RANGE-RECURSIVE PRE-DOPPLER STAP ALGORITHM TO REJECT RANGE DEPENDENT CLUTTER IN AIRBORNE RADAR

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ABSTRACT

This paper addresses the issue of employing pre-Doppler space time adaptive processing (STAP) [1] in non side looking airborne radar. In this configuration, where the platform velocity is misaligned with the radar antenna axis, the clutter possesses specific properties : the space-time repartition of its spectral power depends on the range what induces difficulties to reject it. The range recursive STAP algorithms have already exhibited high performance to track this non stationarity in fully adaptive STAP [2], [3]. We here propose the use of these algorithms in a pre-Doppler architecture. This paper presents significant performance improvements offered by this element space approach compared with a fully architecture. A comparison with SMI [1] and DBU [4] methods associated with a pre-Doppler approach is also studied. We show that the proposed algorithm outperforms them in terms of SINR loss and computational cost.

Index Terms— airborne radar, radar clutter

1. INTRODUCTION

Slowly moving targets detection is a difficult task of airborne radar signal processing. In fact due to the platform motion, the ground clutter Doppler spectrum is spread and thus target signals and ground clutter returns are competing. Space-time adaptive processing (STAP) combines simultaneously spatial and temporal domains [1] and thus allows a better clutter suppression, what improves low-velocity or/and small target detection. The conventional fully adaptive STAP known as the sample matrix inversion (SMI) method as well as the subspace-based eigencanceller are not actually used in practice because of their prohibitive computational cost which prevents their real-time implementation [1], [2]. That is why we are focusing on adaptive algorithms which can recursively compute a subspace-based STAP rejector directly from the data with a linear complexity. In previous work [3], it has also been shown that these adaptive-recursive STAP algorithms are capable of tracking the range non-stationarity of the snapshots induced by a misalignment of the antenna array and the platform direction or more generally a non-side looking configuration. Thanks to the new pre-Doppler algorithm here proposed, the computational cost is further reduced and the performance in severe non stationary environments is improved. It also appears that the proposed approach outperforms the fully derivative-based (DBU) method and the pre-Doppler DBU presented in [4] and [5] respectively to compensate for the range dependency of the data. Section 2 describes the problem statement and the signal model. The pre-Doppler adaptive algorithm is presented in section 3. Simulation results and a discussion are given in section 4. At last concluding remarks are given in section 5.

2. PROBLEM STATEMENT AND SIGNAL MODEL

Figure 1 represents the system under consideration; i.e. a monostatic pulsed Doppler airborne radar in non side looking configuration. The radar antenna array is linear and composed of N half wavelength uniformly spaced elements. It is not aligned (non sidelooking configuration) with the platform velocity vector and forms a crab angle ϕ_a. P pulses are emitted at a constant pulse repetition frequency (PRF) . The radar antenna array is positioned on an airborne platform at the altitude h and moves with constant velocity $\mathbf{v}_a$. The ground clutter is split in rings of constant range R_c from the radar which are split themselves in N_c patches (here $N_c = 360$). Each clutter patch is described by its azimuth ϕ_c and its elevation θ_c. A space time snapshot at range k in the presence of a target is given by

$$\mathbf{x}(k) = \alpha_t \mathbf{v}(\varpi_t, \vartheta_t) + \mathbf{x}_{i+n}(k) \quad (1)$$

α_t is the unknown target amplitude, $\mathbf{v}$ is the target steering vector and $\mathbf{x}_{i+n}(k)$ is the interference plus noise signal vector. The ground clutter is here the only interference component and it is supposed unambiguous in range. The target steering vector is defined by

$$\mathbf{v}(\varpi_t, \vartheta_t) = \mathbf{b}(\varpi_t) \otimes \mathbf{a}(\vartheta_t) \quad (2)$$

where

$$\mathbf{b}(\varpi_t) = [1; e^{j2\pi\varpi_t}; \dots ; e^{j(M-1)2\pi\varpi_t}] \quad (3)$$

is the temporal steering vector, ϖ_t the target Doppler frequency and

$$\mathbf{a}(\vartheta_t) = [1; e^{j2\pi\vartheta_t}; \dots ; e^{j(N-1)2\pi\vartheta_t}] \quad (4)$$

is the spatial steering vector with ϑ_t the target spatial frequency, $\otimes$ denotes the Kronecker product . The interference plus noise $\mathbf{x}_{i+n} = \mathbf{x}_c + \mathbf{x}_n$ (in the absence of jammer) is composed of a noise vector $\mathbf{x}_n$ supposed to be gaussian and spatially and temporally white and a ground clutter component

$$\mathbf{x}_c = \sum_{i=1}^{N_c} \alpha_{c_i} \mathbf{v}\left(\vartheta_{c_i}, \varpi_{c_i}\right)$$

