ENERGY-BASED SOURCE LOCALIZATION WITH NON-IDEAL ENERGY DECAY FACTOR

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ABSTRACT

Free-space propagation with the energy decay factor equal to two is often assumed in energy-based localization algorithms. In practice, non-ideal energy decay factors that are different from two can occur. This paper derives the Cramer-Rao Lower Bound (CRLB) for source localization with non-ideal energy decay factor and performs a sensitivity analysis of the localization algorithm presented in [6] with respect to the energy decay factor. The algorithm in [6] is found to be quite sensitive to the variations in the energy decay factor. This paper then proposes an extended algorithm for [6] that takes the non-ideal energy decay factor into account. Simulations show that the proposed solution reaches the CRLB accuracy for Gaussian noise as the signal-to-noise ratio tends to infinity.

***Index Terms*—** Source localization, microphone array, energy decay factor, CRLB

1. INTRODUCTION

The passive localization of an acoustic source has received considerable interests in recent years in microphone array processing. Many energy-based algorithms have been proposed to solve this problem [1]-[6]. Compared to time-of-arrival (TOA) and time-difference-of-arrive (TDOA), using energy has less device complexity and cost since no timing and synchronization are needed [5]. Among the algorithms using energy, Sheng and Hu [2] proposed a maximum likelihood method that can achieve the CRLB accuracy. However, it requires iterative search and good initial solution guess close to the true solution. Among the non-iterative solutions, the one proposed in [6] provides better accuracy than others and is shown analytically to reach the CRLB accuracy for Gaussian noise as the signal-to-noise ratio tends to infinity.

The rationale in using signal energy measurements for source localization lies in the fact that the received signal energy is inversely proportional to the power n of the distance between the source and the receiving microphone, where n, called the energy decay factor, is dependent on the signal propagation environment. Most of the existing energy-based algorithms assume that n is equal to two, corresponding to signal propagation in a lossless free-space. Nevertheless, this assumption may not always be valid. Li and Hu [1] indeed conducted a field experiment and found that n could deviate from two in some cases. It is therefore important to investigate the effect of the deviation of n from two on the performance of a source localization algorithm. Here, we perform the study on the localization algorithm given in [6].

Taking into account the effect of the non-ideal energy decay factor (different from two) in the signal energy model, this paper first derives the CRLB for a source location estimate under non-ideal energy decay factor scenario. The paper then conducts a sensitivity analysis of the algorithm in [6] where ideal energy decay factor is assumed but in fact not the case. We find that the accuracy of the method is quite sensitive to the variations in the energy decay factor. We therefore extend the algorithm in [6] and develop a preprocessing step to handle the non-ideal energy decay factor situation. The extended solution remains to be closed-form and not iterative. Simulation results show that the extended solution reaches the CRLB accuracy at low Gaussian noise level.

The remaining of this paper is organized as follows. In the next section, the signal energy model is introduced and the CRLB is derived. Section 3 investigates the performance sensitivity of the algorithm in [6] with respect to the energy decay factor. Section 4 deduces the extended solution and evaluates the covariance matrix of the source location estimate. Simulation results are provided in Section 5 and Section 6 is the conclusion.

2. THE SIGNAL ENERGY MODEL AND THE CRLB

Let us assume that M microphones at $\mathbf{s}_i = [x_i \ y_i \ z_i]^T$ are used to identify the location $\mathbf{u}^o = [x^o \ y^o \ z^o]^T$ of an acoustic source. The signal energy p_i measured at sensor i, $i = 1, 2, \ldots, M$, can be modeled as [2]

$$p_i = \frac{\alpha_i}{r_i^{o\,n}} A^o + \varepsilon_i = \frac{\alpha_i}{\|\mathbf{u}^o - \mathbf{s}_i\|^n} A^o + \varepsilon_i, \qquad (1)$$

where α_i is the known energy gain of receiver i and n is the energy decay factor. We assume that the signal energy at the source A^o is not known, the noise processes at different receivers ε_i are independent of each other and the non-zero mean in ε_i has been subtracted. For a sufficiently long observation time, the measurement noise vector $\boldsymbol{\varepsilon} = [\varepsilon_1 \ \varepsilon_2 \ \ldots \ \varepsilon_M]^T$ can be well approximated by a zero mean Gaussian distributed random vector with covariance matrix $\mathbf{Q}_p = \mathrm{diag}\{\sigma_1^2 \ \sigma_2^2 \ \ldots \ \sigma_M^2\}$ according to the Central Limit Theorem.

