ROBUST MULTIUSER DETECTION BASED ON PROBABILITY CONSTRAINED OPTIMIZATION OF THE MMSE RECEIVER

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ABSTRACT

The performance of multiuser detection (MUD) algorithms for code-division multiple-access (CDMA) systems depends on the accuracy of channel estimates. Such estimates are typically affected by errors, which can lead to significant degradation of the performance. In this paper, we develop a MUD technique which is based on probability constrained optimization of the minimum mean-square error (MMSE) multiuser receiver, and is robust against channel estimation errors. Its relationship to the recently proposed robust worst case optimization based MUD technique is established. The uncertainty parameter of the worst case based design is quantified in terms of the outage probability used in the probability constrained design and second-order statistics of the channel estimation errors. Simulation results demonstrate the potential of the proposed technique to outperform the existing robust techniques.

***Index Terms*—** Code division multiaccess, robustness, signal detection, multiuser channels, optimization methods.

1. INTRODUCTION

Code-division multiple-access (CDMA) schemes have been a focus of extensive research [1]. A large number of multiuser detection (MUD) algorithms have been proposed [1]. The optimal maximum likelihood (ML) detector [2] is prohibitively expensive for practical applications and, hence, suboptimal linear MUD techniques have gained much interest as computationally attractive alternatives to the ML detector [3], [4]. Traditional MUD techniques assume that the channel and, therefore, the signature of the desired user are known precisely [3], [4]. In practice, the channel estimates are obtained either by using training sequences [4] or blindly [5]. Such estimates are typically affected by errors that are ignored in subsequent detection. When the exact channel is unavailable and its erroneous estimate is used, the performance of linear MUD algorithms may degrade severely [6], [7]. Therefore, CDMA MUD methods robust against user signature estimation errors are desired. Such robust MUD procedure has been developed in [7] using the worst case optimization based approach of [8]. In the worst case based approach, the performance of the MUD is optimized for the least favorable user signature estimation error. The least favorable user signature estimation error is introduced by means of bounding the norm of the user signature error vector using an uncertainty parameter that is chosen in ad hoc manner. However, if the training mode is used, the user signature estimation error is known to be Gaussian distributed [9], and, therefore, its norm is χ^2 distributed and not bounded. It has been proposed in [9] to bound the signature estimation error in probability. However, the robust MUD problem formulations in [7] and [9] are essentially the same. Indeed, in both formulations the user signature error vector is assumed to be norm bounded, and the MUD performance is optimized for the least favorable user signature error.

In this paper, we develop a rigorous approach to MUD problem that provides the robustness against desired user signature estimation errors with a certain selected probability. Thus, only the desired user signature estimation errors which occur with sufficiently high probability are considered, while the errors whose probability is low are discarded. The problem which we consider is different from the problems considered in [7] and [9], where the mean-square error (MSE) is minimized for all user signature errors from the known uncertainty region. However, we show that the probability constrained robust MUD is related to the worst case based MUD of [7]. The established relationship leads to a straightforward interpretation of the worst case design parameter in terms of the MUD outage probability and second-order statistics of the channel estimation errors.

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2. BACKGROUND

We consider a K-user synchronous CDMA system.[1] Assuming that there is no inter-symbol interference (ISI), the received discrete-time baseband signal can be modeled as [1]

$$\boldsymbol{x}(n) = \sum_{k=1}^{K} A_k b_k(n) \boldsymbol{s}_k + \boldsymbol{v}(n) \tag{1}$$

where n is the sample index, $\boldsymbol{x}(n) = [x(nT_s), x(nT_s + T_c), \ldots, x(nT_s + (L-1)T_c)]^T$ is the received data vector, T_s is the symbol period, L is the spreading factor, $T_c = T_s/L$ is the chip period, A_k and $b_k(n)$ are the received signal amplitude and the nth data symbol of the kth user, respectively, $\boldsymbol{v}(n) = [v(nT_s), v(nT_s + T_c), \ldots, v(nT_s + (L-1)T_c)]^T$ is the zero-mean additive random uncorrelated Gaussian noise vector with variance σ^2, $\boldsymbol{s}_k = [s_k(0), s_k(T_c), \ldots, s_k((L-1)T_c)]^T$ is the signature vector of the kth user with its normalized signature waveform given as