α_{c_i} is the amplitude of the ith azimuth clutter patch and $\mathbf{v}$ is defined in the same way as (2), (3) and (4) with ϑ_{c_i} and ϖ_{c_i} being the spatial and normalized Doppler frequencies respectively. The space time repartition of the clutter spectral power (called clutter ridge) is obtained by plotting the clutter normalized Doppler frequency as a function of the spatial frequency. Figures 2(a) to 2(c) represent these clutter ridges for different values of the crab angle ϕ_a and for different ranges. We remark the range independence of the clutter ridges in the side looking configuration ($\phi_a = 0^o$) and on the contrary their range dependency in a non side looking configuration ($\phi_a = 10^o$, $\phi_a = 90^o$). This non stationarity in range of the data implies difficulties to reject the clutter plus noise components and more particularly to estimate the clutter plus noise covariance matrix. Indeed the optimum weight vector of the interference plus noise rejection filter given by [1], $\mathbf{w}_{opt} = \kappa \mathbf{R}_{i+n}^{-1} \cdot \mathbf{v}_t$ requires the knowledge of the clutter plus noise covariance matrix $\mathbf{R}_{i+n} = E\left\{\mathbf{x}_{i+n}\mathbf{x}_{i+n}^H\right\} = \mathbf{R_c} + \mathbf{R_n}$ where $\mathbf{R_c} = E\left\{\mathbf{x}_c\mathbf{x}_c^H\right\}$ and $\mathbf{R_n} = \sigma^2\mathbf{I}$ are the clutter and the noise space-time covariance matrices, respectively. In practice $\mathbf{R}_{i+n}$ is unknown and must be estimated from the snapshots. The well-known SMI consists in an estimation of the matrix by averaging over the secondary range cells,

$$\widehat{\mathbf{R}}_{i+n}(k) = \frac{1}{S}\sum_{l=1, l\neq k}^{S} \mathbf{x}(l)\mathbf{x}(l)^H \quad (5)$$

where k is the test range cell and S is the number of secondary range cells. The SMI weight vector is then $\mathbf{w}_{smi} = \kappa \widehat{\mathbf{R}}_{i+n}^{-1}\mathbf{v}(\varpi_t, \vartheta_t)$, where κ is a scalar which imposes to have a unity gain $\mathbf{w}_{smi}^H \cdot \mathbf{v}(\varpi_t, \vartheta_t) = 1$. This estimator is unbiased only in case of iid (independent and identically distributed) data. As the clutter is range independent in the side looking configuration, the covariance matrix $\mathbf{R}_{i+n}$, can thus be estimated through (5). Contrarily in a non side looking configuration, where the clutter is range dependent, this is no longer possible. To mitigate this range non stationarity problem, several methods of compensation ([6] and the references within) have been proposed in the literature but they either are too complex to be implemented or require the knowledge of the radar parameters. In a previous work [3] the use of range recursive fully space time adaptive algorithms (like FAPI [7]) was found to be a good alternative approach to this problem. In the following section, we investigate the use of range recursive pre-Doppler STAP algorithms to further improve the performance in more difficult situations.

3. RANGE-RECURSIVE PRE-DOPPLER STAP ALGORITHM

In this section, we propose a range recursive pre-Doppler STAP algorithm. In this partially approach, the full spatial dimension is maintained and the temporal dimension is reduced. The classical fully adaptive structure is reminded in figure 3(a) and a general partially adaptive structure is presented in figure 3(b). As P pulses are in a CPI, let us define a sub-CPI which is a subset of the CPI and contains K successive pulses. There are then $P - K + 1$ sub CPI. Let us remark that the choice of K is constrained by the following rule [1] : the number of degrees of freedom must be greater than the clutter rank, thus K is an integer such as

