#### **RADAR CLUTTER MITIGATION VIA SPACE-TIME WAVEFRONT ADAPTIVE SENSING**

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# ABSTRACT

Space-time adaptive processing (STAP) in complex radar propagation and inhomogeneous clutter environments is often precluded because neither the target wavefront is sufficiently known nor is signal-free training data available. In recent work, wavefront adaptive sensing (WAS) was proposed to overcome these challenges by combining minimum variance (MV) adaptive processing and blind source separation (BSS) for distributed sources. In this paper, WAS is compared with conventional BSS and MV processing both analytically and via simulation. In an overthe-horizon radar (OTHR) spread-Doppler clutter environment, WAS is shown to avoid MV signal cancellation at high SNR and poor BSS threshold performance at low SNR.

*Index Terms* — Adaptive signal processing, Array signal processing, HF radar, Radar clutter, Ionospheric electromagnetic propagation.

## **1. INTRODUCTION**

Spatially inhomogeneous and Doppler-spread clutter compromises target detection performance of radar systems. Classical space-time adaptive processing (STAP) methods achieve clutter suppression by estimating a clutter-plusnoise covariance matrix from "signal-free" training data. Signal-free snapshots are typically obtained by using data from range bins which are identically distributed but well separated from the hypothesized target range bin "under test". In highly inhomogeneous environments, such as HF over-the-horizon radar (OTHR), the clutter statistics change significantly across neighboring range bins [1,2]. An alternative approach is to estimate the signal-plus-clutterplus-noise covariance matrix at the range bin under test and employ minimum variance distortionless response (MVDR) adaptive beamforming [3,4].

This paper addresses a signal wavefront mismatch scenario, where blind source separation (BSS) [5] and a new hybrid approach, called wavefront adaptive sensing (WAS) were compared to the conventional MVDR. The proposed WAS uses BSS to estimate the strong clutter wavefronts and utilizes a conventional, possibly mismatched, steering vector for the target. The idea behind WAS is to use BSS only on strong components in the data, i.e. the clutter. The novelty of WAS is that BSS is used to estimate the wavefronts of strong spatially distributed clutter while conventional steering vectors model the usually much weaker point target returns. Simulations show that WAS outperforms MVDR in mismatched scenarios.

# 2. STAP MODELING FOR OTH RADAR

Multipath OTHR clutter that arises due to propagation in ionosphere is modeled in this section. Consider a uniformlyspaced linear array (ULA) antenna with *N*, *d*-spaced elements. Let  $\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_M]$  be the space-time snapshot of size  $N \times M$ , sampled at the particular range gate, where  $\mathbf{x}_m, m = 1, ..., M$  is a spatial snapshot at *m*th sample. Using the STAP notation, define the space-time snapshot:  $\boldsymbol{\chi} = vec(\mathbf{X})$  of size  $NM \times 1$  which contains noise, clutter, and possibly a target. The radar echo from a point scatterer, such as a target, with amplitude  $\alpha$ , elevation angle  $\phi$ , and bearing angle  $\theta$ , can be expressed as:

$$\boldsymbol{\chi} = \alpha \mathbf{b}(\boldsymbol{\omega}) \otimes \mathbf{a}(\boldsymbol{\theta}, \boldsymbol{\phi}), \tag{1}$$

where  $\otimes$ stands for Kronecker product,  $\mathbf{b}(\omega) = \left\{ b_m(\omega) = e^{j\omega_s m}, m = 1, ...M \right\}$  is temporal steering vector with Doppler angular frequency  $\omega_d = 2\pi f_d T_r$ , where  $f_d$  is Doppler frequency and  $T_r$  is pulse repetition interval (PRI), and  $\mathbf{a}(\theta, \phi) = \{a_n(\theta, \phi) = e^{jkn}, n = 1, ...N\}$ , is spatial vector with wave-number steering frequency  $k_t = 2\pi \frac{d}{\lambda} \cos \phi \sin \theta$ , and wavelength  $\lambda_0$ . The noise  $\chi_n$ is assumed spatially and temporally white with covariance matrix:  $\tilde{\mathbf{R}}_{n} = E\left[\boldsymbol{\chi}_{n}\boldsymbol{\chi}_{n}^{T}\right] = \sigma_{n}^{2}\mathbf{I}$ , where **I** is an identity matrix.