$$\boldsymbol{s}_k(m) = \sum_{l=0}^{L-1} c_k(l) g_k(m - lT_c) \tag{2}$$

$\boldsymbol{c}_k = [c_k(0), c_k(1), \ldots, c_k(L-1)]^T$ is the spreading code vector of the kth user, $g_k(m)$ is the chip waveform convolved with the kth user channel impulse response, and $(\cdot)^T$ stands for the transpose.

[1] Note that the extension to asynchronous system is straightforward.

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We use here the standard assumptions that the chip sequence period is the same as the symbol period; for each user, the data symbols are zero-mean independent random variables that are drawn from the binary phase shift keying (BPSK) constellation; the channel is a finite impulse response (FIR) filter whose impulse response is much shorter than T_s; the channels are quasistatic; and the data symbols of different users are uncorrelated with each other and with the noise.

The output of a linear multiuser receiver is given by [3]

$$y(n) = \boldsymbol{f}^H \boldsymbol{x}(n) \tag{3}$$

where $\boldsymbol{f} = [f_0, f_1, \ldots, f_{L-1}]^T$ is an $L \times 1$ complex vector of the receiver coefficients, and $(\cdot)^H$ stands for the Hermitian transpose. The received output $y(n)$ is used for symbol detection.

A popular approach to optimize the MUD vector of the receive coefficients $\boldsymbol{f}$ is to minimize the output energy at the receiver, or equivalently, minimize the MSE. Mathematically, assuming that the first user is the desired one, the problem can be formulated as

$$\begin{aligned} &\min_{\boldsymbol{f}} E\{|b_1(n) - \boldsymbol{f}^H \boldsymbol{x}(n)|^2\} \\ &= \min_{\boldsymbol{f}} \{1 + \boldsymbol{f}^H \boldsymbol{R} \boldsymbol{f} - \boldsymbol{d}^H \boldsymbol{f} - \boldsymbol{f}^H \boldsymbol{d}\} \end{aligned} \tag{4}$$

where $\boldsymbol{R} = E\{\boldsymbol{x}(n)\boldsymbol{x}^H(n)\}$ is the data covariance matrix, $\boldsymbol{d} = E\{\boldsymbol{x}(n) b_1^*(n)\}$ is the correlation vector between the received data vector $\boldsymbol{x}(n)$ and the desired user symbol $b_1(n) \in \{-1, 1\}$, and $E\{\cdot\}$ and $(\cdot)^*$ denote the statistical expectation and complex conjugate, respectively.

The solution to the optimization problem (4) is refereed to as the MMSE receiver and can be written as

$$\boldsymbol{f}_{\rm opt} = \boldsymbol{R}^{-1} \boldsymbol{d}. \tag{5}$$

In practice, the exact data covariance matrix is unavailable. Therefore, its sample estimate

$$\hat{\boldsymbol{R}} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{x}(n) \boldsymbol{x}^H(n) \tag{6}$$

is typically used. To provide robustness against finite sample size effect, it has been proposed to use the so-called diagonal loading technique [10], i.e., to replace $\hat{\boldsymbol{R}}$ by $\hat{\boldsymbol{R}} + \gamma \boldsymbol{I}$, where γ is the constant loading factor, and $\boldsymbol{I}$ denotes the identity matrix. It is also easy to check that the correlation vector $\boldsymbol{d}$ can be equivalently expressed as $\boldsymbol{d} = \beta \boldsymbol{s}_1$, where $\beta \triangleq A_1 E\{|b_1(n)|^2\}$. Therefore, the practically applicable diagonal loading based multiuser receiver can be written as

$$\boldsymbol{f}_{\rm dl} = (\hat{\boldsymbol{R}} + \gamma \boldsymbol{I})^{-1} \boldsymbol{s}_1. \tag{7}$$

An essential shortcoming of the MMSE receiver (5) as well as the receiver (7) is that they are not robust against the desired user signature $\boldsymbol{s}_1$ estimation errors. In [7], [8], it has been proposed to explicitly model the actual user signature $\tilde{\boldsymbol{s}}_1$ as

$$\tilde{\boldsymbol{s}}_1 = \boldsymbol{s}_1 + \boldsymbol{e} \neq \boldsymbol{s}_1 \tag{8}$$

where $\boldsymbol{e}$ denotes an unknown complex vector of the desired user signature estimation error.

