Low-Complexity Adaptive Step Size Constrained Constant Modulus SG-based Algorithms for Blind Adaptive Beamforming

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***Abstract*— In this paper, two low-complexity adaptive step size algorithms are investigated for blind adaptive beamforming. Both of them are used in a stochastic gradient (SG) algorithm, which employs the constrained constant modulus (CCM) criterion as the design approach. A brief analysis is given for illustrating their properties. Simulations are performed to compare the performances of the novel algorithms with other well-known methods. Results indicate that the proposed algorithms achieve superior performance, better convergence behavior and lower computational complexity in both stationary and non-stationary environments.**

***Index Terms*–Blind adaptive beamforming techniques, constrained constant modulus (CCM), modified adaptive step size (MASS), time averaging adaptive step size(TAASS).**

I. INTRODUCTION

In recent years, many adaptive filtering algorithms have been used for beamforming, which is a promising and widely investigated technology for rejecting interference and improving the performance of high capacity mobile communications systems [1]. Many methods have been presented in different communication systems [2]-[4]. In contrast to fixed beamforming techniques, an adaptive beamformer has the ability of rejecting interference and pointing its mainbeam in the desired direction with the change of scenarios. Blind adaptive beamforming, which is intended to form the array direction response without knowing users' information beforehand, is an important topic that deals with interference cancellation, tracking improvement and complexity reduction.

The blind adaptive SG method, which is commonly employed in the blind adaptive beamforming area, is a well-known technique for solving optimization problems with different criteria, e.g., minimum mean squared error (MMSE) [5], minimum variance (MV) [6] and constant modulus (CM) [7]. The MV algorithm is a computational efficient approach to estimate the input covariance matrix. The results in [6] prove that the MV criterion leads to a solution identical to that obtained from the minimization of the mean squared error (MSE). The CM algorithm for beamforming exploits the low modulus fluctuation exhibited by communications signals using constant modulus constellations to extract them from the array input. It is well known that the performance of the CM method is superior to that of the MV. A disadvantage of both two methods is that they are sensitive to the step size. The small value of the step size will lead to slow convergence rate, whereas a large one will lead to high misadjustment or even instability. Besides, the CM cost function may have local minima, and CM receivers do not have closed-form solutions. Xu and Liu [8] developed a SG algorithm on the basis of the CCM technique to sort out the local minimum problem and obtain the global minima. But the problem incurred by the step size cannot be solved properly.

For accelerating the convergence, recursive least squares (RLS) algorithms were introduced by Xu and Tsatsanis using the constrained minimum variance (CMV) criterion [9] and then developed by de Lamare and Sampaio-Neto with the CCM approach [10]. The latter, which improves the performance significantly, optimizes a quadratic cost function based on the CM criterion subject to linear constraints for the array weight adaptation. Combining with the constrained condition, this method reaches an optimal solution. Nevertheless, the RLS based beamformers cannot avoid complicated computations caused by the required correlation matrix inversion.

Comparing SG algorithms, which represent simple and low-complexity solutions but subject to slow convergence, with RLS methods, which possess fast convergence but high computational load, it is preferable to adopt SG beamformers due to complexity and cost issues. For this reason, the improvement of blind SG techniques is an important topic and has been investigated for several decades. In this research area, the works in [6] and [9] employ standard SG methods with fixed step size (FSS) that are not efficient with respect to convergence and steady-state performance. The performance of the beamformer is strongly dependent on the choice of the step size [11]. It reflects a tradeoff between misadjustment and the convergence rate. Actually, the communication systems are non-stationary environments, which make it very difficult to predetermine the step size. It is necessary to make the step size track the change of the system automatically and so obtain good convergence behavior. Previous researches have focused on this aim and some good results have been reported. The adaptive step size (ASS) mechanism was employed in both the MV [12], [13] and the CM [14] criteria for improving the performance. Because of requiring an additional update equation for the gradient of the weight vector with respect to the step size, which increases the extra computational load, the applications of these algorithms are limited. The authors of [15] propose two novel variable step size mechanisms for MV algorithms. The simulation experiments show that the new mechanisms are superior to previously reported methods and have a reduced complexity.