At HF frequencies, assuming no ionospheric Doppler modulation, the sea clutter patch from azimuth angle  $\theta_i$ , i = 1, ..., I, and elevation angle  $\phi_j$ , j = 1, ..., J with a

Doppler spectrum consisting of multiple Bragg lines can be expressed as:

$$\boldsymbol{\chi}_{c}(\boldsymbol{\theta}_{i},\boldsymbol{\phi}_{j}) = \mathbf{H}(\boldsymbol{\omega})\boldsymbol{\alpha}_{ij} \otimes \mathbf{a}(\boldsymbol{\theta}_{i},\boldsymbol{\phi}_{j}), \qquad (2)$$

where  $\mathbf{a}_{ij} \forall i = 1, ..., I$ , j = 1, ..., J are zero-mean complex Gaussian-distributed random variables with  $\mathbf{a}_{ij} \sim N^c(\mathbf{0}, \xi \mathbf{I})$ , and  $\mathbf{H}(\boldsymbol{\omega})$  contains coefficients of the Bragg-line sea clutter spectrum which is constant over all azimuth and elevation angles. The ionospheric modulation of each clutter patch is modeled here as a complex zeromean Gaussian-distributed modulation function that in general can vary across elevation (or mode) and azimuth, i.e.  $\gamma_{ij} \sim N^c(\mathbf{0}, \Gamma_{ij})$ . The received modulated radar echo from a sea clutter patch is then:

$$\boldsymbol{\chi}_{c}(\boldsymbol{\theta}_{i},\boldsymbol{\phi}_{j}) = \left(\mathbf{H}\left(\boldsymbol{\omega}\right)\boldsymbol{\alpha}_{ij} \odot \boldsymbol{\gamma}_{ij}\right) \otimes \mathbf{a}(\boldsymbol{\theta}_{i},\boldsymbol{\phi}_{j}), \quad (3)$$

where  $\odot$  stands for Hadamard product which has covariance matrix:

$$\mathbf{R}_{c}(\theta_{i},\phi_{j}) = Cov[\boldsymbol{\chi}_{c}(\theta_{i},\phi_{j})] = Cov[((\mathbf{H}(\omega)\boldsymbol{\alpha}_{ij})\odot\boldsymbol{\gamma}_{ij})\otimes\mathbf{a}(\theta_{i},\phi_{j})]$$
$$= [\boldsymbol{\xi}(\mathbf{H}(\omega)\mathbf{H}^{H}(\omega))\odot\boldsymbol{\Gamma}_{ij}]\otimes[\mathbf{a}(\theta_{i},\phi_{j})\mathbf{a}^{H}(\theta_{i},\phi_{j})].$$
(4)

The entire radar sea clutter return is then obtained by summing over all modes (elevations) and azimuths. STAP is most effective the ionospheric modulation is constant over azimuth i.e.  $\Gamma_{ij} = \Gamma_j \quad \forall i = 1, ..., I$ . Physically, this treats the ionosphere as a vertically layered medium where different raymodes are subject to Doppler shifts which are constant across a bearing sector at least as wide as the transmit mainlobe. In this case, (4) becomes:

$$\mathbf{R}_{c} = \sum_{ij} \mathbf{R}_{c}(\theta_{i}, \phi_{j}) = \sum_{j} \tilde{\mathbf{\Gamma}}_{j} \otimes \tilde{\mathbf{P}}_{j}, \qquad (5)$$

where  $\tilde{\mathbf{\Gamma}}_{j} = \xi (\mathbf{H}(\omega) \mathbf{H}^{H}(\omega)) \odot \mathbf{\Gamma}_{j}$  is the Doppler-spread temporal covariance for the  $j^{th}$  mode and  $\tilde{\mathbf{P}}_{j} = \sum_{i} \mathbf{a}(\theta_{i}, \phi_{j}) \mathbf{a}^{H}(\theta_{i}, \phi_{j})$ .

