PERFORMANCE ANALYSIS OF A TWO-ELEMENT LINEARLY CONSTRAINED MINIMUM VARIANCE BEAMFORMER WITH SENSOR DELAY-LINE PROCESSING

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ABSTRACT

The bandwidth performance of a two-element linearly constrained minimum variance beamformer with sensor delay-lines (SDLs) attached is studied in terms of the directions of the interference signals, the inter-spacing between delay-line sensors and the length of the SDL. Compared with broadband beamformers with tapped delay-lines (TDLs), the SDL-based structure performs better in two ways: its output SINR drops less as the inter-delay within delay-lines increases and with the same number of delays and weights it can achieve a better performance than the TDL one.
Index Terms: Adaptive systems, array signal processing, delay lines

1. INTRODUCTION

Tapped delay-lines (TDLs) (or FIR/IIR filters in its discrete form) are often employed to improve the bandwidth performance of an adaptive beamformer [1, 2, 3, 4], where the length of the TDL, J, is dependent on the bandwidth of the impinging signals and the larger the bandwidth the more delay-line taps are required [5, 6]. The delays between taps decrease due to increasing signal bandwidth and frequency. As a result, very high speed analogue TDLs or digital sampling circuits have to be employed for signals with very high frequency and bandwidth.

As a solution, a new broadband beamforming structure was proposed, where the conventional TDLs are replaced by sensor delay-lines (SDLs) and correspondingly the required wired delays between taps in conventional broadband beamforming are replaced by spatial propagation delays between sensors of the SDLs [7]. For the case of a conventional broadband linear array system with TDLs, it will change to a planar array system without TDLs, as shown in Fig. 1. Note originally the conventional planar array system without TDLs is used for narrowband beamforming with steering capability in both the elevation and azimuth angles and now the structure in Fig. 1 is used for broadband beamforming with steering capability in the azimuth angle only, which is similar to that of the conventional broadband linear array system with TDLs. Since there is only one coefficient required for each of the received sensor signals and no temporal processing is required, it is a broadband beamforming system with spatial-only information.

In this paper, we will provide a detailed analysis of its bandwidth performance as a linearly constrained minimum variance (LCMV) beamformer [2]. In Section 2, the LCMV beamformer is briefly reviewed in the context of the SDL-based structure. The bandwidth performance of a two-element beamformer is then studied as an example in Section 3 and conclusions are drawn in Section 4.

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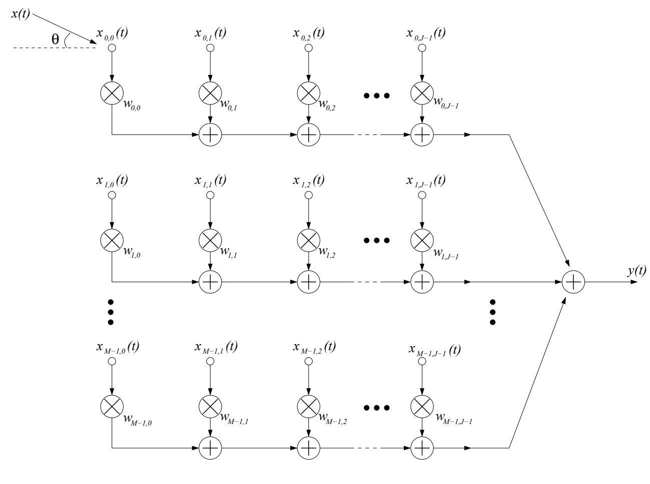

Fig. 1: A broadband beamformer with sensor delay-lines

2. LCMV BEAMFORMER WITH SENSOR DELAY-LINES

Consider a broadband beamformer with M original omnidirectional array sensors and each one is followed by a SDL of $J-1$ omnidirectional sensors, as shown in Fig. 1. Let $x_{m,j}(t)$ denote the signal received at the j-th sensor at the m-th SDL. We can then combine signals received from all J sensors at the m-th SDL into the *element* signal vector X_m. Furthermore, we can use the *total* signal vector to summarize all of the M element signal vectors. i.e.