From the energy measurement model in (1) and using the approach in [6], the CRLB for a source location can be found to be

$$CRLB(\mathbf{u}^o) = \left(\mathbf{J}_{11} - \frac{\mathbf{J}_{12}\mathbf{J}_{12}^T}{J_{22}}\right)^{-1}, \qquad (2)$$

where

$$\mathbf{J}_{11} = (nA^o)^2 \sum_{i=1}^{M} \frac{\alpha_i^2}{\sigma_i^2} \frac{(\mathbf{u}^o - \mathbf{s}_i)(\mathbf{u}^o - \mathbf{s}_i)^T}{r_i^{o\,2(n+2)}}, \qquad (3)$$

$$\mathbf{J}_{12} = -nA^o \sum_{i=1}^{M} \frac{\alpha_i^2}{\sigma_i^2} \frac{(\mathbf{u}^o - \mathbf{s}_i)}{r_i^{o\,2n+2}}, \qquad (4)$$

$$J_{22} = \sum_{i=1}^{M} \frac{\alpha_i^2}{\sigma_i^2} \frac{1}{r_i^{o\,2n}}. \qquad (5)$$

The CRLB will be used in the sensitivity analysis and the performance evaluation of the proposed solution.

3. PERFORMANCE SENSITIVITY WITH RESPECT TO THE ENERGY DECAY FACTOR

The energy-based solution in [6] reaches the CRLB accuracy for Gaussian noise as the signal-to-noise ratio tends to infinity. However, it assumes the energy decay factor n is ideal which is two. In practice, n may deviate from two if the condition of free-space propagation is violated. Here we investigate how sensitive the algorithm in [6] is to the variations in the energy decay factor.

To gain some insight, we use an energy decay factor different from two to generate the energy measurements. The method in [6] is then applied to estimate a source location. The scenario considered here is as follows. The number of receiving sensors is $M = 9$ and the sensor positions are tabulated in Table 1. All the gains α_i are set to unity. The source is located at (3.0,4.5,4.0)m and the source signal energy A^o is 100. The covariance matrix of the noise vector $\boldsymbol{\varepsilon}$,$\mathbf{Q}_p$, is set as $\sigma^2\mathbf{I}_M$. The true energy decay factor is $n = 2.03$. Fig.1 depicts the localization accuracy as the noise power σ^2 increases. The MSE of the source location estimate from the method in [6] deviates more from the CRLB when the noise power becomes smaller. The trace of the covariance matrix of the solution from [6] is also plotted and it attains the CRLB without significant difference when the noise power is low. This implies that the degradation in performance mainly comes from bias.

We now derive the theoretical MSE of the source location estimate using the Taylor series expansion approach [7]. Define the unknown vector as $\boldsymbol{\theta} = [\mathbf{u}^{oT}\ A^o]^T$ and the signal energy vector as $\mathbf{g}(\boldsymbol{\theta})$ whose ith element is $g_i(\boldsymbol{\theta}) = \frac{\alpha_i}{{r_i}^2}A = \frac{\alpha_i}{\|\mathbf{u}-\mathbf{s}_i\|^2}A$. When an initial solution $\boldsymbol{\theta}_o$ close to the true value is available, $\mathbf{g}(\boldsymbol{\theta})$ can be expanded with respect to $\boldsymbol{\theta}_o$ through Taylor series as

$$\mathbf{g}(\boldsymbol{\theta}) \simeq \mathbf{g}(\boldsymbol{\theta}_o) + \mathbf{G}(\boldsymbol{\theta}_o)(\boldsymbol{\theta} - \boldsymbol{\theta}_o), \tag{6}$$

where only the linear term is kept and

$$\mathbf{G}(\boldsymbol{\theta}_o) = \left.\frac{\partial \mathbf{g}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}_o} = \left.\begin{bmatrix} -\frac{2\alpha_1 A}{r_1^4}(\mathbf{u}-\mathbf{s}_1)^T & \frac{\alpha_1}{r_1^2} \\ \vdots & \vdots \\ -\frac{2\alpha_M A}{r_M^4}(\mathbf{u}-\mathbf{s}_M)^T & \frac{\alpha_M}{r_M^2} \end{bmatrix}\right|_{\boldsymbol{\theta}_o}. \tag{7}$$