# 3. WAVEFRONT ADAPTIVE SENSING

The key idea of WAS is to obtain spatial covariance matrix without requirements for signal-free data, by separating clutter and signal wavefronts using BSS methods which exploit their different Doppler characteristics. Among its several features, WAS extends blind source separation (BSS) techniques to the separation of *distributed* sources (i.e. clutter) from point targets of interest.

A STAP clutter space-time snapshot realization from the covariance of Eq. (5) can be expressed as  $\overline{\chi} = \sum \overline{\gamma}_j \otimes \overline{\mathbf{p}}_j$ ,

where  $\overline{\gamma}_{j}$  and  $\overline{\mathbf{p}}_{j}$  are respectively temporal and spatial vectors, generated from the zero-mean circular complex Gaussian distributions with covariance matrices  $\overline{\Gamma}_{j}$ ,  $\overline{\mathbf{P}}_{j}$ , respectively. Alternatively, the space-time snapshot can be represented as  $\mathbf{X} = \sum_{j} \overline{\mathbf{p}}_{j} \overline{\gamma}_{j}^{H}$ , where  $\overline{\chi} = vec(\mathbf{X})$  or in a matrix form as:

$$\mathbf{X} = \mathbf{A}\mathbf{S},\tag{6}$$

where columns of  $\mathbf{A} = \begin{bmatrix} \overline{\mathbf{p}}_1 & \dots & \overline{\mathbf{p}}_J \end{bmatrix}$  are the beam-space wavefronts corresponding to each raymode and the rows of  $\mathbf{S} = \begin{bmatrix} \overline{\mathbf{\gamma}}_1^H & \cdots & \overline{\mathbf{\gamma}}_J^H \end{bmatrix}^T$  are the corresponding Doppler coefficients. The formulation of (6), wherein A and S are jointly estimated, fits neatly within the framework of BSS methods.

Consider, without loss of generality that the first source is a target:  $\mathbf{s} = [t \ \mathbf{c}]^{H}$ , therefore the structure of the mixing matrix is:  $\mathbf{A} = [\tilde{\mathbf{a}}_{t} \ \mathbf{A}_{c+n}]$ . The target wavefront can be obtained by:  $\tilde{\mathbf{a}}_{t} = \mathbf{A}\mathbf{e}_{t}$ , where  $\mathbf{e}_{t} = [1, 0, ..., 0]^{H}$ . Consider block-diagonal structure of spatial covariance matrix of independent sources:  $\Gamma_{s} = E[\mathbf{S}\mathbf{S}^{H}] = \begin{bmatrix} \sigma_{t}^{2} \ \mathbf{0}^{H} \\ \mathbf{0} \ \Gamma_{c} \end{bmatrix}$ , where  $\sigma_{t}^{2}$  is a target power and  $\Gamma_{c} = E[\mathbf{c}\mathbf{c}^{H}]$  is clutter

covariance matrix. The spatial input covariance matrix is:

$$\mathbf{R}_{\mathbf{x}} = \tilde{\mathbf{a}}_{t} \left( \mathbf{e}_{t}^{H} \boldsymbol{\Gamma}_{\mathbf{s}} \mathbf{e}_{t} \right) \tilde{\mathbf{a}}_{t}^{H} + \mathbf{R}_{\mathbf{c}+\mathbf{n}}, \tag{7}$$

where clutter plus noise covariance matrix is

$$\mathbf{R}_{\mathbf{c}+\mathbf{n}} = \mathbf{A}_{\mathbf{c}+\mathbf{n}} \boldsymbol{\Gamma}_{\mathbf{c}} \mathbf{A}_{\mathbf{c}+\mathbf{n}}^{H} + \boldsymbol{\sigma}_{n}^{2} \mathbf{I} .$$
 (8)