$$X_m = [x_{m,0}(t), x_{m,1}(t), \cdots, x_{m,J-1}(t)]^T, \tag{1}$$

$$X = \begin{bmatrix} X_0^T & X_1^T & \cdots & X_{M-1}^T \end{bmatrix}^T. \tag{2}$$

Each sensor has a corresponding weight $w_{m,j}$. Then we have the corresponding *element* weight vector W_m and *total* weight vector W, as given by

$$W_m = [w_{m,0}, w_{m,1}, \cdots, w_{m,J-1}]^T, \tag{3}$$

$$W = \begin{bmatrix} W_0^T & W_1^T & \cdots & W_{M-1}^T \end{bmatrix}^T. \tag{4}$$

The LCMV beamformer [2] applies linear constraints on the beamformer weights so that signals from desired directions can pass with specified gain while contributions to the output variance (or power) due to interferences from directions other than desired ones are minimized,

$$\min_W W^H \Phi_x W \qquad \text{subject to} \qquad C^H W = f, \tag{5}$$

where $\{.\}^H$ denotes the Hermitian transpose, Φ_x is the signal correlation matrix ($MJ \times MJ$), C is the constraint matrix ($MJ \times J$) and f is the response vector ($J \times 1$). The solution to (5) can be computed using Lagrange multipliers [2],

$$W_{opt} = \Phi_x^{-1} C (C^H \Phi_x^{-1} C)^{-1} f. \tag{6}$$

We assume that the received signal at each sensor consists of three uncorrelated components: desired signal $d(t)$ from θ_d, interference signal $i(t)$ from θ_i and noise $n(t)$. Then the signal correlation matrix Φ_x can be decomposed into three matrices corresponding to the desired signal, interference and noise components, respectively. i.e.

$$\Phi_x = \Phi_d + \Phi_i + \Phi_n. \tag{7}$$

Assume $d(t)$ has a flat power spectral density of $2\pi p_d/\Delta\omega_d$ and a limited bandwidth $\Delta\omega_d$ centered at ω_o, the correlation function of the desired signal is then given by [4],

$$R_d(\tau) = p_d \text{sinc}\left(\frac{\Delta\omega_d \tau}{2}\right) e^{i\omega_0 \tau}, \tag{8}$$

where τ is the delay. Let T_e be the unit propagation delay between adjacent array elements and T_s be the unit propagation delay between adjacent delay-line sensors, the correlation of the desired signal at the (m, j)-th and the (n, k)-th sensor is then

$$[\Phi_{d_{m,n}}]_{j,k} = R_d[(m-n)T_e + (j-k)T_s]. \tag{9}$$

To avoid spatial aliasing the adjacent original array sensors (sensor (m,0) and sensor (m+1,0)) are set to be *half* a wavelength apart at the maximum signal frequency ω_{max} [3],

$$T_e = \frac{L}{c}\sin(\theta_d) = \frac{\pi}{\omega_{max}}\sin(\theta_d), \tag{10}$$

where θ_d is the direction of arrival (DOA) of the desired signal. The adjacent delay-line sensors are set to be r times a *quarter* wavelength apart at ω_{max},

$$T_s = \frac{D}{c}\cos(\theta_d) = \frac{\pi}{2\omega_{max}} r \cos(\theta_d). \tag{11}$$

Substituting (8) (10) and (11) into (9) we now have

$$[\Phi_{d_{m,n}}]_{j,k} = p_d \text{sinc}\left\{\frac{\Delta\omega_d}{2}[(m-n)T_e + (j-k)T_s]\right\} e^{i\omega_o[(m-n)T_e + (j-k)T_s]}. \tag{12}$$