Let the cost function to be minimized be $\xi = (\mathbf{p} - \mathbf{g}(\boldsymbol{\theta}))^T \mathbf{Q}_p^{-1} (\mathbf{p} - \mathbf{g}(\boldsymbol{\theta}))$, where $\mathbf{p} = [p_1\, p_2 \ldots p_M]^T$ is the measurement vector. Substituting (6), taking derivative with respect to $\boldsymbol{\theta}$ and setting it to zero yield the solution

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_o + \left[\mathbf{G}(\boldsymbol{\theta}_o)^T\mathbf{Q}_p^{-1}\mathbf{G}(\boldsymbol{\theta}_o)\right]^{-1}\mathbf{G}(\boldsymbol{\theta}_o)^T\mathbf{Q}_p^{-1}(\mathbf{p}-\mathbf{g}(\boldsymbol{\theta}_o)). \tag{8}$$

If $\boldsymbol{\theta}_o$ is set to be the true solution $\boldsymbol{\theta}^o$, then the estimation error is

$$\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^o = \left[\mathbf{G}(\boldsymbol{\theta}^o)^T\mathbf{Q}_p^{-1}\mathbf{G}(\boldsymbol{\theta}^o)\right]^{-1}\mathbf{G}(\boldsymbol{\theta}^o)^T\mathbf{Q}_p^{-1}(\mathbf{p}-\mathbf{g}(\boldsymbol{\theta}^o)). \tag{9}$$

In (9), $\mathbf{p} - \mathbf{g}(\boldsymbol{\theta}^o)$ can be expressed as $\mathbf{c} + \boldsymbol{\varepsilon}$, where $\mathbf{c}$ is a vector with ith element $\frac{\alpha_i A^o}{r_i^{on}} - \frac{\alpha_i A^o}{r_i^{o2}}$ and $\boldsymbol{\varepsilon}$ is the noise vector in energy measurements. $\mathbf{c}$ represents the mismatch of the true signal energy model with the assumed one caused by the deviation of n.

Taking expectation of (9) yields the bias

$$\mathbf{b} = \left[\mathbf{G}(\boldsymbol{\theta}^o)^T\mathbf{Q}_p^{-1}\mathbf{G}(\boldsymbol{\theta}^o)\right]^{-1}\mathbf{G}(\boldsymbol{\theta}^o)^T\mathbf{Q}_p^{-1}\mathbf{c}. \tag{10}$$

Subtracting (10) from (9) and multiplying its transpose gives the covariance matrix

$$\mathrm{cov}(\hat{\boldsymbol{\theta}}) = \left[\mathbf{G}(\boldsymbol{\theta}^o)^T\mathbf{Q}_p^{-1}\mathbf{G}(\boldsymbol{\theta}^o)\right]^{-1}. \tag{11}$$

Table 1. Microphone sensor positions in meters

i	1	2	3	4	5	6	7	8	9
x_i	0	1.4	2.9	-1.0	0.4	-1.8	2.5	-2.0	2.9
y_i	0	-1.3	0.1	-0.5	2.2	-1.1	0.1	-0.7	-2.4
z_i	0	2.6	-2.5	1.6	2.5	0.7	1.4	-0.7	0.3

Note that (11) is equal to the CRLB when $n = 2$. The MSE of the source position is obtained by adding the trace of the upper left 3×3 submatrix of $\mathrm{cov}(\hat{\boldsymbol{\theta}})$ and the sum of the squares of the first three elements of $\mathbf{b}$.