Assume that the target wavefront is normalized such that:  $\tilde{\mathbf{a}}_t^H \tilde{\mathbf{a}}_t = N$ , therefore, the input SNR is:

$$SNR_{in} = \frac{trace(\tilde{\mathbf{a}}_{t}(\mathbf{e}_{t}^{H}\boldsymbol{\Gamma}_{s}\mathbf{e}_{t})\tilde{\mathbf{a}}_{t}^{H})}{trace(\mathbf{A}_{c+n}\boldsymbol{\Gamma}_{c}\mathbf{A}_{c+n}^{H}+\sigma_{n}^{2}\mathbf{I})} = \frac{N\sigma_{t}^{2}}{\sum_{n=1}^{N}\sigma_{i}^{2}+N\sigma_{n}^{2}} = \frac{\sigma_{t}^{2}}{\overline{\sigma}_{i}^{2}+\sigma_{n}^{2}},$$

where  $\overline{\sigma}_i^2 = \sum_{n=1}^N \sigma_i^2$  is average power of interference signal.

The array output is  $\mathbf{z} = \mathbf{w}^H \mathbf{x}$ , with covariance matrix:

 $\mathbf{R}_{z} = \mathbf{w}^{H} \tilde{\mathbf{a}}_{t} \mathbf{e}_{t}^{H} \mathbf{\Gamma}_{s} \mathbf{e}_{t} \tilde{\mathbf{a}}_{t}^{H} \mathbf{w} + \mathbf{w}^{H} \mathbf{A}_{c+n} \mathbf{\Gamma}_{c} \mathbf{A}_{c+n}^{H} \mathbf{w} + \sigma_{n}^{2} \mathbf{w}^{H} \mathbf{w}$  (9) Widely used criterion for adaptive beamformer performance evaluation is array gain:

$$G = \frac{SNR_{out}}{SNR_{in}} = \frac{\mathbf{w}^{H}\tilde{\mathbf{a}}_{t}\mathbf{e}_{t}^{H}\Gamma_{\mathbf{s}}\mathbf{e}_{t}\tilde{\mathbf{a}}_{t}^{H}\mathbf{w}}{\mathbf{w}^{H}\mathbf{a}_{\mathbf{c}+\mathbf{n}}\Gamma_{\mathbf{c}}\mathbf{A}_{\mathbf{c}+\mathbf{n}}^{H}\mathbf{w}+\sigma_{n}^{2}\mathbf{w}^{H}\mathbf{w}} \frac{\mathbf{A}_{\mathbf{c}+\mathbf{n}}\Gamma_{\mathbf{c}}\mathbf{A}_{\mathbf{c}+\mathbf{n}}^{H}+\sigma_{n}^{2}\mathbf{I}}{\tilde{\mathbf{a}}_{t}(\mathbf{e}_{t}^{H}\Gamma_{\mathbf{s}}\mathbf{e}_{t})\tilde{\mathbf{a}}_{t}^{H}} (10)$$

Array gain of various beamformers varies with selection of weight vector. The weight vector of the well-known optimal beamformer is given by [3]:  $\mathbf{w}_{opt} = \mathbf{R}_{c+n}^{-1}\mathbf{a}_{t}$ , where  $\mathbf{a}_{t}$  is

target steering vector in target direction  $\theta_i$ . In the absence of signal-free training data, the sample covariance matrix of the test data may be used with an additional constraint to avoid signal cancellation to form the adaptive MVDR beamformer weight estimate:

$$\hat{\mathbf{w}}_{\text{MVDR}} = \frac{\hat{\mathbf{R}}_{x}^{-1} \mathbf{a}_{t}}{\mathbf{a}_{t}^{H} \hat{\mathbf{R}}_{x}^{-1} \mathbf{a}_{t}},$$
(11)

where  $\hat{\mathbf{R}}_{\mathbf{x}} = \frac{1}{M} \mathbf{X} \mathbf{X}^{H}$ . The array gain of the MVDR is:

$$G_{MVDR} = (\overline{\sigma}_i^2 + \sigma_n^2) \mathbf{a}_t^H \mathbf{R}_{\mathbf{x}}^{-1} \mathbf{a}_t$$
(12)