$$KN \geq rank(\mathbf{R}_{c_{red}}) + 1$$

where $\mathbf{R}_{c_{red}}$ is the reduced clutter covariance matrix. It follows that the minimal size of the sub-CPI is 2 pulses in the side looking configuration and $K \geq \frac{2N-1}{N-2}$, $K \in \mathcal{N}$ ($\mathcal{N}$ denotes the set of natural numbers). The $NP \times 1$ space-time snapshot in (1) $\mathbf{x}$ is reduced to $P-K+1$ sub CPI $KN \times 1$ snapshot vectors $\tilde{\mathbf{x}}$. The i^{th} vector is then

$$\tilde{\mathbf{x}}_\mathbf{i} = (\mathbf{J}_i \otimes \mathbf{I}_N)^H \cdot \mathbf{x} \quad (6)$$

where

$$\mathbf{J}_i = \begin{pmatrix} \mathbf{0}_{i\times K} \\ \mathbf{I}_K \\ \mathbf{0}_{(P-K-i)\times K} \end{pmatrix} \text{ with } i \in [0, P-K]$$

where $\mathbf{I}_N$ and $\mathbf{I}_K$ are the identity matrices of size $N \times N$ and $K \times K$ respectively. The reduced clutter covariance matrices on each sub CPI are also rank deficient [1] and the reduced data subspace can be partitioned into reduced clutter and noise subspaces. We apply the FAPI algorithm [7] for each reduced snapshot vector to obtain $P - K + 1$ sub CPI weight vectors [1]. A basis of the clutter subspace is obtained as the solution of an exponentially least square problem which is solved by a recursive computation. This algorithm is based on an approximation of the projection on the clutter subspace at two consecutive ranges. More details are given in [7] and in table 1. The corresponding STAP filters computed for each snapshot and for each sub CPI i are obtained through $\tilde{\mathbf{w}}_\mathbf{i} = \left(\mathbf{I} - \tilde{\mathbf{W}}_\mathbf{i}(k)\tilde{\mathbf{W}}_\mathbf{i}(k)^H\right) \cdot \tilde{\mathbf{v}}(\varpi_t, \vartheta_t)$ where $\tilde{\mathbf{v}}$ is the reduced target steering vector obtained as in (6).

Then the outputs from all sub CPI are collected into an $(P - K + 1) \times 1$ vector which is passed through a standard Doppler filter bank.