Fig. 2 shows the theoretical MSE and the corresponding CRLB as n increases from 2, where the measurement noise power σ^2 is 10^{-4}. The location scenario is the same as in Fig. 1. As the mismatch of the energy decay factor increases, the deviation of the MSE from the CRLB becomes larger and larger, and is due to the increase of the bias. The MSE using the algorithm in [6] is also shown to verify the theoretical derivation. The theoretical and simulated MSEs match very well when the mismatch of n is not large. The difference increases as n increases and is about 1 in log-scale [10 log(MSE)] when $n = 2.1$. The difference is due to ignoring the higher order terms in (6) that become significant when the solution is deviating more from the true value as n increases.

In summary, the accuracy of the solution in [6] is quite sensitive to the variations in the energy decay factor. Hence the effect of the energy decay factor has to be taken into account to design a better localization algorithm.

4. THE EXTENDED CLOSED-FORM SOLUTION

We shall extend the algorithm in [6] to account for non-ideal energy decay factor. We start from the signal energy model in (1) and develop a pre-processing step to handle the non-ideal energy decay factor situation. After the pre-processing step, the algorithm in [6] is applied to generate an estimate of the source location.

4.1. Pre-processing

The energy measurements are converted to the energy ratio measurements since the energy measurements are dependent on the signal energy at the source A^o that is not known. We first take the ratio of the energy measurements to eliminate A^o and then normalize the energy decay factor with respect to $n = 2$ by taking the power of $-\frac{2}{n}$ in the ratio. To be specific, when the receiver 1 is set as the reference sensor and using (1), we have the energy ratio q_{i1}, $i = 2, 3, \ldots, M$,

$$q_{i1} = \left(\frac{p_i/\alpha_i}{p_1/\alpha_1}\right)^{-\frac{2}{n}} = \frac{r_i^{o2}}{r_1^{o2}}\left(1 + \frac{\varepsilon_1 r_1^{on}}{A^o\alpha_1}\right)^{\frac{2}{n}}\left(1 + \frac{\varepsilon_i r_i^{on}}{A^o\alpha_i}\right)^{-\frac{2}{n}}. \tag{12}$$

When the signal-to-noise ratio is large enough, that is $(\varepsilon_i r_i^{on})/(A^o\alpha_i) \ll 1$, we have the approximation

$$\begin{aligned} q_{i1} &\simeq \left(\frac{r_i^o}{r_1^o}\right)^2 + \left(\frac{2}{n}\frac{\varepsilon_1 r_i^{o2} r_1^{on-2}}{A^o\alpha_1} - \frac{2}{n}\frac{\varepsilon_i r_i^{on+2}}{A^o\alpha_i r_1^{o2}}\right) \\ &\stackrel{\text{def}}{=} q_{i1}^o + \Delta q_{i1}, \end{aligned} \tag{13}$$

where only the linear noise terms have been kept. $(*)^o$ denotes the true value of $(*)$ and Δq_{i1} represents the additive noise.

Let the energy ratio vector be $\mathbf{q} = [q_{21}\ q_{31}\ \ldots\ q_{M1}]^T$ and the energy ratio error vector be $\Delta\mathbf{q} = [\Delta q_{21}\ \Delta q_{31}\ \ldots\ \Delta q_{M1}]^T$.

Then using the condition that ε_i and ε_j are independent for $i \neq j$, the elements of the covariance matrix of $\mathbf{q}$, $\mathbf{Q}_q$, are found to be

$$\mathbf{Q}_q[i-1, j-1] = \begin{cases} \frac{4}{n^2} \frac{r_1^{o2(n-2)} r_i^{o2} r_j^{o2}}{A^{o2} \alpha_1^2} \sigma_1^2, & i \neq j \\ \frac{4}{n^2} \frac{r_1^{o2(n-2)} r_i^{o4}}{A^{o2} \alpha_1^2} \sigma_1^2 + \frac{4}{n^2} \frac{r_i^{o2(n+2)}}{A^{o2} \alpha_i^2 r_1^{o4}} \sigma_i^2, & i = j \end{cases} \tag{14}$$

for $i, j = 2, 3, \ldots, M$. (14) is the general formula for $\mathbf{Q}_q$ with arbitrary n and it reduces to the one in [6] when n is two.

4.2. The closed-form solution

After the pre-processing step, the closed-form solution in [6] can be applied to obtain an estimate of the source location.