By replacing the absolute bandwidth and center frequency with their relative counterparts $B_d = \Delta\omega_d/\omega_{max}$ and $\Omega_o = \omega_o/\omega_{max}$ respectively, (12) becomes

$$[\Phi_{d_{m,n}}]_{j,k} = p_d \text{sinc}\left\{\frac{B_d}{2}\tau_d\right\} e^{i\Omega_o \tau_d}, \tag{13}$$

where the delay τ_d is given by

$$\tau_d = \pi[(m-n)\sin(\theta_d) + (j-k)\frac{r}{2}\cos(\theta_d)]. \tag{14}$$

Φ_i can be determined in the same way by assuming $i(t)$ has a similar flat power spectral density as $d(t)$. As each sensor in the beamformer is an independent analog device, the noise signals are uncorrelated with each other. Φ_n is then a diagonal matrix with noise power σ^2 lying on the diagonal elements, i.e $\sigma^2 I$.

In the following study, we will assume $\theta_d = 0$. In this case, the constraint matrix C can be expressed as a combination of M identity matrices I_J ($J \times J$) [2],

$$C = \underbrace{[I_J \quad \cdots \quad I_J]}_{M}^T, \tag{15}$$

and f will be a vector with only one non-zero element, which is 1 for a distortion-less response to the desired signal,

$$f = \underbrace{[0 \quad \cdots \quad 1 \quad \cdots \quad 0]}_{J}^T. \tag{16}$$

The beamformer's output power is given by

$$P = E\left[\|y(t)\|^2\right] = W^H \Phi_x W. \tag{17}$$

With the optimized array weights, output powers due to the three signal components defined above now can be found as,

$$P_d = W_{opt}^H \Phi_d W_{opt} \tag{18}$$

$$P_i = W_{opt}^H \Phi_i W_{opt} \tag{19}$$

$$P_n = \sigma^2 W_{opt}^H W_{opt}\ . \tag{20}$$

Finally, the output signal to interference plus noise ratio (SINR) is

$$SINR = P_d/(P_i + P_n). \tag{21}$$

3. BANDWIDTH PERFORMANCE OF A TWO-ELEMENT BEAMFORMER WITH SENSOR DELAY-LINES

To examine the bandwidth performance of the proposed SDL structure we now consider a simple two-element LCMV beamformer receiving a desired signal from $\theta_d = 0$ and an arbitrary interference from θ_i. Both the desired signal and the interference have the same bandwidth and center frequency ω_o. Moreover, the input SIR and SNR are assumed to be -20dB and 20dB respectively.

Equation (11) indicates that the SDL introduces an FIR filter with a sampling period essentially depending on the DOA of the impinging signals for a given r. To understand the effect of θ_i on the bandwidth performance we shall examine the transfer function of a two-element beamformer fed with the interference only. The transfer function of the m-th SDL with $J-1$ delay-line sensors is

$$H_m(\omega) = w_{m,0} + w_{m,1}e^{-i\omega T_s} + \cdots + w_{m,J-1}e^{-i(J-1)\omega T_s}, \tag{22}$$

and the transfer function for the whole two-element beamformer is

$$H(\omega) = H_0(\omega) + H_1(\omega)e^{-i\omega T_e}, \tag{23}$$

where T_e and T_s are defined in (10) and (11) respectively but with θ_d being replaced by θ_i. In order to null the interference completely $H(\omega)$ must be zero over the whole signal bandwidth,

$$H_0(\omega) = -H_1(\omega)e^{-i\omega T_e}. \tag{24}$$

To meet (24) we must ensure that $H_0(\omega)$ and $H_1(\omega)$ have the identical amplitude response and a phase shift varying linearly with frequency over the signal bandwidth, i.e.

