NETWORK BEAMFORMING BASED ON SECOND ORDER STATISTICS OF THE CHANNEL STATE INFORMATION

Veria Havary-Nassab[†] *Shahram Shahbazpanahi*[†] *Ali Grami*[†] *Zhi-Quan Luo*[‡]

[†]Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, Oshawa, ON, Canada, L1H 7K4
[‡]Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455, US

ABSTRACT

The problem of distributed beamforming is considered for a network which consists of a transmitter, a receiver, and r relay nodes. Assuming that the second order statistics of the channel coefficients are available, we design a distributed beamforming technique via maximization of the receiver signal-to-noise ratio (SNR) subject to individual relay power constraints. We show that using semi-definite relaxation, this SNR maximization can be turned into a convex feasibility semi-definite programming problem, and therefore, it can be efficiently solved using interior point methods. We also obtain a performance bound for the semi-definite relaxation and show that the semi-definite relaxation approach provides a c-approximation to the (nonconvex) SNR maximization problem, where $c = O((\log r)^{-1})$ and r is the number of relays.

Index Terms— Distributed beamforming, relay networks, semi-definite programming, convex feasibility problem, distributed signal processing.

1. INTRODUCTION

Emerging wireless technologies, such as sensor and relay networks, have found applications in cooperative communications [1]-[3]. In fact, users of a wireless network can cooperate by relaying each other messages thus improving the communications reliability. However, the limited communication resources, such as battery lifetime of the devices and the scarce bandwidth, challenge the design of such cooperative communication schemes. Therefore, while ensuring that each user receives a certain quality of service (QoS), one is often confronted with the challenge that communication resources are subject to stringent constraints.

Various cooperative communication schemes have been presented in the literature. Examples are amplify-and-forward [3], coded cooperation [4], and compress-and-forward [5] schemes. Among these schemes, the amplify-and-forward approach is of particular interest as it can be easily implemented.

In [6], a distributed beamforming strategy has been developed for the case where the relaying nodes cooperate to build a beam towards the receiver under individual relay power constraints. The authors of [6] assume that each relay knows the *instantaneous* CSI for both backward (transmitter to the relay) and forward (relay to the receiver) links. Using such an assumption, the network beamforming approach is simplified to a distributed power control method. In fact, each relay matches the phase of its weight vector to the total phase of the backward and forward links. Therefore, only the amplitudes of the complex weights remain to be determined. These amplitudes are then obtained through maximizing the signal-to-noise ratio (SNR) at the receiver while guaranteeing that the individual relay powers meet the corresponding constraints. Interestingly enough, such a maximization results in relay powers that are not necessarily at their maximum allowable values. The relaying scheme developed in [6] is based on the availability of instantaneous CSI, and therefore, it does not allow any uncertainty in the channel modeling.

In this paper, we consider the problem of distributed beamforming under the assumption that the *second order statistics* of the channel coefficients are available. Such an assumption allows us to consider uncertainty in the channel modeling through introducing the covariance matrices of the channel coefficients. Based on such an assumption, we develop a distributed beamforming algorithm through maximization of the receiver SNR subject to individual relay power constraints. We show that in the case of individual relay power constraints, the beamforming problem can be turned into a semi-definite programming (SDP) problem which can be efficiently solved using interior point methods. We also analyze the performance of the semi-definite relaxation and prove that the semi-definite relaxation technique provides a c-approximation to the (nonconvex) SNR maximization problem, where $c = O((\log r)^{-1})$ and r is the number of relays.

2. SYSTEM MODEL

Consider a wireless network which consists of a transmitter, a receiver, and r relay nodes. We assume that due to the poor quality of the channel between the transmitter and receiver, there is no direct link between them. As a result, the transmitter deploys the relay nodes to communicate with the receiver. Each relay has a single antenna for both transmission and reception. Assuming a flat fading scenario, let f_i denote the channel coefficient from the transmitter to the ith relay and g_i represent the channel coefficient from the ith relay to the receiver. We also assume that the second order statistics of the channel coefficients $\{f_i\}_{i=1}^r$ and $\{g_i\}_{i=1}^r$ are known. In fact, we model f_i and g_i as random variables with known second order statistics. Considering a two-step amplify-and-forward (AF) protocol, the transmitter broadcasts to the relays, during the first step, the signal $\sqrt{P_0}s$, where s is the information symbol and P_0 is the transmit power. It is assumed that $E\{|s|^2\} = 1$, where $E\{\cdot\}$ represents the statistical expectation, and $|\cdot|$ denotes the amplitude of a complex number. The signal x_i received at the ith relay is given by

