ITERATIVE JOINT DECODING AND SPARSE CHANNEL ESTIMATION FOR SINGLE-CARRIER MODULATION

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ABSTRACT

Single carrier (SC) modulation over a sparse channel motivated by underwater acoustic communications (UAC) is considered. A BER analysis for deterministic channels is first presented that identifies conditions under which SC with zero forcing detection outperforms OFDM. The analysis also motivates the use of MMSE detection in Rayleigh fading channels. An iterative receiver is then developed combining LDPC decoding, channel estimation under a numerosity constraint and CFO tracking. It is shown that MMSE Turbo equalization arises naturally from the marginalization in the Factor Graph when the decoder extrinsic densities are approximated as Gaussian. A frequency-domain MMSE turbo equalizer is then combined with Matching Pursuits (MP) channel estimation. Simulation results for a realistic UAC channel are given showing consistent improvement using iterative processing.

***Index Terms*—** Decision feedback equalizers, iterative methods, decoding, parameter estimation.

1. INTRODUCTION

The use of single-carrier frequency-domain equalization (SC-FDE) [1] is considered here for the UAC application, where the sparse channel makes time-domain equalization impractical. SC-FDE is advantageous for UAC due to the reduced peak-to-average power ratio (PAPR) compared with OFDM, which can reduce the cost/size of amplifiers and transducers. Results in the literature show that SC-FDE performance is comparable to that of OFDM when combined with coding [1]. However, sparse channel and CFO estimation are challenging problems in single-carrier UAC due to the long multipath spreads.

Reduced transmit power for a target BER is obtained by combining SC-FDE detection with LDPC codes. This appoach requires joint iterative decoding and estimation of the sparse channel/CFO. As suggested by the Factor Graph approach [2], the MP algorithm for sparse channel estimation [3] is combined with MMSE turbo equalization [4] and belief propagation decoding. It is shown that the MMSE turbo equalizer arises directly from the Factor Graph marginalization when the decoder extrinsic densities are modelled as Gaussian.

The remainder of the paper is as follows. Signal and channel models, along with the deterministic channel BER analysis are given in Section 2. The relationship of MMSE turbo equalization to the Factor Graph is developed in Section 3. The overall iterative receiver with channel/CFO estimation is presented in Section 4 and results and conclusions follow in Section 5.

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2. SIGNAL MODELS AND BER FOR SC-FDE

The SC system proposed here is based on a generic underwater acoustic channel with Doppler spread < 1 Hz and temporal multipath spread of < 25 msec. Thus, a cyclic prefix of $T_p = 25$ msec. length and packet length of $T_d = 105$ msec. are used. The QPSK symbol rate is $1/T_{sym} = 9600$ sps and a $(1008, 504)$ Gallager LDPC code [5] is employed. For realism in simulations, a detailed signal model incorporating pulse shaping, Doppler shift/spread and multipath spread is considered. The transmit waveform during one packet is thus

$$s(t) = \sum_{k=-N_p}^{N_d-1} c_k g(t - kT_s), \tag{1}$$

where $g(t)$ is a raised-cosine pulse with energy $2E_b$ and bandwidth $(1+\alpha)/(2T_{sym})$. The sequence $\{c_k\}$ includes pilot and code symbols, and corresponds to the vector $\mathbf{c} = [c_{N_d-1}c_{N_d-2}\dots c_0]^T$. QPSK modulation is assumed, so that $c_k = c_{k,1} + jc_{k,2}$, where $c_{k,i} \in \{-1, +1\}$. Each bit $c_{k,i}$ is a pilot or output of an LDPC encoder.

The channel impulse response including the effect of Doppler shift is

$$h(t, \tau) = \sum_{l=1}^{N_c} \alpha_l(t) e^{i2\pi f_c \beta(t) t} \delta(\tau - \tau_l(t) + \beta(t)t). \tag{2}$$