$$\|H_0(\omega)\| = \|H_1(\omega)\|, \tag{25}$$

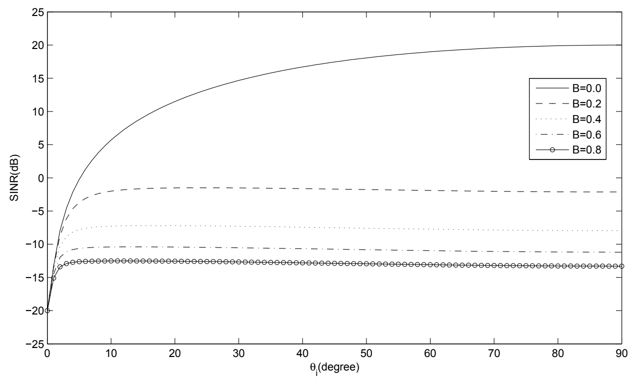

Fig. 2: Output SINR versus θ_i for a narrowband beamformer: $M = 2, J = 1$

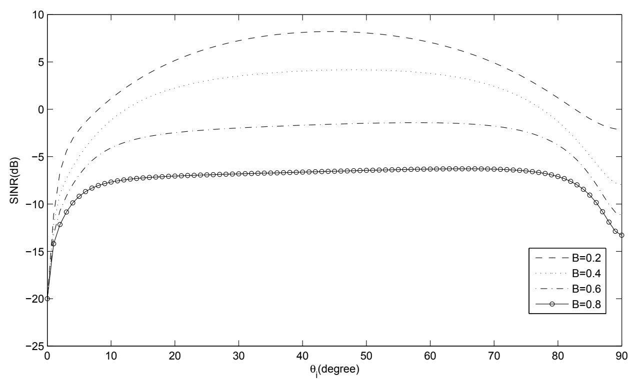

Fig. 3: Output SINR versus θ_i for the SDL-based structure: $M = 2, J = 2, r = 1$

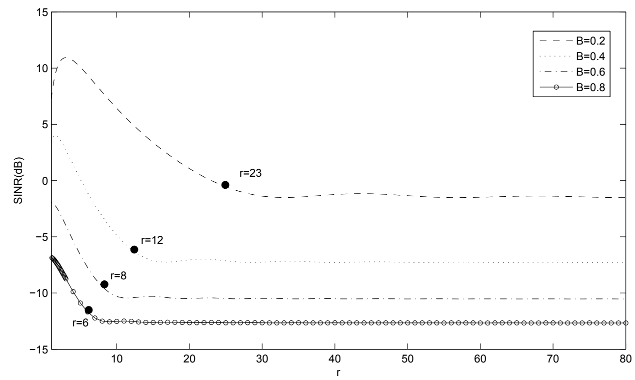

Fig. 4: Output SINR versus r for the SDL-based structure: $M = 2, J = 2, \theta_i = 30^o$

$$\angle H_0(\omega) = \angle H_1(\omega) - \pi - \omega T_e. \tag{26}$$

In a narrowband beamformer ($J = 1$) there is no delay line sensors attached and (24) becomes

$$w_{0,0} = -w_{1,0} e^{-i\omega T_s}, \tag{27}$$

which can only be satisfied at a single frequency ($B = 0$). As shown in Fig. 2, larger bandwidth rapidly worsens the output SINR. In contrast, with one single delay-line sensor ($J = 2$) attached to each original array sensor, $H_m(\omega)$ is then

$$H_m(\omega) = w_{m,0} + w_{m,1} e^{-i\omega T_s}. \tag{28}$$

The extra $e^{-i\omega T_s}$ term allows $H_0(\omega)$ and $H_1(\omega)$ to meet (24) over the signal bandwidth which yields a better bandwidth performance. As shown in Fig. 3 with one delay-line sensor being a quarter wavelength apart ($r = 1$) attached to each original array sensor the output SINR improves significantly.