Table 1 pre-Doppler FAPI Algorithm

Initialization : $\tilde{\mathbf{W}}(0) \leftarrow \mathbf{I}_{K\times N}$, $\tilde{\mathbf{Z}}(0) \leftarrow \mathbf{I}_{K\times N}$
for $i = 1$ to $P - K + 1$ **do**
  for $k = 1$ to Nbr snapshot **do**
    $\tilde{\mathbf{y}}_\mathbf{i}(k) = \tilde{\mathbf{W}}_\mathbf{i}(k-1)^H \cdot \tilde{\mathbf{x}}_\mathbf{i}(k)$
    $\tilde{\mathbf{h}}_\mathbf{i}(k) = \tilde{\mathbf{Z}}_\mathbf{i}(k-1) \cdot \tilde{\mathbf{y}}_\mathbf{i}(k)$
    $\tilde{\mathbf{g}}_\mathbf{i}(k) = \frac{\tilde{\mathbf{h}}_\mathbf{i}(k)}{\beta + \tilde{\mathbf{y}}_\mathbf{i}^H(k)\cdot\tilde{\mathbf{h}}_\mathbf{i}(k)}$
    $\tilde{\mathbf{e}}_\mathbf{i}(k) = \tilde{\mathbf{x}}_\mathbf{i}(k) - \tilde{\mathbf{W}}_\mathbf{i}(k-1) \cdot \tilde{\mathbf{y}}_\mathbf{i}(k)$
    $\varepsilon_i^2(k) = \|\tilde{\mathbf{x}}_\mathbf{i}(k)\|^2 - \|\tilde{\mathbf{y}}_\mathbf{i}(k)\|^2$
    $\tau_i(k) = \frac{\varepsilon_i^2(k)}{1+\varepsilon_i^2(k)\|\tilde{\mathbf{g}}_\mathbf{i}(k)\|^2 + \sqrt{1+\varepsilon_i^2(k)\|\tilde{\mathbf{g}}_\mathbf{i}(k)\|^2}}$
    $\eta_i(k) = 1 - \tau_i(k)\,\|\tilde{\mathbf{g}}_\mathbf{i}(k)\|^2$
    $\tilde{\mathbf{y}}_\mathbf{i}'(k) = \eta_i(k)\tilde{\mathbf{y}}_\mathbf{i}(k) + \tau_i(k)\tilde{\mathbf{g}}_\mathbf{i}(k)$
    $\tilde{\mathbf{h}}_\mathbf{i}'(k) = \tilde{\mathbf{Z}}_\mathbf{i}(k-1)^H\tilde{\mathbf{y}}_\mathbf{i}'(k)$
    $\tilde{\epsilon}_\mathbf{i}(k) = \frac{\tau_i(k)}{\eta_i(k)}(\tilde{\mathbf{Z}}_\mathbf{i}(k-1)\tilde{\mathbf{g}}_\mathbf{i}(k) - (\tilde{\mathbf{h}}_\mathbf{i}'(k)\tilde{\mathbf{g}}_\mathbf{i}(k))\tilde{\mathbf{g}}_\mathbf{i}(k))$
    $\tilde{\mathbf{Z}}_\mathbf{i}(k) = \frac{1}{\beta}(\tilde{\mathbf{Z}}_\mathbf{i}(k-1) - \tilde{\mathbf{g}}_\mathbf{i}(k)\tilde{\mathbf{h}}_\mathbf{i}'(k)^H + \tilde{\epsilon}_\mathbf{i}(k)\tilde{\mathbf{g}}_\mathbf{i}(k)^H)$
    $\tilde{\mathbf{e}}_\mathbf{i}'(k) = \eta_i(k)\tilde{\mathbf{x}}_\mathbf{i}(k) - \tilde{\mathbf{W}}_\mathbf{i}(t-1)\tilde{\mathbf{y}}_\mathbf{i}'(k)$
    $\tilde{\mathbf{W}}_\mathbf{i}(k) = \tilde{\mathbf{W}}_\mathbf{i}(k-1) + \tilde{\mathbf{e}}_\mathbf{i}'(k) \cdot \tilde{\mathbf{g}}_\mathbf{i}(k)^H$
  end for
end for

4. SIMULATION RESULTS

For all simulations, a pulsed Doppler monostatic X-band radar is considered. The antenna array is an ULA composed of 8 elements spaced of half a wavelength. 16 pulses are transmitted during a CPI. The platform is moving at 9000 m with a constant velocity of $100 m.s^{-1}$. The sub-CPI length is 5 pulses and 30 range cells are used for training in case of pre-Doppler algorithms. The FAPI's forgetting factor β, appearing in Table 1, is set to 0.95. The algorithms are compared in terms of SINRloss which represents what could happen in absence of interference and is defined by

$$SINRloss = \frac{\sigma^2 \cdot \left|\mathbf{w}^H\mathbf{v}(\varpi_t, \vartheta_t)\right|^2}{NP \cdot \mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}} \quad (7)$$

where $\mathbf{w}$ is the weight vector calculated according to each algorithm and σ^2 is the noise variance. The optimum SINR loss is

$$SINRloss_{opt} = \frac{\sigma^2 \cdot \mathbf{v}(\varpi_t, \vartheta_t)^H\mathbf{R}_{i+n}^{-1}\mathbf{v}(\varpi_t, \vartheta_t)}{NP}$$

[1]There are many different range recursive algorithms [3], but we here focus on this one because during previous studies it showed the best performance