The Doppler compression factor is $\beta(t) = v(t)/c$, where $v(t)$ is the instantaneous platform velocity and c is the sound speed. Long-range transmission (≥ 1 km) is assumed so that the arrival angles are approximately equal, resulting in a common Doppler compression on each multipath. In designing a robust SC-FDE receiver, it is assumed that the $\alpha_l(t) \in \mathbb{C}$ and Doppler $\beta(t)$ are constant only during the symbol interval $T_d + T_p$, and are independent between intervals. The delays $\tau_l(t)$ are relatively constant for stationary platforms, and can be predicted by ray-tracing. The received signal during one packet is

$$r(t) = \sum_{k=-N_p}^{N_d-1} \sum_{l=1}^{N_c} \alpha_l c_k e^{i2\pi f_c \beta t} g((1+\beta)t - kT_s - \tau_l) + n(t), \tag{3}$$

for $t \in [nT_d - T_p, (n+1)T_d)$. Ocean ambient and preamplifier noise are represented by $n(t)$. This noise is modelled as circular Gaussian with spectral density $2N_0$. The focus here is on stationary platforms, where the Doppler compression β is sufficiently small that its effect on the pulse waveform $g(t)$ can be neglected for purposes of algorithm development. However, the pulse time-compression in (3) due to β is accurately modelled in the simulations in Section 5. Residual Doppler due to waves and buoy/anchor motion can generate significant carrier frequency offsets βf_c which cannot be neglected in the receiver design.

The signal $r(t)$ is low-pass filtered to the Nyquist bandwidth. The narrowband Doppler assumption, in which $\beta << 1$ results in a Nyquist sampled signal $r(q) = r(qT_s)$ which from (3) is

$$\begin{aligned} r(q) &= \sum_{k=-N_p}^{N_d-1} f_{q-k} e^{i2\pi f_c \beta q T_s} c_k + n(q), \\ f_q &= \sum_{l=1}^{N_c} \alpha_l g(qT_s - \tau_l), q = 0, 1, \ldots, N_f - 1. \end{aligned} \tag{4}$$

The effective channel length is then approximately $N_f = \max(\tau_l)/T_s + T_g/T_s$ where T_g is the duration of pulse $g(t)$. Now form the vector $\mathbf{r} = [r(N_d-1) r(N_d-2) \ldots r(0)]^T$ with the cyclic prefix component truncated. Due to the cyclic prefix, with $c_{-l} = c_{N_d - l}$ for $l = 1, \ldots N_p$, the effective channel matrix $\mathbf{F} \in \mathbb{C}^{N_d \times N_d}$ is formed by N_d circular shifts of the vector $\mathbf{f} = [f_0, f_1, \ldots, f_{N_f-1}]$. The vector received signal is

$$\begin{aligned} &\mathbf{r} = \mathbf{V}(\beta)\mathbf{F}\mathbf{c} + \mathbf{n}, \\ &\mathbf{V}(\beta) = \text{diag}\{e^{i2\pi\beta f_c (N_d-1)T_s}, e^{i2\pi\beta f_c (N_d-2)T_s}, \ldots 1\}^T. \end{aligned} \tag{5}$$

The noise $\mathbf{n}$ has covariance matrix $\sigma_n^2 \mathbf{I}$ where $\sigma_n^2 = 2N_0/T_s$.

Zero-forcing equalization for SC-FDE is next reviewed to obtain a simple comparison of uncoded SC-FDE and OFDM BERs for classes of deterministic channels. Assume perfect channel state information (CSI), with $\mathbf{f}$ known a-priori. Let $\mathbf{W} \in \mathbb{C}^{N_d \times N_d}$ be the FFT matrix. Then $\mathbf{W}^H \mathbf{F} \mathbf{W} = \mathbf{H}$, where the diagonal frequency response matrix elements are

$$H_{ii} = \sum_{l=0}^{N_f-1} f_l \exp(-j2\pi l(i-1)/N_d). \tag{6}$$

Thus, ZF demodulation with perfect CSI and zero Doppler corresponds to computing

$$\hat{\mathbf{c}} = \mathbf{W}\mathbf{H}^{-1}\mathbf{W}^H \mathbf{r} = \mathbf{c} + \mathbf{n}'. \tag{7}$$

Note that the noise $\mathbf{n}'$ has covariance matrix $\frac{2N_0}{T_s}\mathbf{W}^H \mathbf{H}^{-1}(\mathbf{H}^*)^{-1}\mathbf{W}$.