Now let's consider the effect of the delay between delay-line sensors (or r) on the performance. In principle, to cancel the interference $H_0(\omega)$ and $H_1(\omega)$ must meet (24) over the signal bandwidth regardless of r. However $H_m(\omega)$ is a periodic function with a period depending on both r and θ_i,

$$\Omega = \frac{2\pi}{T_s} = \frac{4\omega_{max}}{r\cos(\theta_i)}. \tag{29}$$

With the $\cos(\theta_i)$ term the period Ω is limited from $4\omega_{max}/r$ to $+\infty$. For large r values the lower bound of Ω is much smaller than the signal bandwidth. But lower bound will increase as r decreases and it will finally equal to the signal bandwidth when $r = 4/(B\cos(\theta_i))$. If r is small enough and $4\omega_{max}/r$ is large, then Ω will always be much larger than the signal bandwidth. Therefore the phase shift $\angle H_0(\omega) - \angle H_1(\omega)$ can be linear. As r increases the lower bound of Ω approaches the signal bandwidth and it gets more difficult for $H_0(\omega)$ and $H_1(\omega)$ to satisfy (24) within the signal bandwidth and the beamformer performance drops. When r exceeds its critical value $4/(B\cos(\theta_i))$, $4\omega_{max}/r$ eventually drops below the signal bandwidth and $\angle H_0(\omega) - \angle H_1(\omega)$ may repeat periodically within the signal bandwidth (depending on the value of θ_i) therefore (24) cannot be met any more. At this point there will be little change to a poor bandwidth performance with large r. Above discussions are illustrated in Fig. 4.

Moreover, in the study of SINR versus the DOA of the interferences (Fig. 5) it is found that as θ_i increases the SINR tends to rise at first but then falls rapidly when θ_i is very close to 90^o regardless of r values. These can be understood when we consider the delay τ_i. From (11) it is clear that as $\theta_i \rightarrow 90^o$ T_s approaches 0 and the attached SDL fails to null the broadband interference. On the contrary, T_e in (10) increases significantly regardless of r as T_s decreases. Thus τ_i and equivalently the correlation of the interference within the whole beamformer is restored and the beamformer is still able to cancel the interference even without the help of SDL at large θ_i values. However once θ_i gets very close to 90^o the beamformer seen by the interference becomes a narrowband one and the constraints on weights now must be taken into account. With C and f, weights are chosen to ensure that only signals from the broadside with no delay can pass through the beamformer with a unit response ($f_0 = 1$) while all the others will be attenuated, i.e.

$$\begin{cases} w_{0,0} + w_{1,0} = 1 \\ w_{0,1} + w_{1,1} = 0 \end{cases}. \tag{30}$$

When the broadband beamformer seen by the interference approximates a narrowband one as $\theta_i \rightarrow 90^o$, each weight of the equivalent narrowband beamformer is the sum of weights in the corresponding delay line,

$$\begin{cases} w_0 = w_{0,0} + w_{0,1} \\ w_1 = w_{1,0} + w_{1,1} \end{cases}. \tag{31}$$

Since they must follow the constraint on optimal weights selection, we have

$$w_0 + w_1 = 1. \tag{32}$$

Therefore the performance drops to the level of the corresponding narrowband LCMV beamformer.

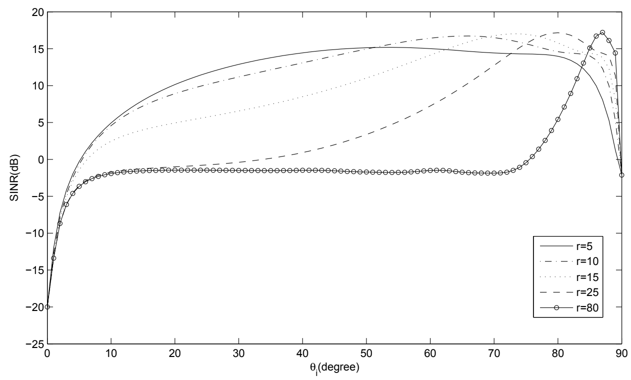

Fig. 5: Output SINR versus θ_i for the SDL-based structure: $M = 2, J = 2, B = 0.2, r \in (5, 80)$

We shall now examine the effect of adding extra delay-line sensors on the bandwidth performance. A signal with non-zero bandwidth can remain correlated with itself for a time shift up to $1/B$ in the TDL system [4]. We can apply this result to our SDL case and expect that adding delay-line sensors will be effective only if T_s is short compared to $1/B$. If T_s is too large due to large r values the interference fails to remain correlated within the delay-line and the beamformer is unable to cancel it further in spite of the extra number of delay-line sensors added.