A popular approach to robust designs is based on the idea of worst case performance optimization [8]. In application to multiuser detection problem, this approach aims at minimizing the MSE assuming that $\boldsymbol{e}$ is an unknown deterministic vector that is bounded in its norm by some known positive constant ε, i.e.,

$$\|\boldsymbol{e}\| \leq \varepsilon^2 \tag{9}$$

where $\|\cdot\|$ denotes the Euclidian norm of a vector.

Then the robust worst case based MMSE linear receiver problem formulation can be expressed in terms of the following optimization problem [7]

$$\min_{\boldsymbol{f}} \left\{ 1 + \boldsymbol{f}^H \hat{\boldsymbol{R}} \boldsymbol{f} + \max_{\|\boldsymbol{e}\| \leq \varepsilon^2} \{ -(\boldsymbol{d} + \beta \boldsymbol{e})^H \boldsymbol{f} - \boldsymbol{f}^H (\boldsymbol{d} + \beta \boldsymbol{e}) \} \right\}. \tag{10}$$

The solution to the problem (10) is given by [7]

$$\boldsymbol{f}_{\rm rob} = \left(\hat{\boldsymbol{R}} + \varepsilon^2 \tau \boldsymbol{I} \right)^{-1} \boldsymbol{s}_1 \tag{11}$$

where the positive constant β is omitted because it does not affect the probability of error at the output of the symbol detector, $\tau \triangleq 1/\|\boldsymbol{f}\| > 0$, and the optimal value $\tau_{\rm opt}$ is the solution of the following equation

$$\tau^2 \left\| (\hat{\boldsymbol{R}} + \varepsilon^2 \tau \boldsymbol{I})^{-1} \boldsymbol{s}_1 \right\|^2 = 1. \tag{12}$$

The receiver (11) is similar to the diagonal loading based receiver (7), however, the diagonal loading coefficient $\varepsilon^2 \tau$ in (11) is not a constant. Therefore, the receiver (11) can be viewed as an *adaptive* diagonal loading based receiver, where the *optimal* diagonal loading coefficient is the solution of the equation (12). The disadvantage of the worst case based approach as applied to the robust MUD problem is that the uncertainty coefficient ε in (9) is unknown and is typically chosen in *ad hoc* way.

3. PROBABILITY CONSTRAINED ROBUST MMSE LINEAR RECEIVER

It is more natural to model the desired user signature error $\boldsymbol{e}$ in statistical sense, rather than deterministic. Indeed, if the training mode is used, the user signature estimation error is known to be Gaussian distributed [9], and, therefore, its norm is χ^2 distributed and not bounded. It indicates that, strictly speaking, the bound (9) (the coefficient ε) used in the worst case based approach is *ad hoc*. In this section, we propose an alternative approach to robust design for MUD. Specifically, using the earlier developed probability constrained optimization based approach [11], we can provide the robustness against desired user signature estimation errors with a certain selected probability.

Mathematically, the robust formulation of the MMSE linear receiver based on the probability constrained optimization approach can be written as

$$\begin{aligned} &\min_{\boldsymbol{f}, t} \ 1 + \boldsymbol{f}^H \hat{\boldsymbol{R}} \boldsymbol{f} + t \\ &\text{subject to} \quad \Pr_{\boldsymbol{e}} \left\{ -\tilde{\boldsymbol{d}}^H \boldsymbol{f} - \boldsymbol{f}^H \tilde{\boldsymbol{d}} \leq t \right\} \geq p \end{aligned} \tag{13}$$

where p is a certain probability value, $\tilde{\boldsymbol{d}} = \boldsymbol{d} + \beta \boldsymbol{e} = \beta \tilde{\boldsymbol{s}}_1$, and $\Pr\{\cdot\}$ stands for the probability operator. Note that the problems of this type are referred in the optimization literature as chance constrained or probability constrained stochastic programming problems [12]. Also note that we search for nontrivial minimizer $\boldsymbol{f}$.