The uncoded BER for SC-FDE with QPSK modulation and deterministic channels is readily shown from (7) to equal.

$$P_b = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0 \frac{1}{N_d}\sum_{i=0}^{N_d-1}\frac{1}{|H'_{ii}|^2}}}\right). \tag{8}$$

Note the rescaling $H_{ii} = \sqrt{\frac{2E_b}{T_s}} H'_{ii}$, where $\sqrt{\frac{2E_b}{T_s}} = g(0)$ is the amplitude of the raised-cosine pulse with energy $2E_b$. For an all pass unit-energy channel, this corresponds to setting the channel gain $E\{|f_0|^2\}$ to unity.

For OFDM, the received signal is identical to (5), except that the code vector $\mathbf{c}$ is replaced by its IFFT $\mathbf{W}^H\mathbf{c}$. The average BER over N_d subcarriers is then

$$P_b = \frac{1}{N_d}\sum_{i=0}^{N_d-1}\frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}|H'_{ii}|^2}\right). \tag{9}$$

The following Proposition compares SC-FDE and OFDM for zero-forcing detection.

Proposition 1 *Consider case (a) where $|H'_{ii}|^2 < 3/(2\gamma_b)$ for $i = 0, \ldots, N_d - 1$, where $\gamma_b = E_b/N_0$. In case (a), the zero-forcing SC-FDE BER is lower-bounded by the OFDM BER. Now for case (b), let $|H'_{ii}|^2 > 3/(2\gamma_b)$ for all i. Then the OFDM BER is lower bounded by the ZF SC-FDE result. The OFDM and SC-FDE system performance is equivalent when the channel is allpass.*

Proof: Define $\alpha_i = \frac{1}{|H'_{ii}|^2}$. For case (a), we can show that erfc$(\sqrt{\gamma_b/\alpha_i})$ is concave in α_i in the region $\{\alpha_i\} \in [2\gamma_b/3, \infty)^{N_d}$. Then using Jensen's inequality

$$\frac{1}{N_d}\sum_{i=0}^{N_d-1}\frac{1}{2}\text{erfc}\left(\sqrt{\frac{\gamma_b}{\alpha_i}}\right) < \frac{1}{2}\text{erfc}\left(\sqrt{\frac{\gamma_b}{\frac{1}{N_d}\sum_{i=0}^{N_d-1}\alpha_i}}\right). \tag{10}$$

But the expression on the r.h.s. of (10) is just the SC-FDE error rate. Equality in (10) is attained when $\alpha_i = c$ for all i, that is, for an all-pass channel. For case (b), we have $\alpha_i < 2\gamma_b/3$ for all i, and erfc$(\sqrt{\gamma_b/\alpha_i})$ is a convex function. Thus the inequality is reversed in (10), and for case (b) SC-FDE BER is upper bounded by the OFDM error rate.

Proposition 1 implies that uncoded SC-FDE with ZF is superior to OFDM as long as all channel gains are lower bounded by $|H'_{ii}|^2 > 3/(2\gamma_b)$. Thus, for strongly Rician channels we would expect SC-FDE-ZF performance to be comparable to OFDM. However, the performance of SC-FDE with zero-forcing can be especially poor in Rayleigh fading. Assume that H'_{ii} are i.i.d. circular Gaussian with unit variance for $i = 0, \ldots, N_d - 1$. This is the case when the channel coefficients f_i are likewise circular Gaussian i.i.d. Then as $N_d \to \infty$, the BER in (8) tends to $1/2$. This is due to the infinite variance of the inverse of an exponential r.v. of the form $1/|H'_{ii}|^2$, which when combined with the strong law of large numbers, shows that $\sum_{i=0}^{N_d-1}|H'_{ii}|^2/N_d \to \infty$ a.s. Thus, MMSE detection of SC-FDE, combined with coding is required for adequate performance over Rayleigh-fading channels.

3. RELATIONSHIP OF MMSE TURBO EQUALIZATION AND THE FACTOR GRAPH FOR SC-FDE

An overall iterative receiver is sought combining LDPC decoding, channel and CFO estimation. The Factor Graph method in [2] has been used to justify such iterative structures. Furthermore, MMSE detection/cancellation in a Turbo equalizer has also been shown to correspond to minimization of Kullback-Leibler distance between the true APP and a constrained factorizable APP [6]. Here, it is shown that Turbo equalization corresponds directly to the Factor Graph marginalization when the decoder extrinsics are viewed as Gaussian.

