5) Set $m \leftarrow m ||\Phi_0||$, $\Phi_1 \leftarrow \Phi_1 \cup \Phi_0$. If m = 0, end the recursion; otherwise go to 2) and continue.

Theorem 2: The complexity of the water-filling algorithm is $O(2^{M})$.

In each recursion, the complexity is $O(2^m)$. In the worst case the size of *m* is reduced by 1 every time. That gives us an overall complexity of $O(2^M + 2^{M-1} + ... + 2^1) = O(2^M)$.

As a comparison, the optimal rate point can also be found by using the Lagrangian multipliers. Since there are $2^M - 1$ inequality constraints in (1), the optimal point can be found by exhaustively searching over *any* combination of them, which leads to a complexity of $O(2^{2^M})$. Although the search can be refined if the

geometrical description of the SW region is known, it is not likely that the refinement can reduce the complexity to $O(2^{M})$. And to our best knowledge, the geometrical description of a SW region for more than 3 sources has never been discussed in the literature.

SIMULATIONS

In this simulation, three zero-mean jointly-Gaussian sources are generated with the covariance matrix:

$$Cov = \begin{bmatrix} 1 & 0.9 & 0.9 \\ 0.9 & 1 & 0.9 \\ 0.9 & 0.9 & 1 \end{bmatrix}.$$
 (16)

The discrete sources $\{X_1, X_2, X_3\}$ are generated by uniformly quantizing the Gaussian sources using a step-size of 0.1. The test set contains 10⁶ samples. The path weights are assumed to be $w_1 = 1$, $w_2 = \exp(1)$ and $w_3 = \exp(2)$.

We calculate the entropies / conditional entropies of the sources from the differential entropy using the same method as in [8]. Then the 3-D SW region is illustrated in Fig. 1(a). We can see that the minimum sum-of-rate plane $R_1+R_2+R_3 = H(X_1X_2X_3)$ is a hexagon in the SW region. We are particularly interested in the cost performance of the rate-points in the hexagon given Corollary 1. The hexagon is projected to the R_1R_2 plane in Fig. 1(b) (the rectangular region sliced by two oblique lines). The cost

$$C = w_1 \exp(R_1) + w_2 \exp(R_2) + w_3 \exp(H(X_1 X_2 X_3) - R_1 - R_2)$$

(17)

is densely sampled inside the region and the contour lines are drawn. The min-cost rate point is found numerically at $R_1 = 5.27$, $R_2 = 4.27$ (and $R_3 = 3.96$).

On the other hand, if we apply the water-filling algorithm, the tube $(R_1 + R_2)$ is the 1st to be overflowed, then $R_1 = (H(X_1X_2) + \ln w_1 + \ln w_2)/2 - \ln w_1 = 5.27$, $R_2 = (H(X_1X_2) + \ln w_1 + \ln w_2)/2 - \ln w_2 = 4.27$ are found; the only remaining source X_3 is coded at the rate $R_3 = H(X_3|X_1X_2) = 3.96$. This result matches the numerical result and supports the optimality of our algorithm.

At the optimal rate point, the cost is 7.76×10^2 . As a comparison, the mean cost inside the hexagon is 1.08×10^3 . This means, instead of randomly picking a point in the SW region with the minimum sum-of-rate constraint, working at the optimal rate point achieves roughly 30% saving in power consumption on average.



Fig. 1: (a) The SW rate region of 3 sources. The white hexagon is the minimum sum-of-rate plane; and (b) the hexagon is projected to the R_1R_2 plane, with the cost contours illustrated. The min-cost rate-point is marked with a circle.

CONCLUSION

In the SWC of multiple correlated sources, careful rate allocation among the sources can achieve the minimum overall power consumption in the communication over a WSN. If we model the cost metric as the weighted sum of exp(*rate*) of the multiple sources, the optimal rate point can be found using a recursive water-filling algorithm, which results in significant reduction in computational complexity in comparison to the conventional approach using the Lagrangian multipliers. Future work will include jointly considering the quantizer design and the rateallocation to achieve the best cost-distortion tradeoff.

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