The problem (13) aims at minimizing the MSE which includes also an unknown desired user signature estimation error $\boldsymbol{e}$ with a certain probability p. Thus, only the desired user signature estimation errors which occur with sufficiently high probability are considered in the formulation (13), while the user signature estimation errors whose probability is low are discarded. The constraint in (13) can be alternatively viewed as a non-outage probability constraint, where outage takes place every time when the constraint is not satisfied. The problem (13) is different from the problem (10), where the MSE is minimized for all user signature errors from the known uncertainty region, no matter how likely they occur in practice.

Following [9], we assume that the desired user signature estimation error $\boldsymbol{e}$ is complex circularly symmetric Gaussian distributed, i.e.,

$$\boldsymbol{e} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma^2). \quad (14)$$

Indeed, for the optimum maximum likelihood (ML) estimate of the user signature, the estimation error is Gaussian distributed (conditioned on the training symbols) [9]. Moreover, while the Gaussian assumption is not satisfied in the presence of residual interference, this is still suitable assumption if the statistical property of the residual interference is unknown.

Using the assumption (14), it is easy to show that the random variable $-\tilde{\boldsymbol{d}}^H\boldsymbol{f} - \boldsymbol{f}^H\tilde{\boldsymbol{d}}$ has the following real Gaussian distribution

$$-\tilde{\boldsymbol{d}}^H\boldsymbol{f} - \boldsymbol{f}^H\tilde{\boldsymbol{d}} \sim \mathcal{N}_{\mathbb{R}}\left(-\boldsymbol{d}^H\boldsymbol{f} - \boldsymbol{f}^H\boldsymbol{d},\ 2\beta^2\|\boldsymbol{C}_{\boldsymbol{e}}^{1/2}\boldsymbol{f}\|^2\right) \quad (15)$$

where $\boldsymbol{C}_{\boldsymbol{e}} \triangleq E\{\boldsymbol{e}\boldsymbol{e}^H\}$ is the covariance matrix of the user signature estimation error. Note that the covariance matrix $\boldsymbol{C}_{\boldsymbol{e}}$ can be typically found in closed form and depends on the method used for signature estimation. For example, the covariance matrix $\boldsymbol{C}_{\boldsymbol{e}}$ can be expressed as a function of the spreading code, the sample size, and the noise variance if the blind subspace-based method of [5] is used to estimate the user signature. Similarly, if the least square (LS) method of [9] (equivalent to the ML method) is used for estimating the desired user signature $\boldsymbol{s}_1$, the covariance matrix $\boldsymbol{C}_{\boldsymbol{e}}$ can be accurately approximated for large sample size M as [9]

$$\boldsymbol{C}_{\boldsymbol{e}} \approx \frac{\sigma^2 L}{M}\boldsymbol{I}. \quad (16)$$

Using the error function for any real Gaussian random variable x

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \quad (17)$$

we can find the probability $\Pr\{x \le c\}$ as follows

$$\Pr\{x \le c\} = \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{c - E\{x\}}{\sqrt{2E\{(x - E\{x\})^2\}}}\right). \quad (18)$$

For the random variable $-\tilde{\boldsymbol{d}}^H\boldsymbol{f} - \boldsymbol{f}^H\tilde{\boldsymbol{d}}$, the probability in (13) can be equivalently written as

$$\Pr_{\boldsymbol{e}}\left\{-\tilde{\boldsymbol{d}}^H\boldsymbol{f} - \boldsymbol{f}^H\tilde{\boldsymbol{d}} \le t\right\} = \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{t + \boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d}}{2\beta\|\boldsymbol{C}_{\boldsymbol{e}}^{1/2}\boldsymbol{f}\|}\right) \quad (19)$$