In the FG approach, variable nodes $c_{k,i}$ represent coded bits, code constraints are represented by the LDPC parity check matrices and the likelihood of the received vector $\mathbf{r}$ in (5) is represented by a separate function block. The key problem in the FG is approximating the density-to-variable messages $\mu_{p \to c_{k,i}}$, defined by the marginalization

$$\mu_{p\to c_k} = \sum_{c_l=\pm 1 \pm i 1: l\neq k} \int \mathcal{CN}(\mathbf{r}; \mathbf{Fc}, \sigma_n^2 \mathbf{I}) \prod_{l\neq k} q_l(c_l) d\mathbf{F}. \quad (11)$$

Note that CFO uncertainty has been eliminated from (11) for simplicity. In (11), the QPSK symbol probabilities $q_l(c_l)$ are the LDPC decoder extrinsics.

First, consider marginalization only over the coded QPSK symbols c_l. The c_l are approximated as independent Gaussian with means $\overline{c}_l$ and diagonal covariance matrix $\mathbf{Q}$. The bits $\overline{c}_{l,j}$ are given by the decoder soft bits $\tanh(\lambda^e(c_{l,j})/2)$, where $\lambda^e()$ are the extrinsic likelihoods. Further define $\mathbf{F}_{(k)}$ as the channel matrix formed by deleting column k, and $\mathbf{c}_{(k)}$ as the QPSK symbols excluding c_k. The k-th column of $\mathbf{F}$ is denoted $\mathbf{f}_k$. Then the problem is to compute the message

$$\mu_{p\to c_k} = \int \mathcal{CN}(\mathbf{r}; \mathbf{F}_{(k)}\mathbf{c}_{(k)} + \mathbf{f}_k c_k, \sigma_n^2 \mathbf{I}) \mathcal{CN}(\mathbf{c}_{(k)}; \overline{\mathbf{c}}_{(k)}, \mathbf{Q}) d\mathbf{c}_{(k)}. \quad (12)$$

The MMSE equalizer solution is given by the following proposition.

Proposition 2 *The density function to code variable message for SC modulation, when the decoder extrinsics are approximated as independent Gaussian, is given by*

$$\begin{aligned} &\mu_{p\to c_k} \propto \exp\left(-|c_k - \hat{c}_k|^2/p_k\right) \\ &\hat{c}_k = \frac{1}{\mathbf{f}_k^H \Sigma^{-1} \mathbf{f}_k} \mathbf{f}_k^H \Sigma^{-1} \left(\mathbf{r} - \mathbf{F}_{(k)} \overline{\mathbf{c}}_{(k)}\right) \\ &\Sigma = \mathbf{f}_k \mathbf{Q} \mathbf{f}_k^H + \sigma_n^2 \mathbf{I},\ p_k = (\mathbf{f}_k^H \Sigma^{-1} \mathbf{f}_k)^{-1}. \end{aligned} \quad (13)$$

That is, the message is a Gaussian density with mean and covariance given by a MMSE Turbo equalizer.

Outline of Proof: The integral (12) has a closed form solution identical to that for the Kalman filter innovations likelihood. The result is

$$\mu_{p\to c_k} = \mathcal{CN}(\mathbf{r} - \mathbf{F}_{(k)}\overline{\mathbf{c}}_{(k)}; \mathbf{f}_k c_k, \mathbf{F}_{(k)} \mathbf{Q} \mathbf{F}_{(k)}^H + \sigma^2 \mathbf{I}). \quad (14)$$

The density (14) is then readily manipulated to yield the density (13) in terms of c_k.

The additional marginalization over $\mathbf{F}$ cannot be computed in closed form in general. A practical strategy is to approximate the integral using the value of $\mathbf{F}$ corresponding to the maximum-likelihood solution, conditioned on soft bits $\overline{\mathbf{c}}$ computed using the decoder total APPs. However, the channel $\mathbf{f}$ is typically sparse, hence the Matching Pursuits algorithm is used in the next section to obtained a constrained numerosity solution.