The main idea of the closed-form solution is using $\mathbf{q}$ to obtain an estimate of $\mathbf{u}^o$ and the auxiliary variable r_1^{o2}, where $\mathbf{u}^o$ and r_1^{o2} are assumed to be uncorrelated. It then applies the relationship between $\mathbf{u}^o$ and r_1^{o2} to improve the estimation accuracy. We shall summarize the closed-form solution below. The details can be found in [6].

Stage-1: We form the pseudo linear vector equation as

$$\mathbf{e}_1 = \mathbf{h}_1 - \mathbf{G}_1 \boldsymbol{\phi}_1 = \begin{bmatrix} \mathbf{s}_1^T \mathbf{s}_1 - \mathbf{s}_2^T \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_1^T \mathbf{s}_1 - \mathbf{s}_M^T \mathbf{s}_M \end{bmatrix} - \begin{bmatrix} 2(\mathbf{s}_1 - \mathbf{s}_2)^T & 1 - q_{21} \\ \vdots & \vdots \\ 2(\mathbf{s}_1 - \mathbf{s}_M)^T & 1 - q_{M1} \end{bmatrix} \begin{bmatrix} \mathbf{u}^o \\ r_1^{o2} \end{bmatrix}, \tag{15}$$

where $\mathbf{e}_1 = r_1^{o2} \Delta \mathbf{q}$ is the residual error vector. Here, $\mathbf{u}^o$ and r_1^{o2} are considered uncorrelated. Applying weighted least-squares (WLS) to (15) yields the solution

$$\boldsymbol{\phi}_1 = (\mathbf{G}_1^T \mathbf{W}_1 \mathbf{G}_1)^{-1} \mathbf{G}_1^T \mathbf{W}_1 \mathbf{h}_1, \tag{16}$$

where $\mathbf{W}_1$ is the weighting matrix and defined as

$$\mathbf{W}_1 = \frac{r_1^{o4}}{A^{o2}} E[\mathbf{e}_1 \mathbf{e}_1^T]^{-1} = \frac{1}{A^{o2}} \mathbf{Q}_q^{-1}. \tag{17}$$

Stage-2: Exploring the relationship between $\mathbf{u}^o$ and r_1^{o2} yields the vector equation

$$\begin{aligned} \mathbf{e}_2 &= \mathbf{h}_2 - \mathbf{G}_2 \boldsymbol{\phi}_2 \\ &= \begin{bmatrix} (\boldsymbol{\phi}_1(1:3) - \mathbf{s}_1) \odot (\boldsymbol{\phi}_1(1:3) - \mathbf{s}_1) \\ \phi_1(4) \end{bmatrix} \\ &- \begin{bmatrix} \mathbf{I}_{3\times 3} \\ 1\,1\,1 \end{bmatrix} [(\mathbf{u}^o - \mathbf{s}_1) \odot (\mathbf{u}^o - \mathbf{s}_1)], \end{aligned} \tag{18}$$

where $\mathbf{e}_2 = \mathbf{B} \Delta \boldsymbol{\phi}_1$ is the equation error vector, $\mathbf{B} = \text{diag}\{[2(\mathbf{u}^o - \mathbf{s}_1)^T \ 1]^T\}$ and $\Delta \boldsymbol{\phi}_1$ is the error vector in $\boldsymbol{\phi}_1$. $\odot$ denotes the element by element multiplication.

The WLS solution to (18) is

$$\boldsymbol{\phi}_2 = (\mathbf{G}_2^T \mathbf{W}_2 \mathbf{G}_2)^{-1} \mathbf{G}_2^T \mathbf{W}_2 \mathbf{h}_2, \tag{19}$$

where the weighting matrix $\mathbf{W}_2$ is defined as

$$\mathbf{W}_2 = \frac{r_1^{o4}}{A^{o2}} E[\mathbf{e}_2 \mathbf{e}_2^T]^{-1} = \mathbf{B}^{-1} (\mathbf{G}_1^{oT} \mathbf{W}_1 \mathbf{G}_1^o) \mathbf{B}^{-1}, \tag{20}$$

and $\mathbf{G}_1^o$ is the noise-free version of $\mathbf{G}_1$.