Applying (19) to the the probability constraint in (13), we obtain the following equivalent constraint

$$\operatorname{erf}\left(\frac{t + \boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d}}{2\beta\|\boldsymbol{C}_{\boldsymbol{e}}^{1/2}\boldsymbol{f}\|}\right) \ge 2p - 1. \quad (20)$$

The constraint (20) is convex if and only if its left-hand side is positive. The latter is guarantied if $2p - 1 > 0$ or, equivalently, $p > 0.5$. In this case, the constraint (20) can be rewritten as

$$\tilde{\varepsilon}\|\boldsymbol{C}_{\boldsymbol{e}}^{1/2}\boldsymbol{f}\| \le t + \boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d} \quad (21)$$

where

$$\tilde{\varepsilon} = 2\operatorname{erf}^{-1}(2p - 1)\beta \quad (22)$$

and $\operatorname{erf}^{-1}(\cdot)$ denotes the inverse error function. The constraint (21) is called the second-order cone (SOC) constraint and is convex.

The constraint (21) is tight for any optimal solution (see the formal proof of this fact in [8]), and the problem (13) can be expressed in terms of real-valued quantities as

$$\min_{\boldsymbol{f},t}\ 1 + \boldsymbol{f}^H\hat{\boldsymbol{R}}\boldsymbol{f} + t$$
$$\text{subject to}\quad \tilde{\varepsilon}^2\|\boldsymbol{C}_{\boldsymbol{e}}^{1/2}\boldsymbol{f}\|^2 \le (t + \boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d})^2. \quad (23)$$

The solution to (23) can be found by optimizing the Lagrangian function

$$L(\boldsymbol{f}, t, \lambda) = 1 + \boldsymbol{f}^H\hat{\boldsymbol{R}}\boldsymbol{f} + t + \lambda\left(\tilde{\varepsilon}^2\boldsymbol{f}^H\boldsymbol{C}_{\boldsymbol{e}}\boldsymbol{f} - t^2 - 2t(\boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d}) - (\boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d})^2\right) \quad (24)$$

where λ is a Lagrange multiplier. Differentiating the Lagrangian function $L(\boldsymbol{f}, t, \lambda)$ with respect to $\boldsymbol{f}$, t, and λ; and setting these partial derivatives equal to zero, we obtain, respectively

$$(\hat{\boldsymbol{R}} + \tilde{\varepsilon}^2\lambda\boldsymbol{C}_{\boldsymbol{e}})\boldsymbol{f} = 2\lambda t\boldsymbol{d} + 2\lambda\boldsymbol{d}\boldsymbol{d}^H\boldsymbol{f} + 2\lambda\boldsymbol{f}^H\boldsymbol{d}\boldsymbol{d} \quad (25)$$
$$2t\lambda = 1 - 2\lambda(\boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d}) \quad (26)$$
$$\tilde{\varepsilon}^2\boldsymbol{f}^H\boldsymbol{C}_{\boldsymbol{e}}\boldsymbol{f} = t^2 + 2t(\boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d}) + (\boldsymbol{d}^H\boldsymbol{f} + \boldsymbol{f}^H\boldsymbol{d})^2. \quad (27)$$

Inserting (25) into (26) and solving the equation, we find that

$$\boldsymbol{f}_{\text{prob}} = (\hat{\boldsymbol{R}} + \tilde{\varepsilon}^2\lambda\boldsymbol{C}_{\boldsymbol{e}}^{1/2})^{-1}\boldsymbol{s}_1 \quad (28)$$

where the positive constant β is omitted because it does not affect the probability of error at the output of the symbol detector. Similarly, the analytical solution for t can be easily found form (26), and written as

$$t = 1/2\lambda - \boldsymbol{d}^H\boldsymbol{f} - \boldsymbol{f}^H\boldsymbol{d}. \quad (29)$$

Applying (28) and (29) to (27) yields

$$4\tilde{\varepsilon}^2\lambda^2\left\|(\hat{\boldsymbol{R}} + \tilde{\varepsilon}^2\lambda\boldsymbol{C}_{\boldsymbol{e}}^{1/2})^{-1}\boldsymbol{s}_1\right\|^2 = 1. \quad (30)$$

The optimal value of the Lagrange multiplier λ_{opt} is then a zero of (30) that can be solved using the efficient Newton-type numerical procedure developed in [7] for solving similar equation (12) for τ.