This paper proposes two blind CCM beamformers based on two novel adaptive step size mechanisms. The origins of these mechanisms can be traced back to the works of [16] and [17] where low-complexity adaptive step size mechanisms were developed for LMS algorithms. In contrast to [15]-[17], the mechanisms here are designed for CCM algorithms, since it is well-known that they outperform the CMV algorithms [8]. Because of this reason, the simulations here just compare the algorithms related to the CCM criterion. The additional number of operations of the proposed algorithms does not depend on the number of sensor elements. In addition, the results are presented for stationary and non-stationary environments, proving that the new mechanisms could reach better performance and faster convergence behavior than those of previous methods.

The remaining of this paper is organized as follows. In the next section, we present a system model for smart antennas. Based on this model, the blind adaptive CCM beamformer design using the SG method is presented in Section III. In Section IV, the proposed adaptive step size mechanisms are derived. Simulation results are provided in Section V, and conclusions are drawn in Section VI.

II. SYSTEM MODEL

In order to describe the system structure, let us make two simplifying assumptions for the transmitter and receiver models [18]. First, the propagating signals are assumed to be produced by point sources; that is, the size of the source is small with respect to the distance between the source and the sensors that measure the signal. Second, the sources are assumed to be in the "far field," namely, at a large distance from the sensor array, so that the spherically propagating wave can be reasonably approximated with a plane wave. Besides, we assume a lossless, nondispersive propagation medium, i.e., a medium that does not attenuate the propagating signal further and the propagation speed is uniform so that the waves travel smoothly.

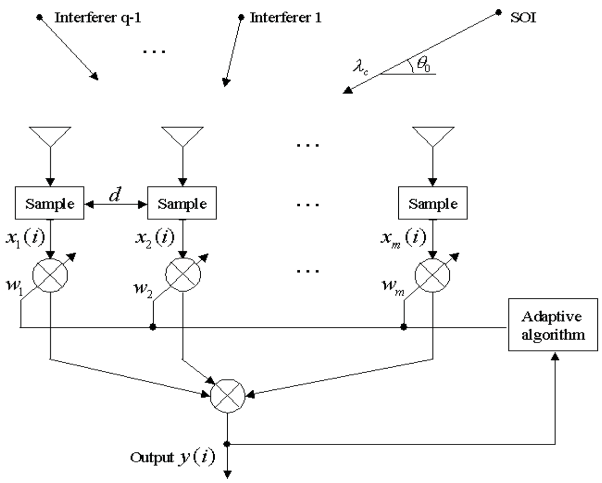

Fig. 1. Adaptive beamforming structure for ULA.

Let us consider the adaptive beamforming scheme in Fig. 1 and suppose that q narrowband signals impinge on the uniform linear array (ULA) of m ($q \leq m$) sensor elements from the sources with unknown directions of arrival (DOAs) θ_0,...,θ_{q-1}. The ith snapshot's vector of sensor array outputs can be modeled as [19]

$$\boldsymbol{x}(i) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(i) + \boldsymbol{n}(i), \quad i = 1, \ldots, N \tag{1}$$

where $\boldsymbol{\theta} = [\theta_0, \ldots, \theta_{q-1}]^T \in \mathcal{C}^{q\times 1}$ is the vector of the unknown signal DOAs, $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_0), \ldots, \boldsymbol{a}(\theta_{q-1})] \in \mathcal{C}^{m\times q}$ is the complex matrix composed of the signal direction vectors $\boldsymbol{a}(\theta_k) = [1, e^{-2\pi j \frac{d}{\lambda_c} cos\theta_k}, \ldots, e^{-2\pi j(m-1)\frac{d}{\lambda_c} cos\theta_k}]^T \in \mathcal{C}^{m\times 1}$, $(k = 0, \ldots, q-1)$, where λ_c is the wavelength and $d = \lambda_c/2$ is the inter-element distance of the ULA, $\boldsymbol{s}(i) \in \mathcal{R}^{q\times 1}$ is the real value vector of the source data, $\boldsymbol{n}(i) \in \mathcal{C}^{m\times 1}$ is the complex vector of white sensor noise, which is assumed to be a zero-mean spatially and white Gaussian process, N is the number of snapshots, and $(\cdot)^T$ stands for the transpose. The output of a narrowband beamformer is given by

$$y(i) = \boldsymbol{w}(i)^H \boldsymbol{x}(i) \tag{2}$$

where $\boldsymbol{w}(i) = [w_1(i), \ldots, w_m(i)]^T \in \mathcal{C}^{m\times 1}$ is the complex weight vector, and $(\cdot)^H$ stands for the Hermitian transpose.