The main idea of a BSS beamformer is to exploit the temporal differences between the multipath clutter modes and target to estimate clutter wavefronts in A<sub>au</sub>, and signal wavefront in  $\tilde{\mathbf{a}}_{t}$ . In particular, considering data in the form of (6) suggests a whole host of blind source separation methods can be employed to determine A from X based solely on the temporal properties of S. With a well estimated A, separation of the target from Doppler spread multimode clutter can be relatively easily accomplished. In this work, the algebraic second-order BSS method, implemented via the well-known SOBI algorithm that exploits differences in sources spectra, is adopted [5-6]. At the output of the algorithm the estimated mixing matrix  $\hat{\mathbf{A}}$ . whose columns contain wavefronts of separated clutter modes, is obtained. In the BSS framework, the standard approach is to form:

$$\mathbf{S} = \hat{\mathbf{A}}^{-1} \mathbf{X},\tag{13}$$

which in the radar problem considered here gives an estimate of the Doppler spectra of the target and clutter modes in the rows of S. In terms of beamforming on the target, therefore, the weight vector of the BSS beamformer is just:

$$\mathbf{w}_{BSS} = \left[ \hat{\mathbf{A}}^{-1} \right]^{H} \mathbf{e}_{t}.$$
 (14)

Note that this beamformer implicitly assumes that the location of target column of  $\hat{A}^{-1}$  can be determined. In the OTHR application, this knowledge can be obtained using the point-like spatial and single Doppler frequency temporal characteristic of the point-target versus the distributed spatial and Bragg-line Doppler features of the separated clutter modes.

Using (14) in (8), the output covariance matrix is:

$$\mathbf{R}_{z} = \mathbf{e}_{t}^{H} \hat{\mathbf{A}}^{-1} \hat{\mathbf{A}} \mathbf{e}_{t} \mathbf{e}_{t}^{H} \boldsymbol{\Gamma}_{s} \mathbf{e}_{t} \mathbf{e}_{t}^{H} \hat{\mathbf{A}}^{H} \left( \hat{\mathbf{A}}^{H} \right)^{-1} \mathbf{e}_{t} + \mathbf{e}_{t}^{H} \hat{\mathbf{A}}^{-1} \hat{\mathbf{A}}_{c+n} \boldsymbol{\Gamma}_{c} \hat{\mathbf{A}}_{c+n}^{H} \left( \hat{\mathbf{A}}^{H} \right)^{-1} \mathbf{e}_{t} + \sigma_{n}^{2} \mathbf{e}_{t}^{H} \hat{\mathbf{A}}^{-1} \left( \hat{\mathbf{A}}^{H} \right)^{-1} \mathbf{e}_{t}.$$
(15)

Exploiting mixing matrix structure:  $\mathbf{A} = [\tilde{\mathbf{a}}_t \ \mathbf{A}_{c+n}]$ , obtain:

$$\mathbf{I} = \hat{\mathbf{A}}^{-1}\hat{\mathbf{A}} = \hat{\mathbf{A}}^{-1}\left[\tilde{\mathbf{a}}_{t}, \hat{\mathbf{A}}_{c+n}\right] = \left[\hat{\mathbf{A}}^{-1}\tilde{\mathbf{a}}_{t}, \hat{\mathbf{A}}^{-1}\hat{\mathbf{A}}_{c+n}\right].$$
 Note that  $\hat{\mathbf{A}}^{-1}\tilde{\mathbf{a}}_{t}$  is a vector, then by construction:

 $\hat{\mathbf{A}}^{-1}\tilde{\mathbf{a}}_{t} = \begin{bmatrix} 1 & \mathbf{0}^{H} \end{bmatrix}^{H}, \quad \hat{\mathbf{A}}^{-1}\hat{\mathbf{A}}_{c+n} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix}^{H}, \text{ leading to:} \\ \mathbf{e}_{t}^{H}\hat{\mathbf{A}}^{-1}\hat{\mathbf{A}}_{c+n} = \mathbf{0}^{H}. \text{ Using last expressions, (15) can e rewritten as:} \end{cases}$ 

$$\mathbf{R}_{z} = \mathbf{e}_{t}^{H} \mathbf{\Gamma}_{s} \mathbf{e}_{t} + \sigma_{n}^{2} \mathbf{e}_{t}^{H} \hat{\mathbf{A}}^{-1} \left( \hat{\mathbf{A}}^{H} \right)^{-1} \mathbf{e}_{t}$$
(16)

Note that  $\mathbf{e}_{t}^{H}\mathbf{J} = \mathbf{0}^{H}$ , means that the BSS beamformer nulls out interferences at the output of the beamformer:  $\mathbf{R}_{z} = \sigma_{t}^{2} + \sigma_{n}^{2} |\mathbf{e}_{t}^{H} \hat{\mathbf{A}}^{-1}|^{2}$ , providing following output SNR:

$$SNR_{out}^{BSS} = \frac{\sigma_t^2}{\sigma_n^2 \left| \mathbf{e}_t^H \hat{\mathbf{A}}^{-1} \right|^2}$$
(17)

Finally, array gain for BSS beamformer is:

$$G_{BSS} = \frac{\sigma_t^2}{\sigma_n^2 \left| \mathbf{e}_t^H \hat{\mathbf{A}}^{-1} \right|^2} \frac{\overline{\sigma}_i^2 + \sigma_n^2}{\sigma_t^2} = \frac{\overline{\sigma}_i^2 + \sigma_n^2}{\sigma_n^2 \left| \mathbf{e}_t^H \hat{\mathbf{A}}^{-1} \right|^2} \quad (18)$$

This analysis shows that assuming accurate estimate of **A**, the BSS completely mitigates the clutter. The drawback of the BSS beamformer is that its performance degrade dramatically when source separation is imposible.

The WAS consists of forming a signal-free covariance matrix using the BSS-estimated  $\hat{\mathbf{A}}_{c+n}$  and forming an estimate of  $\boldsymbol{\Gamma}_{c}$  in (8), using the following expression:

$$\hat{\boldsymbol{\Gamma}}_{WAS} = diag\left(\hat{\boldsymbol{A}}_{c+n}^{\dagger} \left[\boldsymbol{R}_{\boldsymbol{X}} - \sigma_{n}^{2}\boldsymbol{I}\right]\left(\hat{\boldsymbol{A}}_{c+n}^{\dagger}\right)^{H}\right), \quad (19)$$

where  $\hat{\mathbf{A}}_{c+n}^{\dagger} = \left[\hat{\mathbf{A}}_{c+n}^{H}\hat{\mathbf{A}}_{c+n}\right]^{-1}\hat{\mathbf{A}}_{c+n}^{H}$  is the Moore-Penrose pseudoinverse, and  $diag(\cdot)$  denotes the diagonal matrix of the argument. Using expression (8) in (19), we obtain:

$$\hat{\boldsymbol{\Gamma}}_{WAS} = \boldsymbol{\Gamma}_c + \boldsymbol{\sigma}_t^2 \mathbf{D}$$
(20)

where  $\mathbf{D} = diag\left(\hat{\mathbf{A}}_{c+n}^{\dagger} \tilde{\mathbf{a}}_{t} \tilde{\mathbf{a}}_{t}^{H} \left(\hat{\mathbf{A}}_{c+n}^{\dagger}\right)^{H}\right)$ . Finally the WAS

estimator of the spatial covariance matrix is:

 $\hat{\mathbf{R}}_{\mathbf{c+n}}^{WAS} = \hat{\mathbf{A}}_{\mathbf{c+n}} \hat{\mathbf{\Gamma}}_{WAS} \hat{\mathbf{A}}_{\mathbf{c+n}}^{H} + \sigma_n^2 \mathbf{I} = \mathbf{R}_{\mathbf{c+n}} + \sigma_t^2 \hat{\mathbf{A}}_{\mathbf{c+n}} \mathbf{D} \hat{\mathbf{A}}_{\mathbf{c+n}}^{H}.$  (21) This means, the WAS approximate the true signal-plusnoise covariance matrix based on the clutter wavefronts, obtained by BSS. Expected advantageous performance of the WAS is motivated by requirement to estimate strong interferences only, and not weak target signal.

The resulting WAS beamformer with modeled signal wavefront can be obtained by using

$$\mathbf{w}_{\text{WAS}} = \left(\hat{\mathbf{R}}_{c+n}^{WAS}\right)^{-1} \mathbf{a}_{t}.$$
 (22)

The array gain of the WAS beamformer is further compared to the BSS and the MVDR via simulations. Note that although the WAS beamformer of (22) uses a modeled signal wavefront, unlike conventional MVDR, the covariance matrix employed is nominally signal-free due to the use of clutter wavefronts estimated via BSS.

### 4. SIMULATION RESULTS

In this section, the WAS method is compared with BSS and conventional adaptive beamforming in terms of array gain in a simulated multi-mode OTH HF radar environment. Simulations were conducted for a uniform half-wavelengthspaced linear array of 100 elements. A radar echo with carrier frequency of  $f_c = 25$  MHz and pulse repetition frequency (PRF) of 5Hz were used such that the Bragg line Doppler components of the sea clutter return would be at  $f_{b} = \pm 0.1 \sqrt{f_{c}}$  (MHz) =  $\pm 0.5$  Hz without ionospheric motion. To simulate ionospherically-induced Doppler, an additional 0.5 Hz frequency shift was introduced between two sets of Bragg lines corresponding to multipath propagation of clutter returns via two different layers. A total of 128 array snapshots are used to estimate the beamformer weights for the WAS, BSS, and MVDR methods.

Fig. 1 illustrates the array pattern for the target beam and a horizontal wavenumber-Doppler power spectrum at the target receive beam. The clutter-to-noise-ratio (CNR) is 40 dB and SCNR is -10 dB (SNR of 30 dB). This figure shows that the BSS beamformer has an array pattern peak nearer the correct direction of the target arrival. It also shows that the WAS using the estimated clutter wavefront vectors for covariance matrix estimation and BSS beamformers outperform the MVDR.



Fig 1. Array patterns and Doppler spectra at the target beam.

The array gain loss as a function of the input SNR is shown in Fig. 2 large  $(0.65^{\circ})$  and small  $(0.22^{\circ})$  steering angle mismatch. Note that the WAS method achieves nearly the greater of the BSS and MVDR methods over the entire SNR range. WAS avoids the signal cancellation due to mismatch at higher SNR because it uses the estimated clutter wavefronts in the clutter plus noise covariance matrix. At lower SNR, WAS avoids the problem of poor estimation of the signal wavefront in BSS because it uses the assumed steering vector of MVDR. As seen in the results for larger mismatch, WAS underperforms BSS but this is often a desirable trade-off to avoid the sudden performance degradation of BSS due to signal wavefront estimation error at a lower SNR.

## **5. CONCLUSION**

WAS is a hybrid adaptive beamforming approach which avoids MVDR signal cancellation due to mismatch at high SNR and the deleterious effects of BSS threshold effects of low SNR. Applications include OTHR clutter mitigation.

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Fig 2. Array gain loss for large and low mismatch.

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