$$x_i = \sqrt{P_0}\, f_i\, s + \nu_i \tag{1}$$

where ν_i is the noise at the ith relay whose variance is known to be σ_ν^2. During the second step, the ith relay transmits the signal y_i which can be expressed as

$$y_i = w_i x_i \tag{2}$$

where w_i is the complex beamforming weight used by the ith relay. At the destination, the received signal can be written as

$$z = \sum_{i=1}^{r} g_i y_i + n \tag{3}$$

where z is the received signal and n is the receiver noise whose variance is known to be σ_n^2. Using (1) and (2), we can rewrite (3) as

$$\begin{aligned} z &= \sum_{i=1}^{r} g_i w_i x_i + n \\ &= \underbrace{\sqrt{P_0} \sum_{i=1}^{r} w_i f_i \, g_i s}_{\text{signal component}} + \underbrace{\sum_{i=1}^{r} w_i g_i \nu_i + n}_{\text{total noise, } n_T} . \end{aligned} \tag{4}$$

are used:

Using (4) and assuming that the relay noises $\{\nu_i\}_{i=1}^r$, the receiver noise n, and the channel coefficients $\{g_i\}_{i=1}^r$ are all independent from each other, the total noise power $P_{\rm n}$ can then be obtained as

$$\begin{aligned} P_{\rm n} &= E\{|n_T|^2\} \\ &= E\left\{ \sum_{i,j=1}^{r} w_i w_j^* g_i g_j^* \right\} \underbrace{E\{|\nu_i|^2\}}_{\sigma_\nu^2} + E\{|n|^2\} \\ &= \mathbf{w}^H \mathbf{Q} \mathbf{w} + \sigma_n^2 \end{aligned} \tag{5}$$

where $(\cdot)^*$ represents complex conjugate, $(\cdot)^H$ denotes Hermitian transpose, and the following definitions are used:

$$\begin{aligned} \mathbf{w} &\triangleq [w_1 \ w_2 \ \dots \ w_r]^T \\ \mathbf{Q} &\triangleq \sigma_\nu^2 E\{\mathbf{g}\mathbf{g}^H\} \\ \mathbf{g} &\triangleq [g_1 \ g_2 \ \cdots \ g_r]^T . \end{aligned}$$

Here, $(\cdot)^T$ is the transpose operator.

Also, using (4), the signal component power $P_{\rm s}$ can be obtained as

$$\begin{aligned} P_{\rm s} &= E\left\{ P_0 \left| \sum_{i=1}^{r} w_i f_i g_i \right|^2 |s|^2 \right\} \\ &= P_0 E\left\{ \sum_{i,j=1}^{r} w_i w_j^* f_i g_i f_j^* g_j^* \right\} \underbrace{E\{|s|^2\}}_{1} \\ &= \mathbf{w}^H \mathbf{R} \mathbf{w} \end{aligned} \tag{6}$$

where the following definitions are used:

$$\begin{aligned} \mathbf{h} &\triangleq [f_1 g_1 \ f_2 g_2 \ \dots \ f_r g_r]^T = \mathbf{f} \odot \mathbf{g} \\ \mathbf{R} &\triangleq P_0 E\{\mathbf{h}\mathbf{h}^H\} = P_0 E\{(\mathbf{f} \odot \mathbf{g})(\mathbf{f} \odot \mathbf{g})^H\} . \end{aligned} \tag{7}$$

Here, $\odot$ represents the element-wise Schur-Hadamard product.