III. Blind Adaptive CCM Algorithms

The purpose of SG algorithms is to get an acceptable output performance and reduce the complexity load by avoiding the correlation matrix estimation and inversion. We describe the CCM algorithm on the basis of the SG method, which is called CCM-SG.

For the CCM-SG algorithm, we consider the cost function as the expected deviation of the squared modulus of the array output to a constant, say 1. The CCM cost function is simply a positive measure of the average amount that the beamformer output deviates from the unit modulus condition [4]. By using the constraint condition, the cost function of CCM-SG can be expressed as

$$\begin{aligned} &J_{CM} = (|y(i)|^2 - 1)^2, \quad i = 1, \ldots, N \\ &\text{subject to} \quad \boldsymbol{w}^H(i)\boldsymbol{a}(\theta_0) = 1 \end{aligned} \tag{3}$$

where $\boldsymbol{a}(\theta_0)$ denotes the steering vector of the desired signal. The constrained optimization means that the technique minimizes the contribution of the undesired interferences while maintaining the gain along the look direction to be constant.

The SG blind adaptive algorithm optimizes the Lagrangian cost function described by

$$L_{CCM} = (|y(i)|^2 - 1)^2 + \lambda(\boldsymbol{w}^H(i)\boldsymbol{a}(\theta_0) - 1) \tag{4}$$

where λ is a scalar Lagrange multiplier. The solution can be obtained by setting the gradient terms of (4) with respect to $\boldsymbol{w}$(i) equal to zero and using the constraint. Thus, we obtain

$$\begin{aligned} \boldsymbol{w}(i+1) = \boldsymbol{w}(i) - \mu(|y(i)|^2 - 1)y^*(i) \\ \cdot [\boldsymbol{x}(i) - \boldsymbol{a}^H(\theta_0)\boldsymbol{x}(i)\boldsymbol{a}(\theta_0)] \end{aligned} \tag{5}$$

where μ here is the step size, which is a fixed value for FSS and a variable value for ASS and $*$ denotes complex conjugate.

Because of the shortcomings introduced before for both FSS and ASS algorithms, it is necessary to develop other methods for improving the performance of SG method.

IV. Proposed Adaptive Step Size Mechanisms

In this section, two novel adaptive step size methods are described for adjusting the step size following the change of the communication system. The step size adjustment is controlled by the square prediction error, which means that a large error will cause the step size to increase for providing fast tracking while a small error will result in a decrease of step size to yield smaller misadjustment. The computational complexity is not a problem in these mechanisms.

A. *Modified Adaptive Step Size (MASS) Mechanism*

The first proposed algorithm based on the MASS mechanism employs the prediction error and uses the update rule

$$\mu(i+1) = \alpha\mu(i) + \gamma(|y(i)|^2 - 1)^2 \tag{6}$$

where $0 < \alpha < 1$, $\gamma > 0$ and $y(i)$ is the same as that in (2). The rationale for the MASS is that for large prediction error the algorithm will make the step size increase to track the change of the system whereas a small error will result in a decrease of the step size. The parameter γ is an independent variable for controlling the prediction error and scaling it at different levels. It is worth pointing out that the step size $\mu(i+1)$ should be limited in a range as follows

$$\mu(i+1) = \begin{cases} \mu_{max} & \text{if } \mu(i+1) > \mu_{max} \\ \mu_{min} & \text{if } \mu(i+1) < \mu_{min} \\ \mu(i+1) & \text{otherwise} \end{cases} \tag{7}$$

where $0 < \mu_{min} < \mu_{max}$. The constant μ_{min} is chosen as a compromise between the satisfying level of steady-state misadjustment and the required minimum level of tracking ability while μ_{max} is normally selected close to the point of instability of the algorithm for providing the maximum convergence speed. The MASS is the result of several attempts to devise a simple and effective mechanism.