Note that the probability constrained robust MMSE linear receiver (28), (30) is equivalent to the worst case based linear receiver (11), (12) if the LS method is used for estimating the desired user signature and the covariance matrix $\boldsymbol{C}_{\boldsymbol{e}}$ is given by (16). In this case the probability constrained robust MMSE linear receiver is simplified as

$$\boldsymbol{f}_{\text{prob}} = (\hat{\boldsymbol{R}} + \varepsilon^2\lambda\boldsymbol{I})^{-1}\boldsymbol{s}_1 \quad (31)$$
$$4\varepsilon^2\lambda^2\left\|(\hat{\boldsymbol{R}} + \varepsilon^2\lambda\boldsymbol{I})^{-1}\boldsymbol{s}_1\right\|^2 = 1 \quad (32)$$

where

$$\varepsilon = 2\operatorname{erf}^{-1}(2p - 1)\beta\sigma\sqrt{L}/\sqrt{M}. \quad (33)$$

Therefore, the worst case and the probability constrained optimization based designs are related to each other. Equation (33) explicitly quantifies this relationship providing an interpretation of the worst case design parameter ε (the radius of the uncertainty set) in terms of the outage probability and second-order statistics of the user signature estimation error.

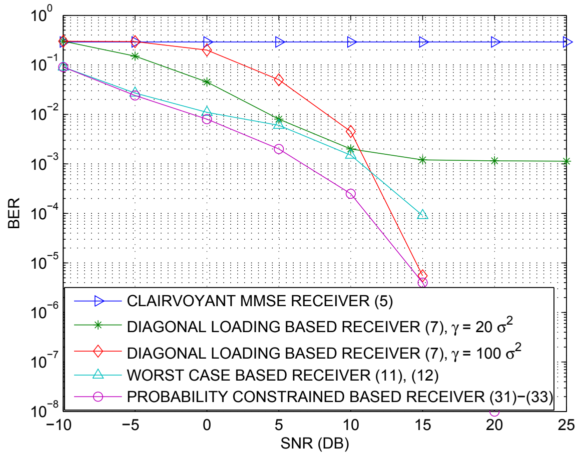

Fig. 1. BERs versus SNR.

4. SIMULATION RESULTS

A seven-user synchronous CDMA system is considered. The BPSK modulation and binary Golden spreading code of the length $L = 31$ are used. The interference-to-noise ratio (INR) is taken to be equal to 20 dB for all interfering users. A total 1000 runs is used to obtain each point of the bit error rate (BER) curves.

The following MUD algorithms are used: the linear MMSE receiver (5) with the sample data covariance matrix (6) and exact $\boldsymbol{s}_1$ (called as clairvoyant MMSE receiver); the receiver (7) with the diagonal loading parameter $\gamma = 20\sigma^2$ and $100\sigma^2$, where $\sigma^2 = 1$ (called as diagonal loading based receiver); the receiver (11), (12) with the uncertainty parameter ε^2 chosen as recommended in [7] and equal to $0.7\sqrt{L}$ (called as worst case based receiver); and the proposed receiver (31)-(33) with the outage probability $p_{\text{out}} = 1 - p = 0.005$ (called as probability constrained based receiver). The blind subspace based estimate of the desired user signature $\boldsymbol{s}_1$, which is obtained from 30 data vectors using the technique of [5], is used for the diagonal loading based, the worst case based, and the probability constrained based receivers.