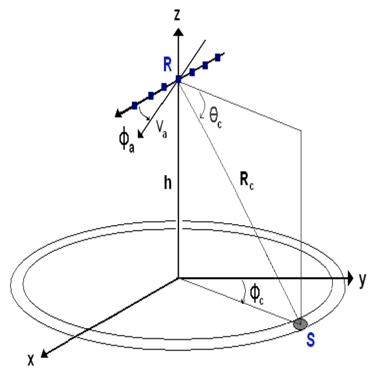

Fig. 1. Geometry of the monostatic non sidelooking configuration

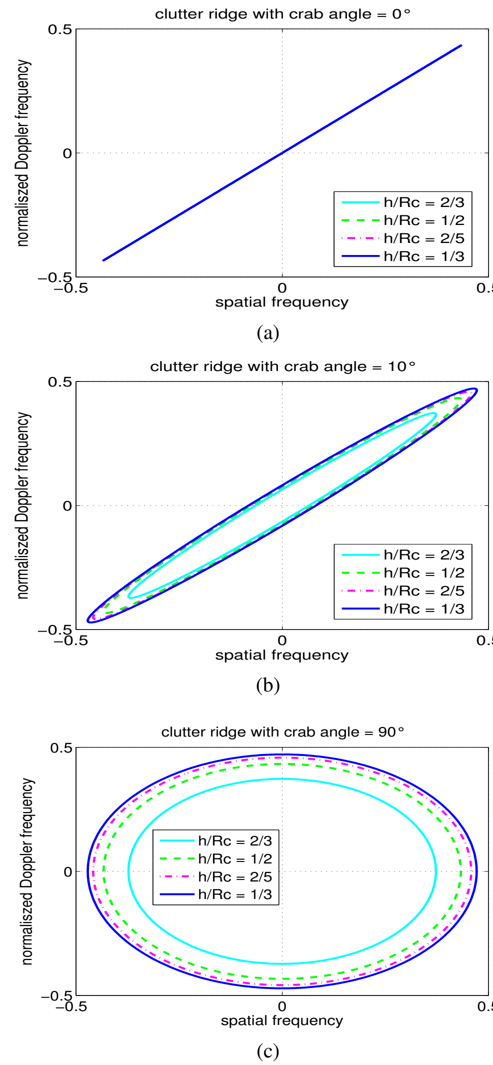

Fig. 2. Examples of clutter ridges. No velocity misalignment (a) ; velocity misalignment of 10^o (b), velocity misalignment of 90^o (c)

For the partially adaptive STAP, the SINRloss at the i^{th} output of the Doppler filter is given by:

$$SINRloss_i = \frac{\sigma^2 \cdot \left|\mathbf{w_i}^H \mathbf{v}(\varpi_t, \vartheta_t)\right|^2}{NP \cdot \mathbf{w_i}^H \mathbf{R}_{i+n} \mathbf{w_i}} \quad (8)$$

where the weight vector of the i^{th} sub-CPI $\mathbf{w_i}$ is defined by $\mathbf{w_i} = \mathbf{W} \cdot \mathbf{f_d}_i$ where $\mathbf{W}$ is the matrix containing the weight vectors from all the sub CPI and $\mathbf{f_d}_i$ is the i^{th} Doppler filter. Finally the maximum SINRloss over all the output of the Doppler filters is taken. At first, we show through a simulation the relevance of using pre-Doppler STAP algorithms in forward looking configuration ($\phi_a = 90^o$). For the classical FAPI, the training data set contains 250 range cells. Figure 4 shows that the performance of the pre-Doppler FAPI algorithm is better than FAPI in its original form. Indeed, the non stationarity of the data is better mitigated. Then, we compare different pre-Doppler STAP algorithms: SMI ([1]), DBU ([4] and [5]) and FAPI in non side looking configurations with a crab angle of 10^o in figure 5(a) and in forward looking configuration ($\phi_a = 90^o$) illustrated by figure 5(b). In all the cases, the range recursive proposed algorithm outperforms the others. The SINR loss curve is closer to the optimal and the width of the notches is thiner than for the others. Moreover, the computational cost of the pre-Doppler FAPI is lower ($O(KNr)$ where r is the rank of the clutter covariance matrix) than the non recursive algorithms under consideration ($O((KN)^3)$ for pre-Doppler SMI and $O((2KN)^3)$ for pre-Doppler DBU).

5. CONCLUSION

In conclusion, we have proposed a range-recursive pre-Doppler STAP algorithm which is capable of mitigating the range non stationarity problem induced by the non side looking configuration. This range-recursive STAP algorithm has a linear complexity and is here associated with a pre-Doppler smoothing which further reduces the computational cost because less range cells than in the case of the full STAP are clearly required for training. Thus the computational complexity is very low and linear in the different parameters ($O(MKr)$). Moreover we have shown that the performance is also improved : the width of the clutter notch is narrower than with other pre-Doppler algorithms.