TABLE I
ADDITIONAL COMPUTATIONAL COMPLEXITY OF PROPOSED ALGORITHMS

Proposed Algorithms	Number of operations per snapshot	
	Additions	Multiplications
MASS	1	3
TAASS	2	6

B. Time Averaging Adaptive Step Size (TAASS) Mechanism

The second mechanism, which is called TAASS, uses a time average estimate of the correlation of $(|y(i)|^2-1)$ and $(|y(i-1)|^2-1)$. The update rule is

$$\mu(i+1)=\alpha\mu(i)+\gamma v^2(i) \tag{8}$$

where $v(i)=\beta v(i-1)+(1-\beta)(|y(i)|^2-1)(|y(i-1)|^2-1)$ and $0<\beta<1$. The limits on $\mu(i+1)$, α and γ are similar to those of the MASS algorithm. The exponential weighting parameter β governs the averaging time constant, namely, the quality of the estimation. Previous samples, in stationary environments, contain information that is related to determine an accurate measure for the proximity of the adaptive beamformer coefficients to the optimal ones. Here, β should be close to 1. For non-stationary environments, the time averaging window should be small for deleting the deep past data and leaving space for the current statistics adaption, so, $\beta<1$.

There are two objectives for using $v(i)$ here. First, it rejects the effect of the uncorrelated noise sequence on the step-size update [17]. In the beginning, because of scarcity of transmitters' information, the error correlation estimate $v^2(i)$ is large and so $\mu(i)$ is large to increase the convergence rate and to track the change of input data. As it approaches the optimum, $v^2(i)$ is very small , resulting in a small step size for ensuring low misadjustment near optimum. Second, the error correlation is generally a good measure of the proximity to the optimum.

C. Computational Complexity and Convergence Analysis

In this part, both the computational complexity and the convergence behavior of the proposed mechanisms based on the CCM criterion are investigated.

1) Computational Complexity: The computational complexities of the proposed MASS and TAASS mechanisms are analyzed. It is well known that the computational complexity of the ASS algorithm is a linear monotonic increasing function of the number of sensor elements (in AWGN model), i.e., the complexity will increase following the increase of the number of sensor elements. Therefore, the computational complexity becomes very large if the array size is big.

An important feature of the proposed algorithms is that they only require a few fixed number of operations for updating the step size compared with that of the ASS method, which is proportional to the number of sensor elements. The additional computational complexity of the proposed adaptive step size mechanisms is listed in Table I.

2) Convergence Analysis: Considering the space limitation, we just give the range of the step size for convergence. For further analysis, we assume that $\mu(i)$ varies slowly around its mean value. This assumption is approximately true if γ is small and α closes to one, which will be shown in the simulations. Under this condition, according to [15], we have

$$\begin{aligned}&E[\mu(i)(|y(i)|^2-1)y^*(i)[\boldsymbol{x}(i)-\boldsymbol{a}^H(\theta_0)\boldsymbol{x}(i)\boldsymbol{a}(\theta_0)]]\\&=E[\mu(i)]E[(|y(i)|^2-1)y^*(i)[\boldsymbol{x}(i)-\boldsymbol{a}^H(\theta_0)\boldsymbol{x}(i)\boldsymbol{a}(\theta_0)]]\end{aligned} \tag{9}$$

and

$$E[\mu(i)(|y(i)|^2-1)\boldsymbol{x}(i)\boldsymbol{x}^H(i)]\boldsymbol{w}(i)=E[\mu(i)]\boldsymbol{R}_{CCM}\boldsymbol{w}(i) \tag{10}$$

where $\boldsymbol{R}_{CCM}=E[(|y(i)|^2-1)\boldsymbol{x}(i)\boldsymbol{x}^H(i)]\in\mathcal{C}^{m\times m}$.