3. SNR MAXIMIZATION

Our goal is to maximize the receiver SNR subject to individual relay power constraints. Such a case is of particular interest when the relay nodes are restricted in their battery lifetimes. The ith relay transmit power is given by $\alpha_i |w_i|^2$, where $\alpha_i \triangleq P_0 E\{|f_i|^2\} + \sigma_\nu^2$. In this case, we aim to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{w}} \quad & \text{SNR} \\ \text{subject to} \quad & \alpha_i |w_i|^2 \leq P_i \quad \text{for } i = 1, 2, \dots, r \end{aligned} \tag{8}$$

or, equivalently:

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\sigma_n^2 + \mathbf{w}^H \mathbf{Q} \mathbf{w}} \\ \text{subject to} \quad & \alpha_i |w_i|^2 \leq P_i \quad \text{for } i = 1, 2, \dots, r \end{aligned} \tag{9}$$

where P_i is the maximum allowable transmit power of the ith relay. Using the definition $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$, the optimization problem in (9) can be written as

$$\begin{aligned} \max_{\mathbf{X}} \quad & \frac{\text{tr}(\mathbf{R}\mathbf{X})}{\sigma_n^2 + \text{tr}(\mathbf{Q}\mathbf{X})} \\ \text{subject to} \quad & \alpha_i |w_i|^2 \leq P_i \quad \text{for } i = 1, 2, \dots, r , \\ & \text{rank } \mathbf{X} = 1, \quad \text{and} \quad \mathbf{X} \succeq 0 \end{aligned} \tag{10}$$

or, equivalently, as

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{subject to} \quad & \text{tr}\left(\mathbf{X}(\mathbf{R} - t\mathbf{Q})\right) \geq \sigma_n^2 t , \\ & \mathbf{X}_{ii} \leq P_i / \alpha_i \quad \text{for } i = 1, 2, \dots, r , \\ & \text{rank } \mathbf{X} = 1, \quad \text{and} \quad \mathbf{X} \succeq 0 \end{aligned} \tag{11}$$

where $\text{tr}(\cdot)$ represents the trace of a matrix and $\mathbf{X} \succeq 0$ means that $\mathbf{X}$ is constrained to be a symmetric positive semi-definite matrix. The optimization problem in (11) is not convex and may not be amenable to a computationally efficient solution. Let us then ignore the rank constraint in (11). That is, using a semi-definite relaxation, we aim to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{X}, t} \quad & t \\ \text{subject to} \quad & \text{tr}\left(\mathbf{X}(\mathbf{R} - t\mathbf{Q})\right) \geq \sigma_n^2 t \\ \text{and} \quad & \mathbf{X}_{ii} \leq P_i / \alpha_i \quad \text{for } i = 1, 2, \dots, r \\ \text{and} \quad & \mathbf{X} \succeq 0 . \end{aligned} \tag{12}$$

Due to the relaxation, the matrix $\mathbf{X}_{\rm opt}$ obtained by solving the optimization problem in (12) will not be of rank one in general. If $\mathbf{X}_{\rm opt}$ happens to be rank one, then its principal component will be the optimal solution to the original problem. Note that the optimization problem in (12) is quasiconvex. In fact, for any value of t, the feasible set in (12) is convex. Let $t_{\max}$ be the maximum value of t obtained by solving the optimization problem (12). If, for any given t, the convex feasibility problem [7]

$$\begin{aligned} \text{find} \quad & \mathbf{X} \\ \text{such that} \quad & \text{tr}\left(\mathbf{X}(\mathbf{R} - t\mathbf{Q})\right) \geq \sigma_n^2 t \\ \text{and} \quad & \mathbf{X}_{ii} \leq P_i / \alpha_i \quad \text{for } i = 1, 2, \dots, r \\ \text{and} \quad & \mathbf{X} \succeq 0 \end{aligned} \tag{13}$$

is feasible, then we have $t_{\max} \geq t$. Conversely, if the convex feasibility optimization problem (13) is not feasible, then we conclude $t_{\max} < t$. Therefore, we can check whether the optimal value $t_{\max}$ of the quasiconvex optimization problem in (12) is smaller than or greater than a given value t by solving the convex feasibility problem (13). If the convex feasibility problem (13) is feasible then we have $t_{\max} \geq t$. If the convex feasibility problem (13) is infeasible, then we know that $t_{\max} < t$. Based on this observation, one can use a simple algorithm to solve the quasiconvex optimization problem (12) using bisection technique, solving a convex feasibility problem at each step. We assume that the problem is feasible, and start with an interval $[l \;\; u]$ known to contain the optimal value $t_{\max}$. We then solve the convex feasibility problem at its midpoint $t = (l+u)/2$, to determine whether the optimal value is larger or smaller than t. We update the interval accordingly to obtain a new interval. That is, if t is feasible, then we set $l = t$, otherwise, we choose $u = t$ and solve the convex feasibility problem in (13) again.