As shown in Fig. 6, for very large values of r, there is almost no any improvement when extra delay-line sensors are added. The only region that the additional delay-line sensor can improve the beamformer performance effectively is $r < 4/(B\cos(\theta_i))$. Within this range the lower bound of Ω is larger than the signal bandwidth. Adding extra delay-line sensors introduces more free terms which leads to a more linear phase shift and a better ability of nulling interference. However the SINR can only be improved significantly with the first few sensors added. Once it reaches a certain level the effect of extra sensors can be ignored.

As a comparison, the change of the output SINR with respect to the number of delay-line taps in the TDL system are shown in Fig. 7. Note in the TDL case r stands for the number of a quarter wavelength delay measured at ω_{max}. By comparing SINR results from both the TDL and SDL structures, we can conclude that the later one performs better in two aspects. Firstly, as r, or the unit delay within a delay-line, increases, the output SINR of the SDL beamformer drops less than the TDL one. Secondly, with the same number of delays and weights the SDL beamformer generally can achieve a better optimal SINR output than the TDL one.

4. CONCLUSION

A detailed analysis of the bandwidth performance of a LCMV broadband beamforming structure with SDL processing has been provided in terms of the DOA of the interference signals, the inter-spacing between delay-line sensors and the length of the SDLs. Compared with the TDL-based beamformer, the SDL-based beamforming structure performs better in two main aspects: its output SINR drops less as the inter-delay within the SDLs increases and a better output SINR can be archived using the same number of delay-line sensors and weights as the TDL one.

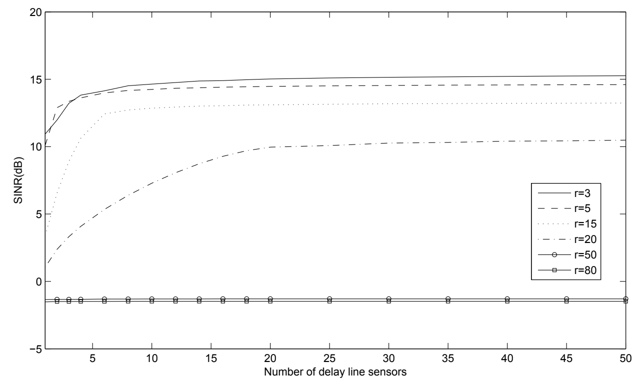

Fig. 6: Output SINR versus J for the SDL-based structure: $M = 2, \theta_i = 30^o, B = 0.2, r \in (3, 80)$

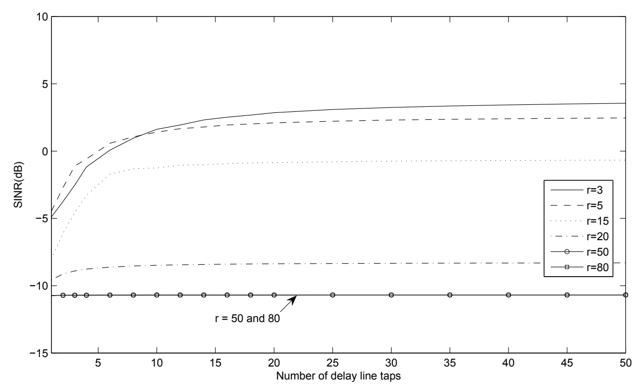

Fig. 7: Output SINR versus J for the TDL-based structure: $M = 2, \theta_i = 30^o, B = 0.2, r \in (3, 80)$

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