4. ITERATIVE DECODING AND CHANNEL ESTIMATION FOR SC-FDE

Unfortunately, the MMSE detector in Proposition 2 does not admit a simple frequency-domain form, in which equalization corresponds to multiplication by a diagonal matrix. To derive a practical iterative receiver, an alternative frequency-domain detector is developed as follows. First, the CFO-corrected IFFT of the received signal (5) is computed yielding

$$\mathbf{r}' = \mathbf{W}^H \mathbf{V}(\hat{\beta})^H \mathbf{r} \approx \mathbf{H}\mathbf{W}^H \mathbf{c} + \mathbf{n}', \quad (15)$$

where $\mathbf{n}'$ still has covariance matrix $\sigma_n^2 \mathbf{I}$. The MMSE detector $\mathbf{M} \in \mathbb{C}^{N_d \times N_d}$ minimizing $||\mathbf{c} - \mathbf{M}\mathbf{r}'||^2$ assuming $E\{\mathbf{c}\mathbf{c}^H\} = 2\mathbf{I}$ is

$$\mathbf{M} = \mathbf{W}\mathbf{H}^H \left[\mathbf{H}\mathbf{H}^H + \frac{N_0}{T_s}\mathbf{I}\right]^{-1}. \quad (16)$$

Now define diagonal matrices $\mathbf{D}, \mathbf{D}'$ with i-th components

$$D_{ii} = \frac{H_{ii}^*}{|H_{ii}|^2 + \frac{N_0}{T_s}},\ D'_{ii} = D_{ii} H_{ii}. \quad (17)$$

The overall MMSE detector is then $\hat{\mathbf{c}} = \mathbf{W}\mathbf{D}\mathbf{W}^H \mathbf{V}(\hat{\beta})^H \mathbf{r}$, which implies that equalization is still in the frequency domain, and only requires multiplication of the IFFT output by a diagonal matrix.

An iterative receiver based on the frequency-domain MMSE equalizer is now developed, based on similar structures for OFDM and SC modulation [6]. The input to the decoder will be the set of log-likelihoods $\lambda(y'_k|c_{k,i})$, where y'_k is derived below and is the MMSE detector output $\hat{c}_k$ with ISI subtracted via soft bits $\hat{c}_{k,i} = \tanh(\lambda^e(c_{k,i})/2)$. The decoding algorithm used here to obtain $\lambda^e()$ is the sum-product algorithm based on the $\tanh$ rule [7].

The likelihood $\lambda(y'_k|c_{k,i}(n))$ is found by rewriting the MMSE frequency-domain detector output as

$$\begin{aligned} y_k &= (\mathbf{W}\hat{\mathbf{D}}\mathbf{W}^H \mathbf{V}(\hat{\beta})^H \mathbf{r})_k \\ &\approx \frac{1}{N_d} \sum_{i=0}^{N_d-1} \frac{|\hat{H}_{ii}|^2}{|\hat{H}_{ii}|^2 + \frac{N_0}{T_s}} c_k + \sum_{j\neq k} (\mathbf{W}\mathbf{D}'\mathbf{W}^H)_{kj} (\hat{c}_j + \tilde{c}_j) + n'_k. \end{aligned} \quad (18)$$

The channel estimates $\hat{H}_{ii}$ are computed via the MP algorithm as discused below. The soft symbol $\hat{c}_j$ is again computed using the decoder extrinsic $\lambda^e(c_{k,i})$, and the soft symbol error $\tilde{c}_j$ is zero mean with covariance

$$E\{\tilde{c}_j^2\} = \sigma_{c_j}^2 = (1 - \hat{c}_{j,1}^2) + (1 - \hat{c}_{j,2}^2). \quad (19)$$

Then define the ISI cancelled signal

$$y'_k = y_k - \sum_{j\neq k} (\mathbf{W}\hat{\mathbf{D}}'\mathbf{W}^H)_{kj} \hat{c}_j, \quad (20)$$

where $\hat{\mathbf{D}}'$ is given by (17), but using the estimates $\hat{H}_{ii}$. The likelihood is

$$p(y'_k|c_k) = \mathcal{CN}\left(y'_k; \frac{1}{N_d} \sum_{i=0}^{N_d-1} \frac{|\hat{H}_{ii}|^2}{|\hat{H}_{ii}|^2 + \frac{N_0}{T_s}} c_k, \sigma_k^2\right), \quad (21)$$

where

$$\sigma_k^2 = \sum_{j\neq k} |(\mathbf{W}\mathbf{D}'\mathbf{W}^H)_{k,j}|^2 \sigma_{c_j}^2 + \frac{2N_0}{T_s} \frac{1}{N_d} \sum_{i=0}^{N_d-1} \frac{|\hat{H}_{ii}|^2}{(|\hat{H}_{ii}|^2 + N_0/T_s)^2}. \quad (22)$$