The channel distortion is modeled as an FIR filter with four taps and the channel impulse response $\boldsymbol{h} = [\delta_0, \delta_1 e^{j\phi_1}, 0, \delta_3 e^{j\phi_3}]^T$, where δ_0, δ_1, and δ_3 are the amplitudes of the first, the second and the fourth taps, respectively, and ϕ_1 and ϕ_3 are the phases of the second and the forth taps. In each simulation run, δ_0 is randomly chosen from the interval $[0.9, 1.1]$, and δ_1 and δ_3 are randomly chosen from the interval $[0.45, 0.55]$. Similarly, ϕ_1 and ϕ_3 are randomly drawn from a uniform distribution over the interval $[0, 2\pi]$.

Fig. 1 shows the BERs of the aforementioned multiuser receivers tested versus the signal-to-noise ratio (SNR) of the desired user. It can be seen that the robust receivers with adaptive diagonal loading, i.e., the worst case based receiver (11), (12) and the probability constrained based receiver (31)-(33) perform better than other receivers, and the probability constrained based receiver outperforms the worst case based receiver, perhaps, due to the better selection of the uncertainty parameter ε.

5. CONCLUSIONS

The problem of robustness of multiuser receivers against user signature estimation error has been addressed, and a new MUD that guarantees the robustness against user signature errors with a certain selected probability has been proposed. The relationship between the proposed MUD and the worst case based MUD is found. The uncertainty parameter of the latter detector is quantified in terms of the outage probability of the proposed probability constrained based MUD and the second-order statistics of the user signature estimation error. Simulation results have validated an improved performance of the proposed approach as compared to the existing linear MUD algorithms.

6. REFERENCES

[1] S. Verdu, *Multiuser Detection.* Cambridge, U.K.: Cambridge Univ. Press, 1998.

[2] S. Verdu, “Minimum probability of error for asynchronous Gaussian multiple access channels,” *IEEE Trans. Inf. Theory*, vol. 32, pp. 85-96, Jan. 1986.

[3] R. Lupas and S. Verdu, “Linear multiuser detectors for synchronous code-division multiple-access channels,” *IEEE Trans. Inf. Theory*, vol. 35, pp. 123-136, Jan. 1989.

[4] M. K. Tsatsanis and G. B. Giannakis, “Optimal decorrelating receivers for DS-CDMA systems: A bsignal processing framework,” *IEEE Trans. Signal Processing*, vol. 44, pp. 3044-3055, Dec. 1996.

[5] H. Liu and G. Xu, “A subspace method for signature waveform estimation in synchronous CDMA systems,” *IEEE Trans. Communications*, vol. 44, pp. 1346-1354, Oct. 1996.

[6] S. Shahbazpanahi and A.B. Gershman, “Robust blind multiuser detection for synchronous CDMA systems using worst-case performance optimization,” *IEEE Trans. Wireless Communications*, vol. 4, pp. 2232 - 2245, Nov. 2004.

[7] K. Zarifi, S. Shahbazpanahi, A.B. Gershman, and Z-Q. Luo, “Robust blind multiuser detection based on the worst-case performance optimization of the MMSE receiver,” *IEEE Trans. Signal Processing*, vol. 53, pp. 295-305, Jan. 2005.

[8] S.A. Vorobyov, A.B. Gershman, and Z-Q. Luo, “Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem,” *IEEE Trans. Signal Processing*, vol. 51, pp. 313-324, Feb. 2003.

[9] R. Wang, H. Li, and T. Li, “Robust multiuser detection for multicarrier CDMA systems,” *IEEE J. Select. Areas Commun.*, vol. 24 pp. 673-683, Mar. 2006.

[10] Z. Tian, K.L. Bell, and H.L. Van Trees, “Robust constrained linear receivers for CDMA wireless systems,” *IEEE Trans. Signal Process.*, vol. 49 pp. 1510-1522, Jul. 2001.

[11] S.A. Vorobyov, A.B. Gershman, and Y. Rong, “On the relationship between the worst-case optimization-based and probability-constrained approaches to robust adaptive beamforming,” in *Proc. 32nd IEEE Int. Conf. on Acoustics, Speech, and Signal Processing, ICASSP'07*, Honolulu, Hawaii, USA, Apr. 2007, vol. 2, pp. 977-980.

[12] A. Prkopa, *Stochastic Programming.* Dordrecht, The Netherlands: Kluwer, 1995.