This procedure is repeated until the width of the interval is small enough. Below we summarize the bisection technique:

1. Select $l < t_{\max}$, $u > t_{\max}$, and tolerance $\epsilon > 0$.
2. $t := (l+u)/2$.
3. Solve the convex feasibility problem (13).
4. If (13) is feasible $l := t$, otherwise $u := t$.
5. If $u - l < \epsilon$ stop, otherwise go to step 2.

To solve the convex feasibility problem (13), one can use the well-studied interior point based methods. For example, the SeDuMi [8] is an interior point method based package which produces a feasibility certificate if the problem is feasible. Once the maximum feasible value for t is obtained, one can replace it into (12). This turns (12) into a convex problem which can also be solved efficiently using interior point based methods. In semi-definite relaxation, the solution may not be rank one in general. Several randomization techniques have been proposed in the literature which can provide a satisfactory approximation to the problem (11) from the solution to the SDP problem [9]. Interestingly, in our extensive simulation results, we have never encountered a case where the solution to the SDP problem has a rank higher than one.

For cases where SDP problem has a solution with rank higher than one, it is possible to establish a bound for performance of the randomization technique. It can be shown that the SDP relaxation approach provides a $c = O((\log r)^{-1})$ approximation to the nonconvex fractional quadratic optimization problem (9). To show this, consider the problem

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\sigma_n^2 + \mathbf{w}^H \mathbf{Q} \mathbf{w}} \\ \text{subject to} \quad & \mathbf{w}^H \mathbf{G}_i \mathbf{w} \leq 1, \quad i = 1, 2, ..., r \end{aligned} \tag{14}$$

where $\mathbf{G}_i$ is a matrix with all zero entries except for the ith diagonal element which is equal to α_i / P_i. The SDP relaxation can be written as

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{\operatorname{tr}(\mathbf{R}\mathbf{X})}{\sigma_n^2 + \operatorname{tr}(\mathbf{Q}\mathbf{X})} \\ \text{subject to} \quad & \operatorname{tr}(\mathbf{G}_i \mathbf{X}) \leq 1, \;\; \mathbf{X} \succeq 0, \quad i = 1, 2, ..., r. \end{aligned} \tag{15}$$

Using bisection, we can solve the SDP relaxation in polynomial time yielding an optimal $\mathbf{X}^* \succeq 0$ and a μ^* satisfying

$$\operatorname{tr}(\mathbf{R}\mathbf{X}^*) = \mu^* (\operatorname{tr}(\mathbf{Q}\mathbf{X}^*) + \sigma_n^2). \tag{16}$$

Clearly, μ^* is an upper bound for the optimal value of (14). Now consider the nonconvex quadratic optimization problem

$$\begin{aligned} \max_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R} \mathbf{w} - \mu^* (\mathbf{w}^H \mathbf{Q} \mathbf{w} + \sigma_n^2) \\ \text{subject to} \quad & \mathbf{w}^H \mathbf{G}_i \mathbf{w} \leq 1, \quad i = 1, 2, ..., r. \end{aligned} \tag{17}$$

Its SDP relaxation can be written as

$$\begin{aligned} \max_{\mathbf{w}} \quad & \operatorname{tr}(\mathbf{R}\mathbf{X}) - \mu^* (\operatorname{tr}(\mathbf{Q}\mathbf{X}) + \sigma_n^2) \\ \text{subject to} \quad & \operatorname{tr}(\mathbf{G}_i \mathbf{X}) \leq 1, \;\; \mathbf{X} \succeq 0, \quad i = 1, 2, ..., r. \end{aligned} \tag{18}$$