The MMSE detector LLR used by the LDPC decoder for bit $c_{k,1}$ for example is

$$\begin{aligned} &\lambda^m(c_{k,1}) = \lambda(y'_k|c_{k,1}) = 4\alpha_H \mathrm{Re}\{y'_k\}/\sigma_k^2 \\ &\alpha_H = \sum_{i=0}^{N_d-1} \frac{|\hat{H}_{ii}|^2}{|\hat{H}_{ii}|^2 + N_0/T_s}. \end{aligned} \quad (23)$$

The Doppler shift β and channel $\mathbf{f}$ are estimated on a symbol-by-symbol basis. In order to decouple the problems, the conventional least-squares channel estimate is first used to estimate β followed by Matching Pursuits (MP) channel estimation. First, consider Doppler estimation and approximate $\mathbf{r}$ in terms of the pilot and soft symbols as

$$\mathbf{r} \approx \mathbf{V}(\beta)\hat{\mathbf{C}}\mathbf{f} + \mathbf{n}, \tag{24}$$

where $\mathbf{C}$ is defined by N_d truncated circular shifts of its first row $[\hat{c}_{N_d-1}, \hat{c}_{N_d-2}, \ldots, \hat{c}_{N_d-N_f}]$. The estimated symbols or pilots are

$$\begin{aligned} \hat{c}_k &= p_{k,1} + jp_{k,2} \text{ (Pilot)} \\ \hat{c}_k &= \tanh(\lambda^d(c_{k,1})/2) + j\tanh(\lambda^d(c_{k,2})/2) \text{ (Data)}, \end{aligned} \tag{25}$$

where $\lambda^d(c_{k,i})$ is the LLR computed using the a-posteriori probabilities (APPs) from the LDPC decoder. Note that the total APPs rather than extrinsics from the decoder are used in the MP algorithm, as this leads to better quality channel estimates, and direct feedback from the estimator back to the decoder is not an issue. Substitution of the least-squares channel estimate $\hat{\mathbf{f}}_{LS}$ into the cost function $||\mathbf{r} - \mathbf{V}(\beta)\mathbf{C}\mathbf{f}||^2$ yields the Doppler estimate

$$\hat{\beta} = \arg\max_{\beta} \mathbf{r}^H \mathbf{V}(\beta)\hat{\mathbf{C}}(\hat{\mathbf{C}}^H\hat{\mathbf{C}})^{-1}\hat{\mathbf{C}}^H\mathbf{V}(\beta)^H\mathbf{r}. \tag{26}$$

The MP algorithm follows [3]. First compute the sufficient statistics $\mathbf{v}^1 = \hat{\mathbf{C}}^H\mathbf{V}(\hat{\beta})^H\mathbf{r}$ and $\mathbf{A} = \hat{\mathbf{C}}^H\hat{\mathbf{C}}$. The channel is assumed sparse with $N << N_f$ significant coefficients. Then at stage k of MP, the single-coefficient channel which results in the smallest squared error is chosen correpsonding to the steps. The FFT yields the frequency-domain channel estimates $\hat{H}_{k,k}$.

$$\begin{aligned} \mathbf{v}^k &= \mathbf{v}^{k-1} - \mathbf{A}_{p_{k-1}}\hat{f}_{p_{k-1}} \\ p_k &= \arg\max_{l \neq \{p_1,\ldots,p_{k-1}\}} |\mathbf{v}_l^k|^2/A_{p_l,p_l} \\ \hat{f}_{p_k} &= \mathbf{v}_{p_k}^1/A_{p_k,p_k}. \end{aligned} \tag{27}$$

The iterative receiver is summarized as follows. 1) Compute Doppler and MP channel estimates using decoder log-APPs $\lambda^d(c_{k,i})$. 2) Update MMSE Turbo detector likelihoods $\lambda^m(c_{k,i})$ in (23). 3) Run LDPC decoder and update decoder extrinsics $\lambda^e(c_{k,i})$ and decoder log-APPs $\lambda^d(c_{k,i})$.