Finally, the source location estimate is obtained by

$$\hat{\mathbf{u}} = \mathbf{P} \sqrt{\boldsymbol{\phi}_2} + \mathbf{s}_1, \tag{21}$$

and the location accuracy is

$$\text{cov}(\hat{\mathbf{u}}) = \frac{r_1^{o4}}{A^{o2}} \mathbf{D}^{-1} (\mathbf{G}_2^T \mathbf{W}_2 \mathbf{G}_2)^{-1} \mathbf{D}^{-1}. \tag{22}$$

$\mathbf{P} = \text{diag}\{\text{sign}(\boldsymbol{\phi}_1(1:3) - \mathbf{s}_1)\}$ and $\mathbf{D} = 2 \ \text{diag}\{(\mathbf{u}^o - \mathbf{s}_1)\}$.

To summarize, the closed-form solution consists of (16), (19) and (21), where the weighting matrices are given in (17) and (20). Note that the true weighting matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ are not available. However, the approximated ones can be obtained to implement the algorithm and the resulting error is found to be insignificant.

5. SIMULATION

Simulation results are presented to show the performance of the extended algorithm. The same scenario used in the Section 3 is adopted here. Two different values of the energy decay factor n, 2.03 and 2.1, are used to generate the measurements separately. We apply the extended algorithm to locate a near-filed source at (3.0,4.5,4.0)m and a far-field source at (6.5,7.5,-6.0)m, respectively.

Fig. 3 shows the MSE and CRLB for the near-field source as the noise power σ^2 increases when $n = 2.03$. The extended algorithm attains the CRLB accuracy when σ^2 is less than -35 in log-scale [10 log(noise power)] and then it starts deviating gradually from the CRLB as σ^2 increases. After σ^2 reaches -27 in log-scale, the threshold effect occurs. The MSE using the algorithm in [6] is also presented for comparison. We observe that the extended algorithm eliminates the bias and a great performance improvement is obtained. The MSE and CRLB for the near-field source when $n = 2.1$ are illustrated in Fig. 4. The CRLB is about 1 in log-scale higher than the case when $n = 2.03$ because of the increase in n. However, the extended algorithm remains to achieve the CRLB accuracy when σ^2 is less than -35 in log-scale.

The results for the far-field source with $n = 2.03$ are shown in Fig. 5. The observations are similar to those in the near-field case. The differences are that in the far-field case, we achieve the CRLB accuracy at a lower noise power level, saying -55 in log-scale, and the threshold effect occurs earlier. The results for the far-field source with $n = 2.1$ are not shown and they are similar to the far-field source case when $n = 2.03$ except that the curves are all shifted up by about 1.2 in log-scale.

6. CONCLUSION

A non-ideal energy decay factor different from two may be encountered in practice. This paper applies Taylor series expansion approach to perform sensitivity analysis with respect to energy decay factor of the algorithm in [6] and finds that non-ideal energy decay factor introduces a bias and degrades its performance. We therefore extends the algorithm in [6] to account for non-ideal energy decay factor. A pre-processing step is developed to handle this non-ideal situation and then the algorithm in [6] is applied to generate an estimate of the source location. We also derive the CRLB under non-ideal energy decay factor. Simulations show that the proposed solution reaches the CRLB accuracy for low noise power level.

It should be noted that perfect calibration and exact value of energy decay factor are assumed in the paper. The performance of the extended algorithm will degrade with imperfect calibration.

7. REFERENCES

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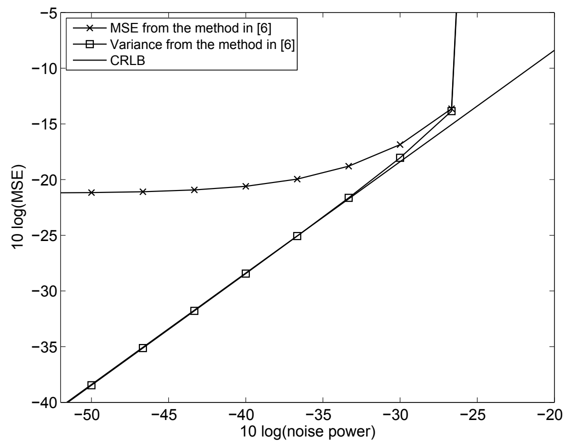

Fig. 1. The localization accuracy as σ^2 increases when ideal energy decay factor is assumed but in fact it is equal to $n = 2.03$.