Now, the weight vector update equation (5) of the blind adaptive CCM beamformer can be written as

$$\boldsymbol{w}(i+1)=(\boldsymbol{I}-\mu(i)(|y(i)|^2-1)\boldsymbol{v}(i)\boldsymbol{x}^H(i))\boldsymbol{w}(i) \tag{11}$$

where $\boldsymbol{v}(i)=(\boldsymbol{I}-\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0))\boldsymbol{x}(i)\in\mathcal{C}^{m\times 1}$.

Define the weight error vector $\boldsymbol{e}_w(i)$ and substitute (11) into the expression, we get

$$\begin{aligned}\boldsymbol{e}_w(i+1)&=\boldsymbol{w}(i+1)-\boldsymbol{w}_{opt}\\&=(\boldsymbol{I}-\mu(i)(|y(i)|^2-1)\boldsymbol{v}(i)\boldsymbol{x}^H(i))\boldsymbol{e}_w(i)\\&\quad-\mu(i)(|y(i)|^2-1)\boldsymbol{v}(i)\boldsymbol{x}^H(i)\boldsymbol{w}_{opt}\end{aligned} \tag{12}$$

Employing (9) and (10) and taking expectations on both sides of (12), we get

$$E[\boldsymbol{e}_w(i+1)]=(\boldsymbol{I}-E[\mu(i)]\boldsymbol{R}_{vx}(i))E[\boldsymbol{e}_w(i)] \tag{13}$$

where $\boldsymbol{R}_{vx}(i)=E[(|y(i)|^2-1)\boldsymbol{v}(i)\boldsymbol{x}^H(i)]=(\boldsymbol{I}-\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0))\boldsymbol{R}_{CCM}$ and $\boldsymbol{R}_{vx}\boldsymbol{w}_{opt}=\boldsymbol{0}$. Therefore, $E[\boldsymbol{w}(i)]\to\boldsymbol{w}_{opt}$ or equivalently, $\lim_{i\to\infty}E[\boldsymbol{e}_w(i)=\boldsymbol{0}]$ represents the stable condition if and only if $\prod_{i=0}^{\infty}(\boldsymbol{I}-E[\mu(i)]\boldsymbol{R}_{vx})\to 0$. Following the idea of the eigenstructure [11] with respect to the correlation matrix $\boldsymbol{R}_{vx}$, the sufficient condition for (13) to hold implies that

$$0\le E[\mu(\infty)]\le\frac{2}{\lambda_{max}^{vx}} \tag{14}$$

where λ_{max}^{vx} is the maximum eigenvalue of $\boldsymbol{R}_{vx}$.

V. SIMULATIONS

The performances of the proposed MASS and TAASS algorithms are compared with other existing algorithms, namely FSS and ASS, in terms of output signal-to-interference-plus-noise ratio (SINR). An ULA containing $m=16$ sensor elements with half-wavelength spacing is considered. The noise is spatially and temporally white Gaussian noise with power $\sigma_n^2=0.01$. For each scenario, $K=1000$ iterations are used to get each simulated curve. In all simulations, the desired signal power is $\sigma_0^2=1$. The BPSK modulation scheme is employed to modulate the signals.

Fig. 2 includes two experiments. Fig. 2(a) shows the output SINR of each method versus the number of snapshots, whose total is 1000 samples. There are five interferers in the system, one interferer with 4 dB above the desired user's power level, one with the same power level of the desired one and three with power 0.5 dB lower than that of the desired user. In this environment, the actual spatial signature of the signal is known exactly. We set the first element of the initial weight vector $\boldsymbol{w}(0)$ equals to the corresponding element of steering vector of SOI $\boldsymbol{a}(\theta_0)$. Other parameters are optimized with $\alpha=0.98$, $\gamma=10^{-3}$, $\mu_0=10^{-5}$, $\mu_{max}=10^{-4}$ and $\mu_{min}=10^{-6}$ for MASS and $\alpha=0.98$, $\beta=0.99$,$\gamma=10^{-3}$, $\mu_0=10^{-4}$, $\mu_{max}=3\times10^{-4}$ and $\mu_{min}=10^{-6}$ for TAASS. We claim that the parameters for the FSS and ASS are tuned in order to minimize the performance, allowing for a fair comparison with the proposed algorithms. The results show that the proposed algorithms converge faster and have better performances than the existing algorithms. The steering vector mismatch scenario is shown in Fig. 2(b). We assume that this steering vector error problem is caused by look direction mismatch [20]. The assumed DOA of the SOI is a constant value 2^o away from the actual direction. Compared with Fig. 2(a), Fig. 2(b) indicates that the mismatch problem leads to a worse performance for all the solutions. The convergence rate of all the methods reduces whereas the devised algorithms are more robust to this mismatch, especially for the TAASS approach, which reaches the steady-state very quickly.