By the definition of μ^*, it follows that $\mathbf{X}^* \succeq 0$ is a global optimal solution for (18). Let us sample from the complex Gaussian distribution $\mathcal{N}(0, \mathbf{X}^*)$. By the result of [10], we can generate in randomized polynomial time an approximate solution $\hat{\mathbf{w}}$ satisfying

$$\hat{\mathbf{w}}^H \mathbf{R} \hat{\mathbf{w}} - \mu^* \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}} \geq c(\operatorname{tr}(\mathbf{R}\mathbf{X}^*) - \mu^* \operatorname{tr}(\mathbf{Q}\mathbf{X}^*)),$$

where $c = O((\log r)^{-1})$ is a constant. In light of (16), we further obtain

$$\hat{\mathbf{w}}^H \mathbf{R} \hat{\mathbf{w}} - \mu^* \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}} \quad \geq \quad c\mu^* \sigma_n^2$$

implying

$$\begin{aligned} \hat{\mathbf{w}}^H \mathbf{R} \hat{\mathbf{w}} - c\mu^* \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}} \quad & \geq \quad c\mu^* \sigma_n^2 + (1-c)\mu^* \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}} \\ & \geq \quad c\mu^* \sigma_n^2, \end{aligned} \tag{19}$$

where the last step follows from the positive semi-definiteness of $\mathbf{Q}$. Rearranging the terms, we obtain

$$\frac{\hat{\mathbf{w}}^H \mathbf{R} \hat{\mathbf{w}}}{\sigma_n^2 + \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}}} \geq c\mu^*$$

implying that $\hat{\mathbf{w}}$ is an c-optimal solution of (14). In other words, the SDP relaxation approach provides a $c = O((\log r)^{-1})$ approximation to the nonconvex fractional quadratic optimization problem (14).

4. SIMULATIONS

In our numerical examples, we consider a network with $r = 20$ relay nodes. The channel coefficients f_i and g_j are assumed to be independent from each other for any i and j. It is also assumed that the channel coefficient f_i can be written as

$$f_i = \bar{f}_i + \tilde{f}_i$$

where $\bar{f}_i$ is the mean of f_i and $\tilde{f}_i$ is a zero-mean random variable. We assume that $\tilde{f}_i$ and $\tilde{f}_j$ are independent for $i \neq j$. For any f_i, we choose $\bar{f}_i = \frac{e^{j\theta_i}}{\sqrt{1+\alpha_f}}$ and $\operatorname{var}(\tilde{f}_i) = \frac{\alpha_f}{1+\alpha_f}$, where θ_i is a uniform random variable chosen from the interval $[0 \;\; 2\pi]$ and α_f is a parameter which determines the level of uncertainty in the channel coefficient f_i. Note that as $E\{|f_i|^2\} = 1$, if α_f is increased, the variance of the random component $\tilde{f}_i$ is increased while its mean is decreased. This, in turn, means that the level of the uncertainty in the channel coefficient f_i is increased. Similarly, we model the channel coefficient g_i as

$$g_i = \bar{g}_i + \tilde{g}_i$$

where $\bar{g}_i$ is the mean of g_i and $\tilde{g}_i$ is a zero-mean random variable. We assume that $\tilde{g}_i$ and $\tilde{g}_j$ are independent for $i \neq j$. For any g_i, we choose $\bar{g}_i = \frac{e^{j\phi_i}}{\sqrt{1+\alpha_g}}$ and $\operatorname{var}(\tilde{g}_i) = \frac{\alpha_g}{1+\alpha_g}$, where ϕ_i is a uniform random variable chosen from the interval $[0 \;\; 2\pi]$ and α_g is a parameter which determines the level of uncertainty in the channel

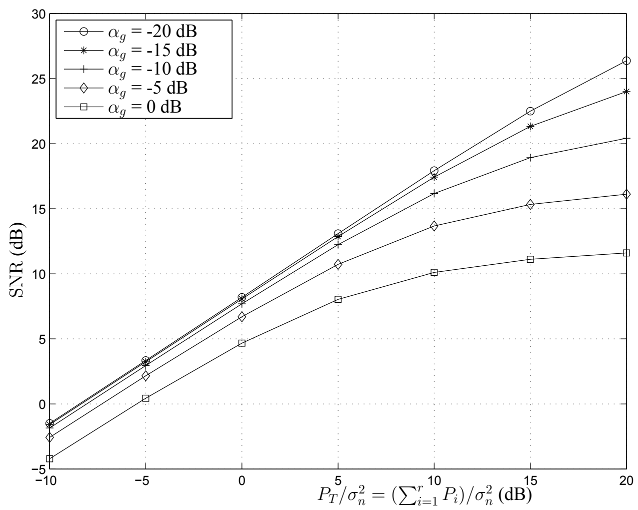

Fig. 1. Maximum achievable SNR versus the transmit power P_T for different values of α_g and for α_f = -5 dB.