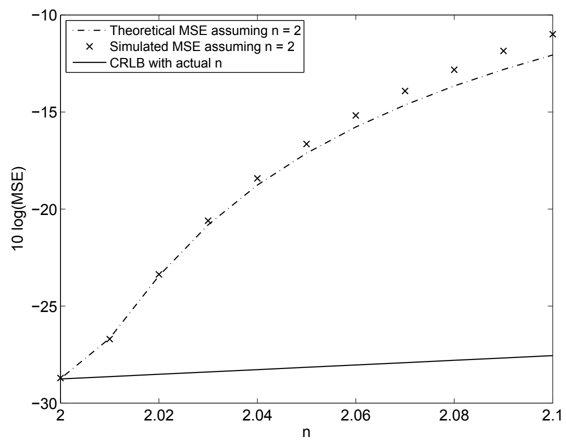

Fig. 2. The localization accuracy from the algorithm in [6] as n increases when $\sigma^2 = 10^{-4}$.

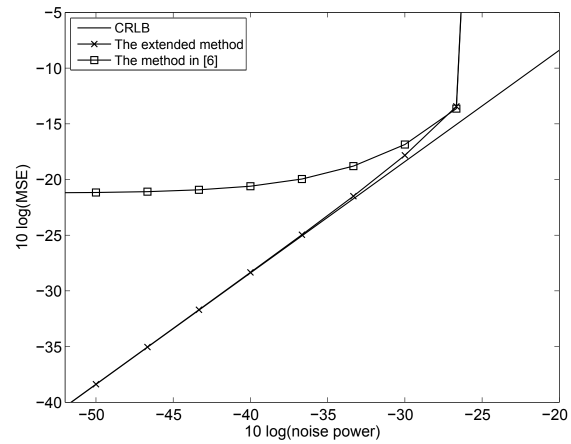

Fig. 3. The localization accuracy of the near-field source when $n = 2.03$.

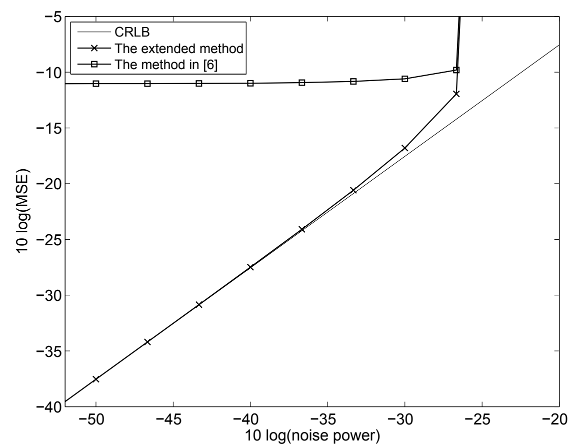

Fig. 4. The localization accuracy of the near-field source when $n = 2.1$.

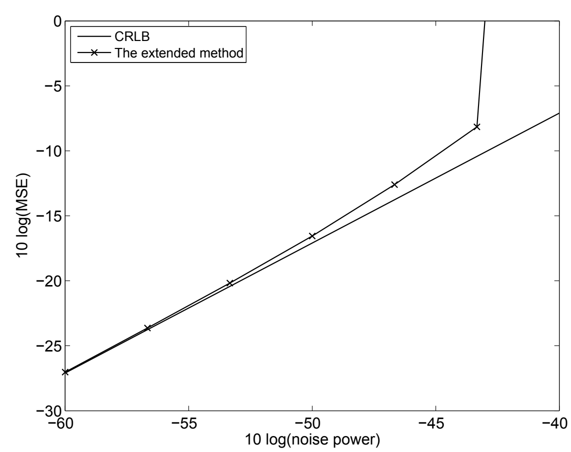

Fig. 5. The localization accuracy of the far-field source when $n = 2.03$.

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