5. RESULTS AND CONCLUSIONS

The iterative SC-FDE receiver was simulated for a 3-ray multipath channel with delays $\tau_l = [0, 17.2, 22.2]$ msec. The coefficients $\alpha_l(t)$ in (2) were generated via AR-1 processes with Rayleigh fading. Fig. 1 illustrates the BER for uncoded MMSE and iterative LDPC decoding for a Doppler spread of .05 Hz and a radial Doppler velocity of .05 m/s. A significant BER gain is obtained for the coded system after 4 outer iterations. In Fig. 2, the Doppler spread was raised to .1 Hz with a radial velocity of .25 m/s. At a BER of 10^{-3}, it is seen that an SNR of 9 dB is required for the higher Doppler spread/velocity scenario in Fig 2. Both the pulse time-compression and time-variation of the $\mathbf{f}$ vector within a symbol would explain this degradation.

To conclude, an iterative SC-FDE receiver was developed for sparse multipath channels. The MMSE Turbo equalizer structure was justified by an approximation marginalization of the Factor Graph messages. The Matching Pursuits channel estimator embedded in the iterative receiver appears to be robust at higher Doppler spreads.

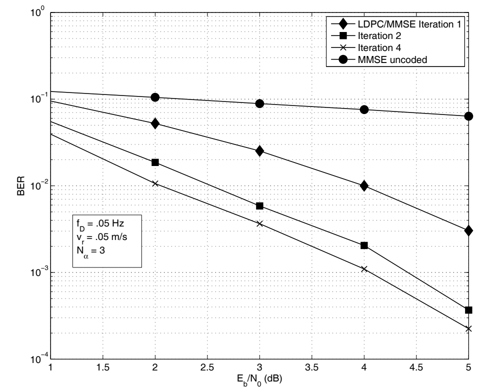

Fig. 1. BER for .05 Hz Doppler spread, .05 m/s radial velocity.

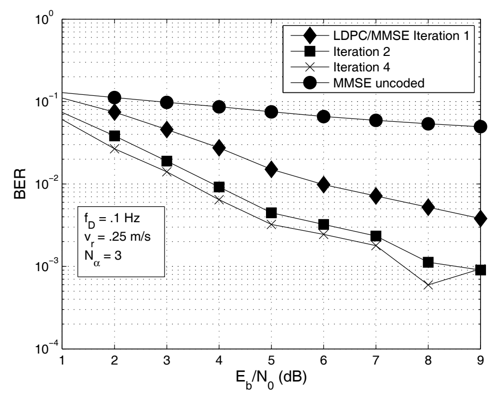

Fig. 2. BER for .1 Hz Doppler spread, .25 m/s radial velocity.

6. REFERENCES

[1] N. Benvenuto and S. Tomasin, "On the comparison between OFDM and single carrier modulation with a DFE using a frequency domain feedforward filter," *IEEE Transactions on Communications*, vol. 50, pp. 947–955, June 2002.

[2] C. Herzet, V. Ramon, and L. Vandendorpe, "A theoretical framework for iterative synchronization based on the sum-product and the expectation-maximization algorithms," *IEEE Transactions on Signal Processing*, vol. 55, pp. 1644–1658, May 2007.

[3] S. Kim and R. A. Iltis, "A Matching Pursuit/GSIC-based algorithm for DS-CDMA sparse channel estimation," *IEEE Signal Processing Letters*, vol. 11, pp. 12–15, Jan. 2004.

[4] M. Tüchler, R. Koetter, and A. C. Singer, "Turbo equalization: Principles and new results," *IEEE Transactions on Communications*, vol. 50, pp. 754–767, May 2002.

[5] D. MacKay, "Encyclopedia of sparse graph codes," http://www.inference.phy.cam.ac.uk/mackay/codes/data.html.

[6] M. Nissilä and S. Pasupathy, "Soft-input soft-ouptut equalizers for turbo receivers: A statistical physics perspective," *IEEE Transactions on Communications*, vol. 55, pp. 1300–1307, July 2007.

[7] J. Chen, A. Dholakia, E. Eleftheriou, M. Fossorier, and X. Hu, "Reduced-complexity decoding of LDPC codes," *IEEE Transactions on Communications*, vol. 53, pp. 1288–1299, Aug. 2005.