6. REFERENCES

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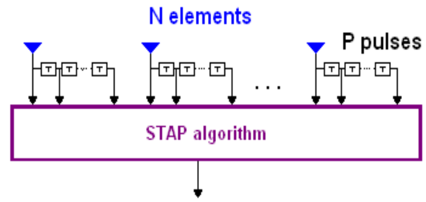
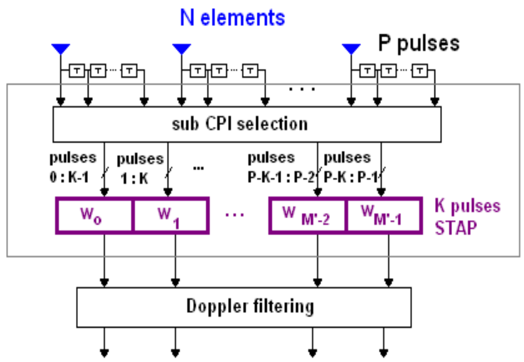


(a)



(b)

Fig. 3. STAP architectures (a) classical , (b) Element-space pre-Doppler

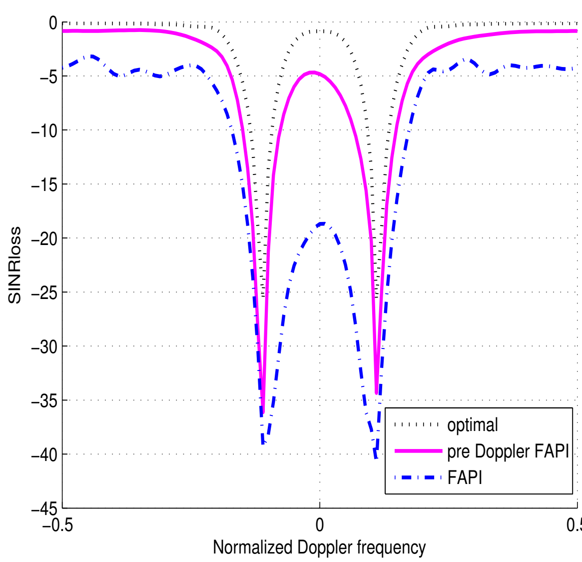
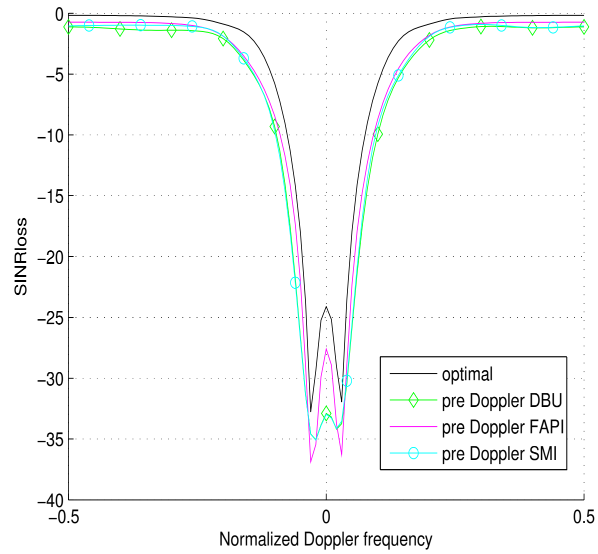
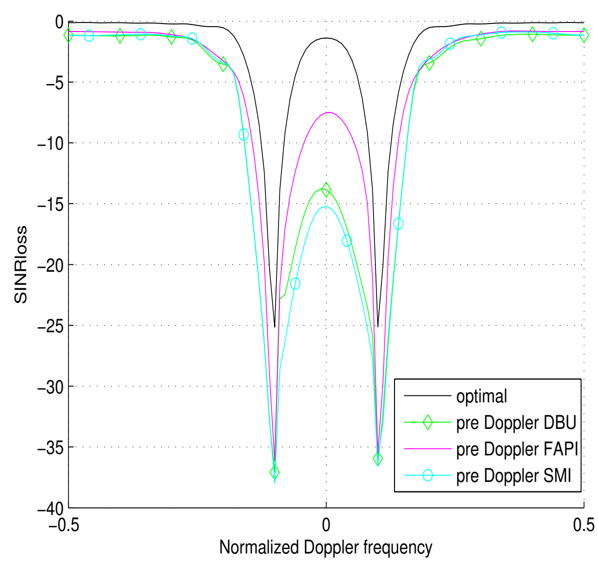


Fig. 4. Relevance of using pre-Doppler STAP algorithms



(a)



(b)

Fig. 5. Performance of element space pre-Doppler STAP algorithms (a) velocity misalignment of 10^o, (b)velocity misalignment of 90^o