In Fig. 3, The system starts with 4 interferers, two of which have the same power as that of the desired signal and the rest of them with the power 0.5 dB lower than the desired one. Two more users with one of them 2 dB above the desired user's power level and the other 0.5 dB lower than that of the desired user, enter the system at 1000 symbols. In this condition, the parameters are set to the same values as those of the previous experiment except $\mu_{max} = 3 \times 10^{-3}$ for MASS and $\mu_{max} = 5 \times 10^{-3}$ for TAASS due to optimization. As can be seen from the figure, SINRs of all the algorithms reduce at the same time. It is clear that the performance degradation of the proposed ones is much less significant than those of the other methods. In addition, MASS and TAASS methods can quickly track the change and recover to a steady-state. This figure illustrates that the proposed algorithms have faster convergence than the reported methods even though they are less complex. The experiment shows that the proposed techniques exhibit better performance after an abrupt change, in a non-stationary environment where the number of users/interferers suddenly changes in the system.

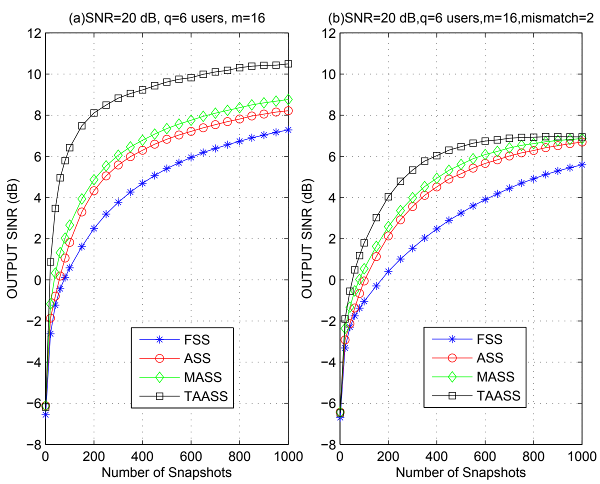

Fig. 2. Output SINR versus number of snapshots for (a) ideal steering vector condition (b) steering vector with mismatch.

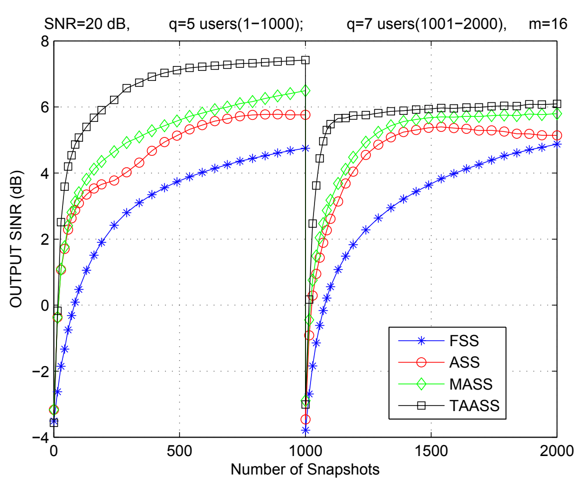

Fig. 3. Output SINR in a scenario where additional interferers suddenly enter and/or leave the system.

VI. Conclusions

In this paper, two novel adaptive step size mechanisms employing SG algorithms have been presented to enhance the performance, improve the convergence property and reduce the computational load of the previously proposed adaptive methods for blind adaptive beamforming. We considered different scenarios to compare the proposed MASS and TAASS algorithms with several existing algorithms. Simulation experiments were conducted to investigate the output SINR. The performances of our new methods are shown to be superior to those of others, both in terms of convergence rate and performance under sudden change in the signal environment even though they are less complex. A complete convergence analysis of the proposed algorithms is under preparation.

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