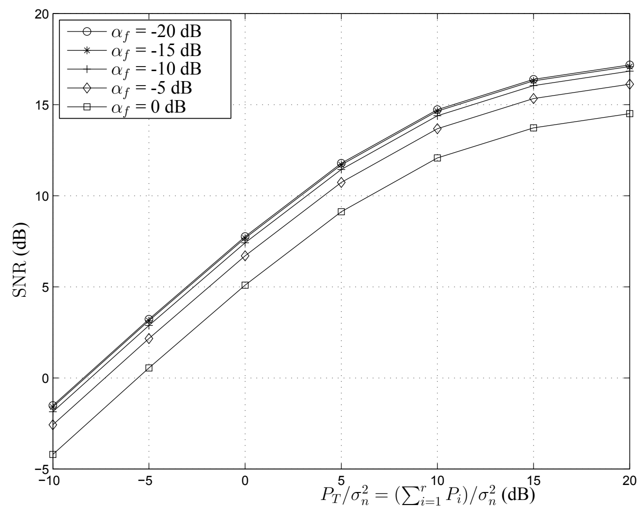

Fig. 2. Maximum achievable SNR versus the transmit power P_T for different values of α_f and for α_g = -5 dB.

coefficient g_i. Based on this channel modeling, we can write the (i, j) entry of the matrices $\mathbf{R}$ and $\mathbf{Q}$, respectively, as

$$
\begin{aligned}
[\mathbf{R}]_{i,j} &= P_0(\bar{f}_i\bar{f}_j^* + \frac{\alpha_f}{1+\alpha_f}\delta_{ij})(\bar{g}_i\bar{g}_j^* + \frac{\alpha_g}{1+\alpha_g}\delta_{ij}) \\
[\mathbf{Q}]_{i,j} &= \sigma_\nu^2(\bar{g}_i\bar{g}_j^* + \frac{\alpha_g}{1+\alpha_g}\delta_{ij})
\end{aligned}
$$

where δ_{ij} is the Kronecker function. Also, throughout our numerical examples, the transmit power P_0 is assumed to be the same as receiver noise power. We assume that the relay nodes are divided into two groups. The relay nodes in each group have the same maximum allowable transmit power, while the maximum allowable transmit power of one group is twice that of the other group, that is, $P_1 = P_2 = \cdots = P_{10} = 2P_{11} = 2P_{12} = \cdots = 2P_{20}$. We use our SDP based technique to obtain the optimum value for matrix $\mathbf{X}$, say $\mathbf{X}_{\rm opt}$. In our intensive simulation examples, we have observed that the matrix $\mathbf{X}_{\rm opt}$ is rank one, and therefore, no randomization technique is required. As a result, the optimum value for the vector $\mathbf{w}$ is the same as the principal eigenvector of $\mathbf{X}_{\rm opt}$. Figure 1 shows the maximum achievable SNRs, when the individual relay nodes have the aforementioned power constraints, versus the total relay transmit power $P_T/\sigma_n^2 = \sum_{i=1}^r P_i/\sigma_n^2$, for $\alpha_f = -5$ dB and for different values of α_g. Figure 2 illustrates the maximum achievable SNRs versus P_T for $\alpha_g = -5$ dB and for different values of α_f. As can be seen from Figures 1 and 2, for any given P_T, the maximum achievable SNR of our SDP based technique is decreased when the uncertainty in f_i (or in g_i) coefficients is increased.

It is also interesting to observe that for $\alpha_f = -5$ dB, the performance gap between the case when the channel coefficients g_i are (almost) perfectly known (i.e., $\alpha_g = -20$ dB) and the cases when these coefficients have significant variances (i.e., $\alpha_g \geq -10$ dB) is increased as P_T/σ_n^2 is increased. However, for $\alpha_g = -5$ dB, the performance gap between the case when the channel coefficients f_i are (almost) perfectly known (i.e., $\alpha_f = -20$ dB) and the cases when these coefficients have significant variances (i.e., $\alpha_f \geq -10$ dB) seems not to change significantly over the range of P_T/σ_n^2.

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